Algebraic Self-Training Algorithms For Multi-Channel System Identification and Equalization

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ABSTRACT: *In this paper we study the problem of identifying and equalizing single input-multiple output (SIMO) linear systems that are common in digital communication systems. We present an adaptive algorithm for the identification of single input-multiple output linear system. We also present an algorithm for the equalization of single input-multiple output linear system. Both algorithms are based on a mild length and zero condition and can be implemented recursively. Simulation results are presented to demonstrate the performances of these two algorithms.*

1 Introduction

The need for identification of single input-multiple output (SIMO) linear systems is common in digital communication, signal processing, and control systems. In quadrature amplitude modulated (QAM) communication systems, for example, if the channel output is oversampled by an integer factor of p, the cyclostationary oversampled output can be viewed as a vector of stationary outputs of p subchannels. Similarly, the SIMO linear system can also be used to model a wideband antenna array with p elements.

In an SIMO system, a common input s_k is sent into p subsystems, each with transfer function $H_i(z)$. The p outputs are corrupted by i.i.d. additive noises $\{w_i[k]\}$ with zero mean and variance σ_w^2. The output signals are given by

$$x_n^{(i)} = \sum_{k=0}^{K} s_k h_i[n-k] + w_i[n] = H_i(z)s_n + w_i[n]. \quad (1.1)$$

where FIR subchannels $H_i(z) = \sum_{k=0}^{K} h_i[k]z^k$ are defined. The objective of SIMO identification is to identify each subsystem $H_i(z)$ without training (or reference) signal s_k. On the other hand, the objective of SIMO equalization is remove the inter-symbol interference of each subchannel to recover the signal sequence $\{s_n\}$ from subchannel outputs.

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In this paper, we present an algebraic method for the identification of SIMO systems. In [1], a similar method was also proposed based on multiple algebraic matching of subchannel transfer functions. It relies on simultaneous elimination of multiple signals and is implemented via a singular value decomposition approach. In order to allow adaptive implementation, we provide a different formulation. We also present an algebraic method for the blind equalization of SIMO systems. Our method originates from the algorithm presented by Slock [2]. However, we provide a criterion for training signal selection that relies on a weaker assumption for the SIMO subchannels. Both algorithms are very simple and can be recursively implemented.

2 SIMO System Identification

2.1 Basic Approach

Here we present the basic system identification idea. To assure the identifiability of the SIMO system, the p FIR subchannels should not have any common zero.

Our approach is based on the outputs of two subsystems with no common zeros. The two filters $G_1(z)$ and $G_2(z)$ will be used to estimate the two subchannels $H_1(z)$ and $H_2(z)$, respectively. The output signals of both filters are to be summed and the result is denoted as y_n.

$$\begin{aligned} y_n &= G_1(z)x_n^{(2)} - G_2(z)x_n^{(1)} \\ &= [H_2(z)G_1(z) - H_1(z)G_2(z)]\, s_n. \quad (2.1) \end{aligned}$$

Algebraically, if we minimize the magnitude of y_n by selecting $G_1(z)$ and $G_2(z)$, then the global and unique minimum would be

$$H_2(z)G_1(z) - H_1(z)G_2(z) = 0 \quad (2.2)$$

under noiseless condition and persistent excitation.

Let all filters and the subchannels have finite impulse response. Since the two subchannels are coprime, the global minimum is reached iff

$$\begin{aligned} G_1(z) &= \zeta(z)H_1(z) \quad (2.3) \\ G_2(z) &= \zeta(z)H_1(z), \quad (2.4) \end{aligned}$$

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where $\zeta(z)$ is a common polynomial factor. Thus both subchannels can be identified by $G_1(z)$ and $G_2(z)$ with an ambigiuous factor $\zeta(z)$. Evidently, if the order of both channels are known, then the two filters can be selected to have minimum orders to avoid the possibility of common factor. If the channel orders are unknown, then either the orders must be estimated from the data, or the common factor has to be eliminated.

