THE H INFINITY DESIGN OF EQUALIZERS

Takeshi Amishima
Ali Jalali
Craig Sims

Department of Electrical and Computer Engineering
West Virginia University
Morgantown West Virginia 26506

ABSTRACT

Kalman filtering has been suggested as a method for the design of equalizers for communication channels with intersymbol interference, and additive random receiver noise. This is possible because a state space structure of the channel model can be constructed. It is appropriate if the noise is truly white, and the finite impulse response model describing the intersymbol interference is short. If the receiver noise is completely arbitrary, except that it has bounded energy, then an H infinity design may be more appropriate, since it is designed to meet these conditions, with a performance guarantee under the worst case situation. An H infinity design can be of full order or reduced order. In this paper we consider the full order case only . The design is not in a stochastic setting, but rather allows one to design for unknown, and possibly very bad disturbances.

INTRODUCTION:

We consider an important problem in the field of digital communications, the recovery of a message when intersymbol interference and noise are both problems. Kalman filtering has been suggested as a method for estimating the symbols in such a context, [1-3]. Here we present similar results using the infinity norm criterion, as developed in [4]. The results are more appropriate if the characteristics of the measurement noise are unknown.The theory will be illustrated on a on a simple binary example and on a 16 QAM signal constellation.

PROBLEM STATEMENT:

We are going to consider the case of having a known channel here, although the case of having an unknown and perhaps continuously changing channel is also possible to address using the ideas presented here. Specifically we assume that at each stage , j , we have available a vector of information whose elements are $\{y(j)\}$, where y is the discrete convolution of h with w.

$$y(j) = h(j) * w(j) = \sum h(i) w(j-i) \quad (1)$$

and where h (j) are the known channel parameters and w(j) are the set of symbols transmitted, which we are trying to estimate. If this were the entire problem, it could be solved by waiting until you had sufficient information to solve the linear algebra problem to get the symbols ,w ,from the data , y. We make the further assumption that the data y is corrupted by noise to give us a received measurement, m(j), where:

$$m(j) = y(j) + v(j)$$

The problem has two parts then. That is, we must both remove the effect of the intersymbol interference and the effect of the noise. This can be done several ways, the most common being the use of a transversal filter. A Kalman filter can also be used. In this paper we indicate a method which is auto-regressive like a Kalman filter, but has the virtue of being appropriate for extremely bad noise, reduced-order versions of this approach may be found in [5].

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THE MODEL AND THE FILTER:

A common state space model, [1-3], for a set of symbols and their past values is:

$$x(j+1) = Ax(j) + Bw(j+1) \qquad (2)$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}'$$

so that $x_1(j+1)$ is the current symbol, $x_2(j+1)$is the last symbol and so on. The state model so described simply operates like a shift register. The channel itself is assumed to have an impulse response h. If one samples faster than the baud rate, or if there are multiple channels, then at each stage there are r signals available of the form indicated in Eq. (1). These can be stacked in a vector of observations, y(j). A noisy measurement vector is then available at each stage:

$$m(j) = y(j) + v(j) = Hx(j) + v(j)$$

where m is the measurement vector, v is a noise vector, and H is comprised of the impulse response elements, $h_k(i)$. If we model the input symbols as a bounded sequence with scaling matrix Q, and the measurement noise sequence, v, as bounded with scaling matrix R, then this is a model which is amenable to H infinity filtering solutions. The filter estimates the vector z(j) =Lx(j) where filter matrices are chosen to ensure that the maximum value of the ratio of the energy in the error to the energy in the disturbances is bound by some positive number gamma squared, i.e.

$$\frac{\sum \|z - z_e\|2}{\sum \{\|w\|2_{Q-1} + \|v\|2_{R-1}\}} < \gamma 2 \qquad (3)$$

where z_e is the estimate. This is the essence of H infinity design. We see that Q and R do not represent covariance matrices here, but rather scaling matrices on the arbitrary disturbances. The significant part of this paper is that the noise can be almost anything, and need not be modeled as a gaussian white process.The noise is required to have the characteristic that;

$$\sum \left\{\|w\|^2_{Q^{-1}} + \|v\|^2_{R^{-1}}\right\} < 1 \qquad (4)$$

This is the setting of the discrete H infinity algorithm that we shall apply to the digital communications problem. However it does not actually fit the description of w, since w is a random sequence of a finite set of symbols.

