FORWARD/INVERSE BLIND EQUALIZATION

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ABSTRACT

A new technique for blind equalization is presented which estimates the inverse (deconvolution) multipath channel and the forward channel as well. This method removes the need for an ad hoc energy constraint on the filter taps and constrains the equalization filter by relating the output back to the equalizer input. The concept of feedback is used to provide a more meaningful constraint for blind equalization filters. The real advantage of the method is increased speed of convergence over traditional stochastic gradient methods, and greater robustness to eigenvalue spread. These benefits come at a modest two times the computational complexity price.

1. INTRODUCTION

Consider the traditional blind equalization problem:

$$y = x * h$$

$$\hat{x} = y * \hat{h}^{-1}.$$

In this problem, a method for blind equalization, which does not have access to the input reference signal x, is used in estimating the inverse filter $\hat{h}^{-1}$. A blind update procedure of the form

$$\hat{h}^{-1} = \hat{h}^{-1} + \mu\ (\hat{x} - g(\hat{x}))y, \tag{1}$$

where $g(\cdot)$ is a nonlinear function, can be used for any Bussgang process [4], i.e. when x is an i.i.d., finite variance signal. The nonlinear function needed can be shown to be the MAP estimate of the signal plus convolutional noise of the system [1], and $\hat{x} - g(\hat{x})$ is a "blind error" which we minimize.

Because the blind error depends only on the output of the equalizer $\hat{x}$, some type of constraint must placed on the norm of the inverse filter in order to avoid the trivial solution of zero output. Current methods of constraining the equalizer tap weights are: holding one of the taps to a fixed value, and normalization by the L_p norm of the taps w, $\frac{1}{(E|w|^p)^{\frac{1}{p}}}$. Using the basic L_2 energy constraint is generally much more robust than the single tap constraint. The *optimal* constraint would be a function of the type of data used, and the charateristics of the channel, but has not been analytically determined to date. In an attempt to provide a more meaningful contraint method, we propose the use of a joint forward and inverse filter estimator which uses feedback.

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2. THE BLIND FORWARD AND INVERSE FILTER ESTIMATOR USING FEEDBACK

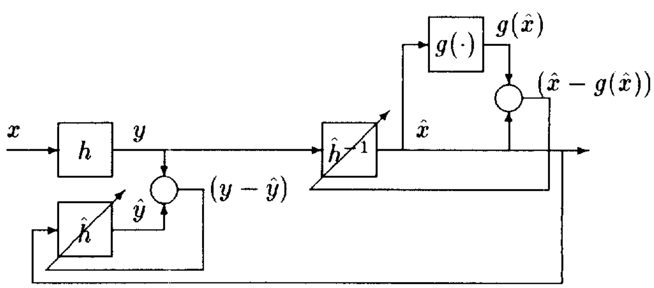

Figure 1: Blind Forward and Inverse Filtering Using Feedback

Refering to Fig. 1, the blind estimate of the original data $\hat{x}$ is then re-filtered to obtain a $\hat{y}$. This is an estimate of the input to the equalizer, and by referencing it to the known y, we form the error given by

$$e = y - \hat{y},$$

which can be used in an update for the forward filter $\hat{h}$.

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By so doing, the energy of the output of the equalizer is referenced back to the input y. The relationship between y and $\hat{y}$ is as follows:

$$y = x * h$$

$$\hat{x} = y * \hat{h}^{-1}$$

$$\hat{y} = \hat{x} * \hat{h}.$$

thus

$$\hat{y} = \hat{h}^{-1} * \hat{h} * y \quad (2)$$

Since both forward and inverse filters are obtained simultaneously in this algorithm, a "self ISI" can be computed with

$$\hat{h}^{-1} * \hat{h},$$

and the algorithm is self-monitoring. This "self ISI" is contrasted with the overall system transfer function from x to $\hat{x}$, which is

$$h * \hat{h}^{-1},$$

and has been the key to two recent blind equalization cost function derivation methods [5, 6]. This paper proposes what is really a time-recursive solution to the minimization of this "self ISI" $= \hat{h}^{-1} * \hat{h}$.