To ensure that no trivial solution $\zeta(z) = 0$ is reached, a linear constraint can be imposed upon the two filters. Once we have minimized $E\{|y_n|^2,\}$ we must first determine (approximately) whether $G_1(z)$ and $G_2(z)$ have any a common factor. Let

$$G_1(z) = \sum_{k=0}^{N_1} g_{1,k} z^k \quad \text{and} \quad G_2(z) = \sum_{k=0}^{N_2} g_{2,k} z^k. \quad (2.5)$$

We can construct a $(N_1 + N_2) \times (N_1 + N_2)$ Sylvester matrix

$$S = \left.\begin{bmatrix} g_{1,0} & \cdots & g_{1,N_1} & 0 & \cdots & 0 \\ 0 & g_{1,0} & \cdots & g_{1,N_1} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & g_{1,0} & \cdots & g_{1,N_1} \\ g_{2,0} & \cdots & g_{2,N_2} & 0 & \cdots & 0 \\ 0 & g_{2,0} & \cdots & g_{2,N_2} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & g_{2,0} & \cdots & g_{2,N_2} \end{bmatrix}\right\} N_1 + N_2. \quad (2.6)$$

It can be checked that

$$N_1 + N_2 - rank(S) = \text{number of common zeros.} \quad (2.7)$$

When there are m common zeros, we can delete $m-1$ rows from the top half and the bottom half of S to yield S_m with rank $N_1 + N_2 - 2(m-1)$. Subsequently, singular value decomposition can be used to remove the ambiguity (common factor) term. In fact if we solve for

$$[b_0 \ \cdots \ b_{N_2-m-1} \ a_0 \ \cdots \ a_{N_1-m-1}]S_m = \mathbf{0}, \quad (2.8)$$

then

$$a(z) = \gamma H_2(z) \quad (2.9)$$

$$b(z) = -\gamma H_1(z), \qquad \gamma \neq 0. \quad (2.10)$$

2.2 Algorithm Implementation

Denote a composite parameter vector and the corresponding composite regressor vector as

$$\mathbf{g} = [g_{1,0} \ \cdots \ g_{1,N_1} \ g_{2,0} \ \cdots \ g_{2,N_2}]', \quad (2.11)$$

$$X_k = [x_k^{(1)} \ \cdots \ x_{k-N_1}^{(1)} \ x_k^{(2)} \ \cdots \ x_{k-N_2}^{(2)}]'. \quad (2.12)$$

We minimize

$$E\{|y_n|^2\} = \mathbf{g}^H E\{X_k X_k^H\}\mathbf{g}, \quad (2.13)$$

subject to

$$\text{(linear constraint)} \qquad \mathbf{c}^H \mathbf{g} = 1, \quad (2.14)$$

$$\text{(nonlinear constraint)} \qquad ||\mathbf{g}||^2 = 1. \quad (2.15)$$

This is equivalent to solving

$$\mathbf{g} = (E\{X_k X_k^H\})^{-1}\mathbf{c}, \quad (2.16)$$

$$\mathbf{g} = \text{minimum eigenvector of } E\{X_k X_k^H\}. \quad (2.17)$$

The minimization of $E|y_n|^2$ can also be easily implemented using the standard recursive least square algorithm with the linear or the nonlinear constraint.

2.3 Multiple Subchannel Systems

The above development applies only to SIMO systems with two subchannels. For general multichannel systems with p subchannels, let

$$G_i(z) = \sum_{k=0}^{m} g_{i,k} z^k, \quad m \geq K, \quad i = 1, 2, \ldots p \quad (2.18)$$

be the p subchannel estimators. Recall that the identification condition is for p subchannels to have no common zeros.

Construct a new signal

$$y_n = \left[\sum_{i=2}^{p} z^{(i-2)m} G_i(z)\right] x_n^{(1)} - G_1(z) \left[\sum_{i=2}^{p} x_{n-(i-2)m}^{(i)}\right].$$

In fact, this signal can be rewritten as

$$y_n = \left[H_1(z) \sum_{i=2}^{p} z^{(i-2)m} G_i(z) - G_1(z) \sum_{i=2}^{p} z^{(i-2)m} H_i(z)\right] s_n. \quad (2.19)$$

Under persistent excitation, the minimization of $E\{|y_n|^2\}$ result in an algebraic cancellation of

$$H_1(z) \sum_{i=2}^{p} z^{(i-2)m} G_i(z) = G_1(z) \sum_{i=2}^{p} z^{(i-2)m} H_i(z).$$