Therefore we are abusing the thory somewhat with regard to w, but the arbitrary nature of the discription of the measurement noise v, may be useful. The full order solution which looks like a Kalman one stage predictor, has a different gain. We are interested in the performance measure indicated by (3). At the oneset, it should be noted that our attempt is to ensure that the ratio of the energy in the error to the energy in the disturbance is bounded by γ^2 as indicated by (3), and that there is a value of γ^2 so small that no solution exists, either for the full or reduced order case.

THE FULL ORDER ALGORITHM

The full order algorithm which was used has the following one stage predictor structure:

$$z_e(j) = Lx_e(j)$$

where:

$$x_e(j+1) = Ax_e(j) + K(j)[m(j) - Hx_e(j)] \qquad (5)$$

$$x_e(j+1) = Ax_e(j) + K(j)[m(j) - Hx_e(j)]$$

and K is computed by the gain expression:

$$K(j) = AP(j)\Lambda^{-1}(j)H'R^{-1}$$

while P propagates as:

$$P(j+1) = AP(j)\Lambda^{-1}(j)A' + BQB'$$

and the matrix $\Lambda(j)$ is defined as :

$$\Lambda(j) = I - \left(\frac{L'L}{\gamma^2} - H'R^{-1}H\right)P(j)$$

QAM EXAMPLE:

This example shows a simulation of an H infinity filter using 16 QAM at a SNR of 20dB. We have 16 possible symbols that are modulated in both amplitude and phase. The measurement noise is added to the received signal. In this example there are 4 measurements at each stage and a 5th order filter is used. Each row of the matrix represents the impulse response of the path from the transmitter to the receiver. The rows of H are:

row 1=[1, .9, .8, .7, 0]
row 2=[.95,.85,.75,.65,0]
row 3=[0, 1.0,.9,. 8,.7, 0]
row 4=[0. .95,.85,.75,.65]

Figure 1 shows the output of the channel before it enters the H infinity Equalizer.Figure 2 shows the result of H infinity equalization with gamma squared equal 1, and R=diag(.0287), and Q=9.746. The H infinity equalizer is clearly able to distinguish the 16 symbols.

BINARY EXAMPLE:

Here we consider a binary channel where the symbol to be estimated is +1 or -1. In this example the order of the state vector is 6, and there is only a single channel with h=[1,0,1]. The measurement noise is R=.01. However 10% of the time we allow a noise of the form v= -Hx to be encountered. This effectively wipes out any signal information. In Figure 3 we see how the Kalman filter estimates the signal which is either 1 or -1,under these circumstances. In Figure 4, we see how the H infinity filter performs with the same data with gamma squared set at 8. There is modest improvement. However if we continue to drop the value of gamma squared, the H infinity problem has no meaningful solution. Moreover as we see in Figure 5, the H infinity filter becomes unstable at $\gamma^2 = 5.67$. So in this example we see that the H infinity approach can enable us to deal with terrible noise somewhat better than the Kalman approach, but as is well known for autoregressive filters, there is the potential for loss of stability, and this happens when we set the value of gamma squared too small. Figure 6 is a result of doing Kalman filtering as in Fig. 3, but now with R=.001. We see that with small expected noise, the filtering performance is even poorer when there is the occasional unexpectedly bad noise to contend with.

CONCLUSION AND SUMMARY:

It appears that H infinity theory has a role to play in Equalization when the noise is arbitrary rather than of a standard spectral description. The user must be aware that there is the potential for a loss of stability however, and that one cannot make guarantees about energy gain ratios for arbitrarily small values of gamma squared.

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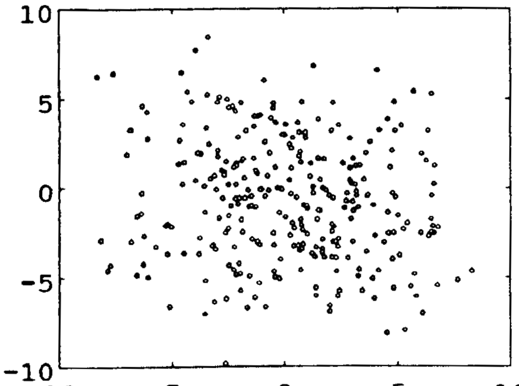