The forward/inverse blind equalization algorithm is simply the use of a traditional blind equalizer to find $\hat{h}^{-1}$ in conjunction with another cost function in order to find $\hat{h}$. In terms of cost functions we propose the use of the sum:

$$J_{tot} = J_b + J_f, \quad (3)$$

where J_b is the blind cost function dependant on $\hat{h}^{-1}$, and J_f is the feedback cost function dependant on $\hat{h}$. The forward and inverse filters are updated according to:

$$\hat{h}^{-1} = \hat{h}^{-1} + \mu\frac{\partial J_{tot}}{\partial \hat{h}^{-1}} \quad (4)$$

$$\hat{h} = \hat{h} + \mu\frac{\partial J_{tot}}{\partial \hat{h}}. \quad (5)$$

One could also use a Newton-type of update:

$$\hat{h}^{-1} = \hat{h}^{-1} + \mu\ (E\{\frac{\partial^2 J_{tot}}{\partial^2 \hat{h}^{-1}}\})^{-1}\frac{\partial J_{tot}}{\partial \hat{h}^{-1}} \quad (6)$$

$$\hat{h} = \hat{h} + \mu\ (E\{\frac{\partial^2 J_{tot}}{\partial^2 \hat{h}}\})^{-1}\frac{\partial J_{tot}}{\partial \hat{h}}. \quad (7)$$

3. RELATION TO FINDING THE FORWARD AND INVERSE FILTERS OF A SYSTEM USING THE REFERENCE

This forward/inverse blind equalization algorithm we are presenting is a blind extension of the simple "with-a-reference" forward and inverse estimator shown in Fig. 2. When the reference signal is given, it becomes a simple matter to identify either the forward or inverse channel as shown. When the reference is not given, we can use the known statistics of the input to find the required nonlinearity $g(\cdot)$ as outlined in [1] , and use the blind error $(\hat{x} - g(\hat{x}))$ instead of the true error $(x - \hat{x})$.

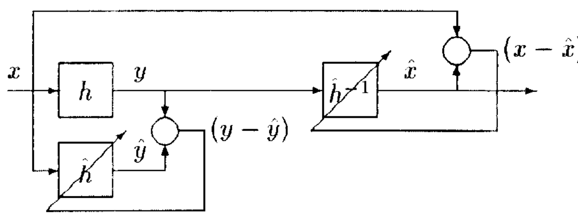

Figure 2: Forward and Inverse Filtering Using a Reference

4. SIMULATIONS

The blind error described above is actually the partial derivative of the blind cost function J_b with respect to the output of the equalizer $\hat{x}$, $\frac{\partial J_b}{\partial \hat{x}}$. In the following simulations we used the O_4^2 cost function which is defined in [2] and is

$$O_s^r = \frac{E|\hat{x}|^r}{(E|\hat{x}|^s)^{\frac{r}{s}}} \quad (8)$$

with $r = 2$ and $s = 4$. For these values of r and s it is equivalent to the Godard cost function (the gradients are equivalent). The blind gradient used in finding the inverse (equalization) filter, is

$$\frac{\partial J_{tot}}{\partial \hat{h}^{-1}} = \frac{\partial O_s^r}{\partial \hat{h}^{-1}} = (\hat{x} - \frac{E|\hat{x}|^2}{E|\hat{x}|^4}\hat{x}^3)y. \quad (9)$$

The forward filter's cost function can be thought of as: $J_f = (y - \hat{y})^2$. The gradient for the forward filter is then:

$$\frac{\partial J_{tot}}{\partial \hat{h}} = \frac{\partial J_f}{\partial \hat{y}}\frac{\partial \hat{y}}{\partial \hat{h}} = (y - \hat{y})\ \hat{x}. \quad (10)$$

The total cost function used for the following tests was therefore:

$$J_{tot} = \frac{E|\hat{x}|^2}{(E|\hat{x}|^4)^{\frac{1}{2}}} + (y - \hat{y})^2. \quad (11)$$

The stochastic gradient updates of (4) and (5) were used.