Because $\{H_i(z)\}$ have no common zero, the result becomes

$$G_1(z) = \zeta(z) H_1(z)$$

$$\sum_{i=2}^{p} z^{(i-2)m} G_i(z) = \zeta(z) \sum_{i=2}^{p} z^{(i-2)m} H_i(z).$$

Similar to the two-channel case, the common factor $\zeta(z)$ can be eliminated by selecting minimum polynomial order in the estimators or by the Sylvester matrix method to yield

$$G_1(z) = \gamma H_1(z) \tag{2.20}$$

$$\sum_{i=2}^{p} z^{(i-2)m} G_i(z) = \gamma \sum_{i=2}^{p} z^{(i-2)m} H_i(z). \tag{2.21}$$

Since $m > \deg\{H_i(z)\}$, we obtain the desired estimation

$$G_i(z) = \gamma H_i(z), \quad i = 1, 2, \ldots p. \tag{2.22}$$

As in the two-channel case, channel estimation based on minimizing $E\{|y_n|^2\}$ can be implemented using the constrained minimization approach or the recursive least square method.

3 SIMO Channel Equalization

3.1 Algebraic Analysis

Suppose that there are p subchannels $\{H_i(z)\}_{i=1}^{p}$ in the SIMO satisfies the length and zero condition. We give a channel equalization criterion that is simply based on the length and zero condition [3].

The equalization system is formed by summing the outputs of p FIR filters, each of which is placed at a subchannel output. When channel noise is zero, the equalizer output signal y_n is given by

$$y_n = \sum_{i=1}^{p} F_i(z) x_n^{(i)} = \left[\sum_{i=1}^{p} H_i(z) F_i(z)\right] s_n. \tag{3.1}$$

We let

$$F_i(z) = f_i + z \sum_{k=0}^{m} g_i[k] z^k = f_i + z G_i(z). \tag{3.2}$$

The overall linear system is simply

$$\begin{aligned} T(z) &\triangleq \sum_{k=0}^{m+K+1} t_k z^k \\ &= \sum_{i=1}^{p} f_i H_i(z) + z\left[\sum_{i=1}^{p} H_i(z) G_i(z)\right] \\ &= \sum_{i=1}^{p} f_i h_i[0] + z \sum_{k=1}^{K} \sum_{i=1}^{p} f_i h_i[k] z^{k-1} \\ &\quad + z \sum_{i=1}^{p} H_i(z) G_i(z) \end{aligned} \tag{3.3}$$

To equalize the SIMO system, we would like to find filters

$$G_i(z) = \sum_{k=0}^{m} g_i[k] z^k \tag{3.4}$$

that result in

$$\sum_{k=1}^{K} \left(\sum_{i=1}^{p} f_i h_i[k]\right) z^{k-1} + \sum_{i=1}^{p} H_i(z) G_i(z) = 0 \tag{3.5}$$

such that all inter-symbol interference is removed to yield

$$T(z) = \sum_{i=1}^{p} f_i h_i[0]. \tag{3.6}$$

To determine the necessary and sufficient condition for such an ISI elimination, define

$$\mathbf{f} \triangleq [f_1 \quad f_2 \quad \ldots \quad f_p]',$$

$$\mathbf{g} \triangleq [g_1[0] \ \ldots \ g_1[m] \ \ldots g_p[0] \ \ldots \ g_p[m]]'$$

$$\bar{H} \triangleq \begin{bmatrix} h_1[1] & h_2[1] & \ldots & h_p[1] \\ \vdots & \vdots & \vdots & \vdots \\ h_1[K] & h_2[K] & \ldots & h_p[K] \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 \end{bmatrix} \tag{3.7}$$

and

$$\mathcal{H} \triangleq \begin{bmatrix} h_1[0] & & & \ldots & h_p[0] & & \\ \vdots & \ddots & & \vdots & \vdots & \ddots & \\ h_1[K] & \ddots & h_1[0] & \ldots & h_p[K] & \ddots & h_p[0] \\ & \ddots & \vdots & \vdots & & \ddots & \vdots \\ & & h_1[K] & \ldots & & & h_p[K] \end{bmatrix}.$$