Figure 1 : measured output y2

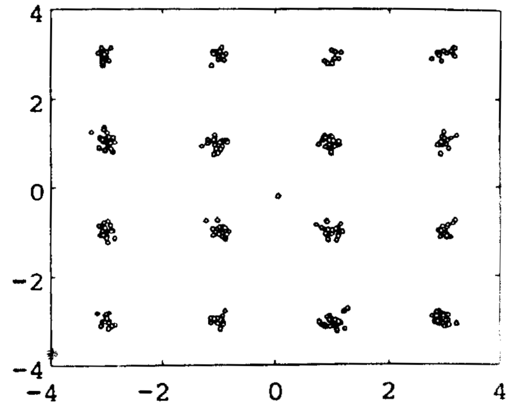

Figure 2 : estimation of x2

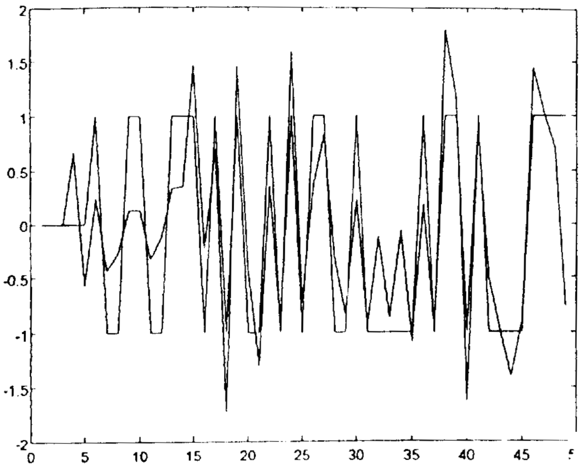

Figure 3: Signal and Kalman Estimate vs. Time

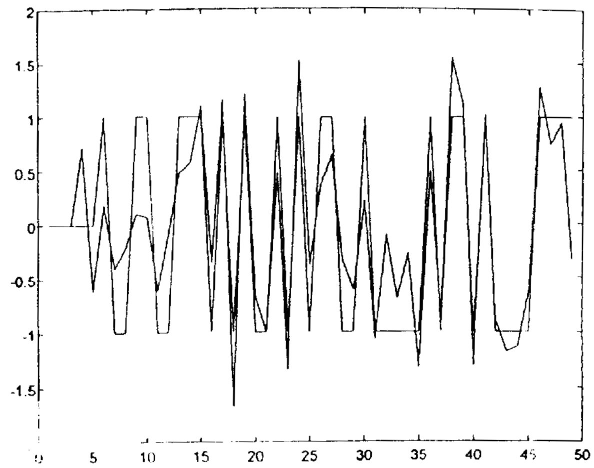

Figure 4: Signal and H infinity Estimate

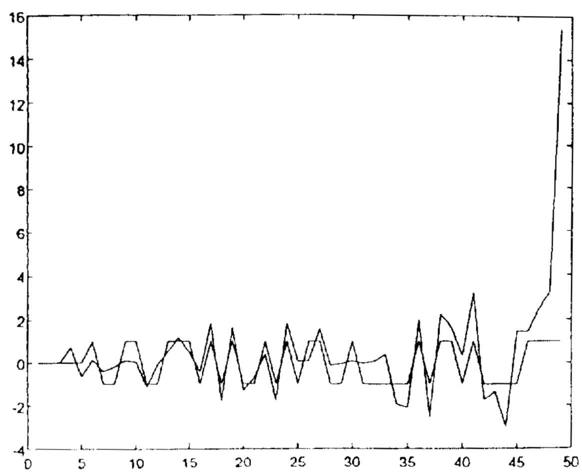

Figure 5: Potential for Stability Loss

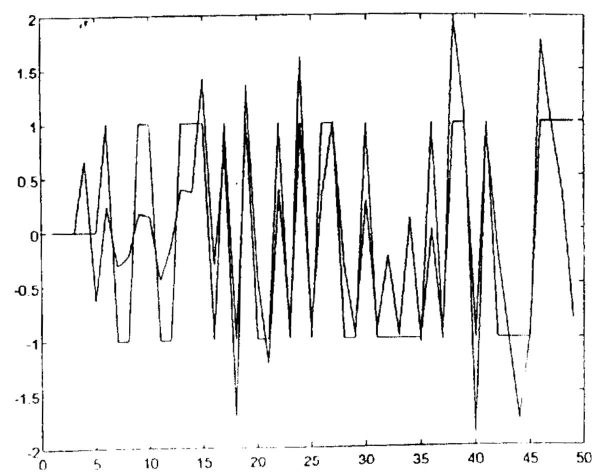

Figure 6: Kalman with Low Noise Expectation