4.1. Simple Test

Using 2-PAM data, a simple test was done in order to verify that both forward and inverse filters can be found in quickly converging fashion and that no arbitrary constraint need be placed on the equalization filter taps. The requirement that also the cost function $J = (y - \hat{y})^2$ be minimized was the means used. Both forward and inverse filters are plotted in Fig. 3 as well their true values. The "self ISI" $\hat{h}^{-1} * \hat{h}$ is also plotted. The filter was a non-minimum phase channel [1 1 -.75], having a correlation matrix eigenvalue spread of 7. The step sizes used were .0001 and the steady state level of ISI was .001. A 16 tap inverse was used and the initial was [0 0 0 0 0 0 0 .1 0 0 0 0 0 0 0 0].

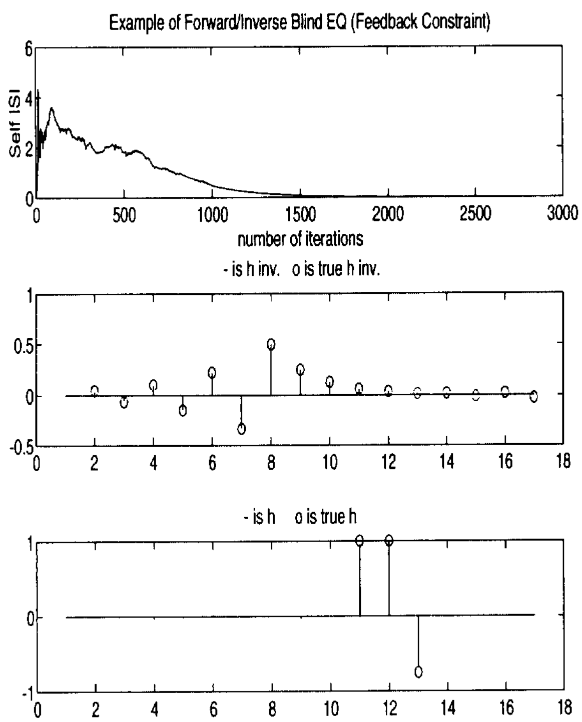

Figure 3: Simple test using 2-PAM data and a non-minimum phase channel [1 1 -.75]. The self-monitoring measure of ISI is plotted.

4.2. Speed of Convergence and Eigenvalue Spread Robustness Tests

Two linear channel models were used in this blind equalization test. Channel 1 was [1 1 − .75] as above and channel 2 was [.4 1 − .7 .6 .3 − .4 .1] with eigenvalue spread of 28. Both 2-PAM data and uniformly distributed unit variance data were used. The step-sizes were adjusted to obtain a steady-state ISI of .015, and the results are plotted in Figs. 4-7. The true ISI measures are plotted for ten realizations for each test. A 16 tap inverse was used for all the runs and the initial was [0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0] for channel 1 and [0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0] for channel 2.

The feedback mechanism does seem to offer some robustness to eigenvalue spread. Comparison to the traditional inverse filter blind stochastic gradient update using an energy constraint was carried out as well. The results are plotted in Figs. 8-11. We see that the forward/inverse method has increased convergence speed compared to the "inverse only" method. Feedback is a more meaningful way to constrain the inverse filter and allows larger step sizes to be used without introducing unstability. For these tests, a larger step size could be used for the forward/inverse algorithm while maintaining the same steady state misadjustment. The interesting result found here is that when using the forward/inverse method, the higher eigenvalue spread tests converged faster than the "inverse only" approach. By comparing Figs. 7 and 11 we see that the forward/inverse method offers speed improvement when the channel has high eigenvalue spread.

For comparison to the exact least squares speed of convergence using the SFTF algorithm, see [8]. The speed of convergence for the forward/inverse algorithm using (4) and (5), lies between the exact least squares and the traditional stochastic gradient methods. It is suspected the forward/inverse algorithm using (6) and (7), the Newton-type updates, would be faster than blind exact least squares methods as tested in [8, 7]. The Newton update forward/inverse algorithm would be closer to the speed of convergence obtainable when the reference is available.

4.3. Avoidance of Local Minima Test

Often researchers have been concerned about the existence of local minima for blind equalizers. It is only of concern when using an equalization filter of low order or when the center of energy of the filter is located near an edge of the filter. It was found with this algorithm, that by initializing the inverse filter with energy in the center, the energy in the converged filter would be centered as desired. It was found for this foward/inverse algorithm, that the avoidance of local minima was only possible through the use of large step sizes. Using the typical example given recently in [7] using the channel [1 .6 .36] and only a three tap inverse which is initialized to [0 0 1], we show that the local minima can be escaped when using step sizes of .065.