The zero ISI equation (3.5) is satisfied when

$$\bar{H}\mathbf{f} + \mathcal{H}\mathbf{g} = \mathbf{0}. \tag{3.8}$$

Thus, in order for solutions to the zero ISI equation exist, $\mathcal{H}$ must have full-row rank. Since $\mathcal{H}$ is an $(m + K + 1) \times (m + 1)p$ matrix, one requirement for it to be full rank is

$$(m+1)p \geq m+K+1, \quad \text{or} \quad (m+1)(p-1) \geq K. \tag{3.9}$$

Thus the equalizer filter length $m+1$ must be sufficient to allow more columns than rows in $\mathcal{H}$. Moreover, it is known that given the above condition, the zero and length condition is the sufficient and necessary condition for the rows of $\mathcal{H}$ to have full rank [3]:

$$\begin{aligned} &h_i[0] \neq 0 \quad \text{for some } 1 \leq i \leq p \\ &h_i[K] \neq 0 \quad \text{for some } 1 \leq i \leq p \\ &\{H_i(z)\} \quad \text{have no common zero.} \end{aligned} \tag{3.10}$$

Hence the zero and length condition is therefore also sufficient and necessary for the existence of parameter vector $\mathbf{g}$ to achieve the zero ISI objective.

3.2 Algorithm Development

The equalizer output signal y_n can then be written as

$$\begin{aligned} y_n &= (\sum_{i=1}^{p} f_i h_i[0]) s_n + \sum_{i=1}^{m+K+1} t_i s_{n-i} \quad (3.11) \\ &+ \sum_{i=1}^{p} f_i w_i[n] + \sum_{i=1}^{p} \sum_{k=0}^{m} g_i[k] w_i[n-k] \end{aligned}$$

Consequently, the average output power is

$$\begin{aligned} E\{|y_n|^2\} &= E\{|s_n|^2\} (\sum_{i=1}^{p} f_i h_i[0])^2 + ||\mathbf{f}||^2 \sigma_w^2 \\ &+ E\{|s_n|^2\} \sum_{\ell=1}^{m+K+1} |t_\ell|^2 + ||\mathbf{g}||^2 \sigma_w^2. \end{aligned}$$

It can be seen that the average power of the equalizer output consists of the following parts:

Signal power: $E\{|s_n|^2\} (\sum_{i=1}^{p} f_i h_i[0])^2$

Residual ISI power: $E\{|s_n|^2\} \sum_{\ell=1}^{m+K+1} |t_\ell|^2$

Noise power: $(||\mathbf{f}||^2 + ||\mathbf{g}||^2) \sigma_w^2$.

To minimize the effect of ISI and noise, the optimum equalizer parameter vector $\mathbf{g}$ should be chosen to minimize

$$E\{|s_n|^2\} \sum_{\ell=1}^{m+K+1} |t_\ell|^2 + ||\mathbf{g}||^2 \sigma_w^2. \quad (3.12)$$

This objective can be accomplished through

$$\min_{\mathbf{g}} E\{|y_n|^2\}. \quad (3.13)$$

The remaining difficulty is the selection of vector $\mathbf{f}$. Unlike in [2], $\mathbf{f}$ should not be chosen arbitrarily because it is crucial for the desired signal to exist, i.e.,

$$\sum_{i=1}^{p} f_i h_i[0] \not\approx 0. \quad (3.14)$$

Since $\{h_i[0]\}$ are unknown, we propose a second step used to determine an appropriate vector $\mathbf{f}$ by maximizing the remaining power of the output signal y_n after equalization. This procedure guarantees the existence of the desired signal component. Thus, our algorithm actually follows a two-step approach:

$$\max_{||\mathbf{f}||=1} \min_{\mathbf{g}} E\{|y_n|^2\}. \quad (3.15)$$

3.3 Algorithm Implementation

Denote

$$\bar{X}_k = \left[x_k^{(1)} \; x_k^{(2)} \; \dots \; x_k^{(p)} \right],$$

$$\mathcal{X}_k = \left[x_{k-1}^{(1)} \; \dots \; x_{k-m}^{(1)} \; \dots \; x_{k-1}^{(p)} \; \dots \; x_{k-m}^{(p)} \right].$$