5. CONCLUSION AND FUTURE RESEARCH

The use of feedback systems has also been proven to be useful in multichannel source separation problems as described in [9, 10]. We have shown a single channel use for feedback as well. When a multichannel system has fewer sensors than sources, feedback becomes a necessary part of the system identification and blind data estimation. We have demonstrated that the use of feedback is of potential use in blind deconvolution algorithms. In this single channel case it has brought increased convergence speed, robustness to eigenvalue spread and removed the need for arbitrary contraints.

6. REFERENCES

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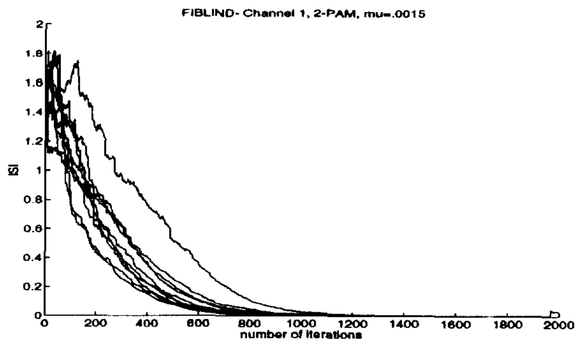

Figure 4: Convergence of ten realizations of 2-PAM data using forward/inverse algorithm. This test used channel 1 with eigenvalue spread of 7.

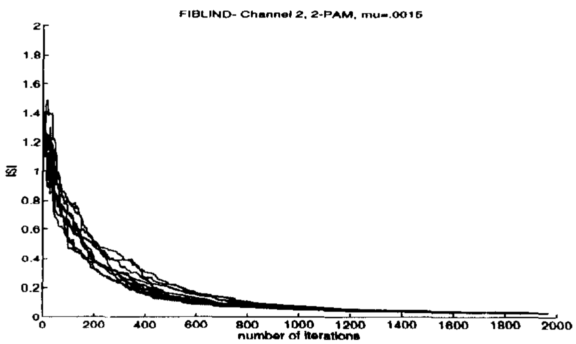

Figure 5: Convergence of ten realizations of 2-PAM data using forward/inverse algorithm. This test used channel 2 with eigenvalue spread of 28.

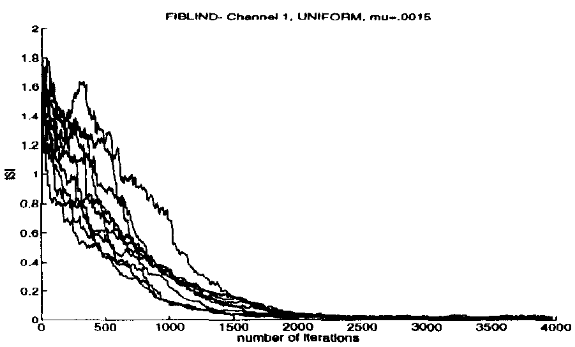

Figure 6: Convergence of ten realizations of uniformly distributed data using forward/inverse algorithm. This test used channel 1 with eigenvalue spread of 7.

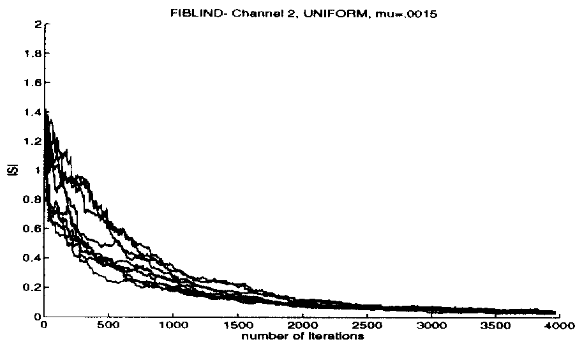

Figure 7: Convergence of ten realizations of uniformly distributed data using forward/inverse algorithm. This test used channel 2 with eigenvalue spread of 28.

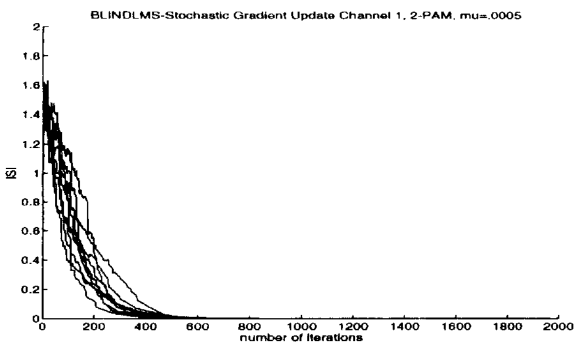

Figure 8: Convergence of ten realizations of 2-PAM data using the "inverse only" stochastic gradient algorithm. This test used channel 1 with eigenvalue spread of 7.

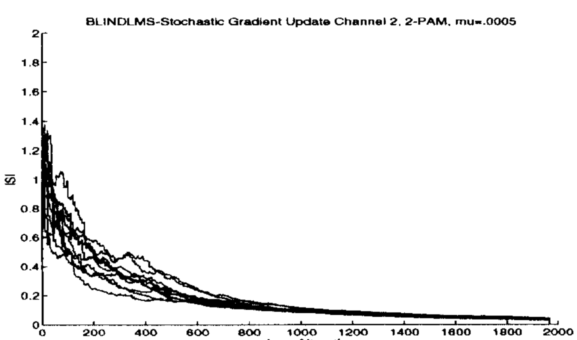

Figure 9: Convergence of ten realizations of 2-PAM data using the "inverse only" stochastic gradient algorithm. This test used channel 2 with eigenvalue spread of 28.

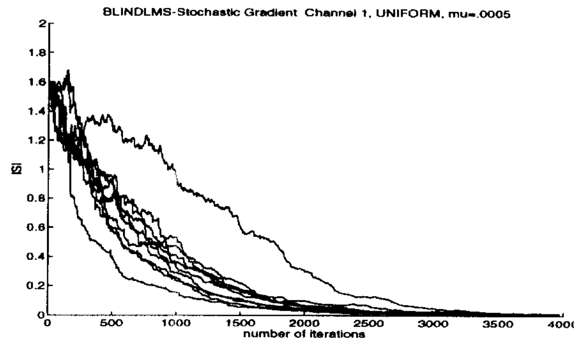

Figure 10: Convergence of ten realizations of uniformly distributed data using the "inverse only" stochastic gradient algorithm. This test used channel 1 with eigenvalue spread of 7.

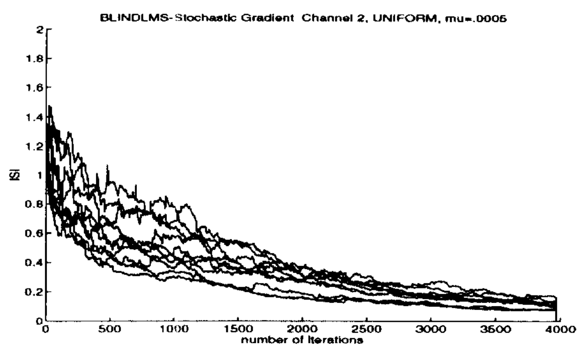

Figure 11: Convergence of ten realizations of uniformly distributed data using the "inverse only" stochastic gradient algorithm. This test used channel 2 with eigenvalue spread of 28.

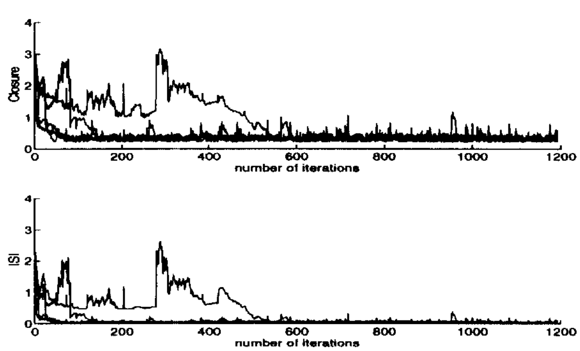

Figure 12: Avoidance of local minima test. The plots show the convergence of ten realizations of 2-PAM data using the [1 .6 .36] channel and step sizes=.0065.