The objective is to determine the optimal parameter vectors $\mathbf{g}$ and $\mathbf{f}$ via

$$\begin{aligned} & \max_{||\mathbf{f}||=1} \min_{\mathbf{g}} \sum_{n=0}^{M} |y_{k-n}|^2 \\ = & \max_{||\mathbf{f}||=1} \min_{\mathbf{g}} \sum_{n=0}^{M} |\bar{X}_{k-n}\mathbf{f} + \mathcal{X}_{k-n}\mathbf{g}|^2 \quad (3.16) \end{aligned}$$

It is straightforward to find that the optimal vector $\mathbf{g}$ must satisfy the normal equation

$$\left(\sum_{n=0}^{M} \mathcal{X}_{k-n}^H \mathcal{X}_{k-n} \right) \mathbf{g} = - \left(\sum_{n=0}^{M} \mathcal{X}_{k-n}^H \bar{X}_{k-n} \right) \mathbf{f}. \quad (3.17)$$

Substitute (3.17) into (3.16), the objective becomes

$$\max_{||\mathbf{f}||=1} \mathbf{f}^H \mathcal{R} \mathbf{f}, \quad (3.18)$$

where

$$\begin{aligned} \mathcal{R} &= \left[\sum_{n=0}^{M} \bar{X}_{k-n}^H \bar{X}_{k-n} - \sum_{n=0}^{M} \bar{X}_{k-n}^H \mathcal{X}_{k-n} \right. \quad (3.19) \\ &\times \left. \left(\sum_{n=0}^{M} \mathcal{X}_{k-n}^H \mathcal{X}_{k-n} \right)^{-1} \sum_{n=0}^{M} \mathcal{X}_{k-n}^H \bar{X}_{k-n} \right]. \end{aligned}$$

Thus, $\mathbf{f}$ can be solved by finding the maximum eigenvector of the matrix $\mathcal{R}$. Given $\mathbf{f}$, the optimum equalizer vector $\mathbf{g}$ can be obtained either by directly solving the least square equation (3.17), or through recursive least square adaptation based on (3.17).

4 Simulation Examples

In our SIMO identification example, we use a typical three ray multipath channel [4]

$$h(t) = p(t) - b e^{j2\pi(f_n - f_c)\tau} p(t - \tau), \quad (4.1)$$

where $p(t)$ is a raised cosine rolloff

$$p(t) = A\frac{\sin(\pi t/T)}{\pi t/T} \cdot \frac{\cos(\alpha \pi t/T)}{1 - 4\alpha^2 t^2/T^2} \quad (4.2)$$

truncated to $8T$ with $\alpha = 0.11$. We selected the symbol duration $T = 50ns$ and the delay $\tau = 6.3ns$. $b = 0.9$ and $f_c = f_n$. The channel output is oversampled by $p = 2$ with $SNR = 30dB$. Channel orders are known and only 100 iterations (data samples) were taken for an 8-level PAM input. True system impulse response (dashed line) and twenty independently identified responses (solid lines) are shown in Fig. 1.

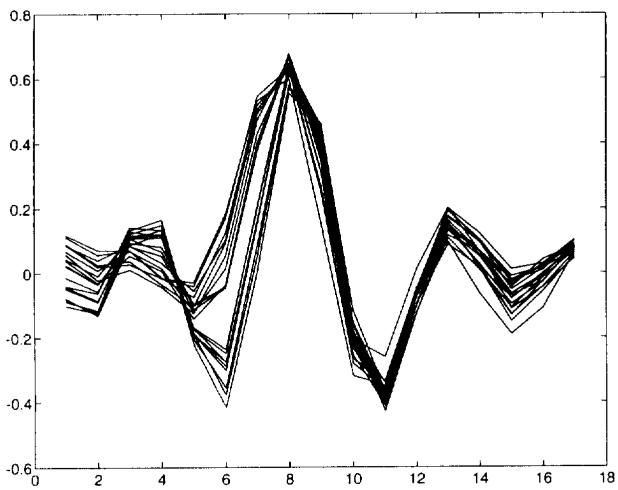

Figure 1: True and Estimated system responses.

In our SIMO equalization example, we consider a lowpass channel with oversampled ($p = 2$) impulse response shown in Fig. 2. Different data lengths are considered under two SNR levels. The residual ISI and noise power (MSE) in the equalizer output normalized by the desired signal power is shown in Fig. 3.

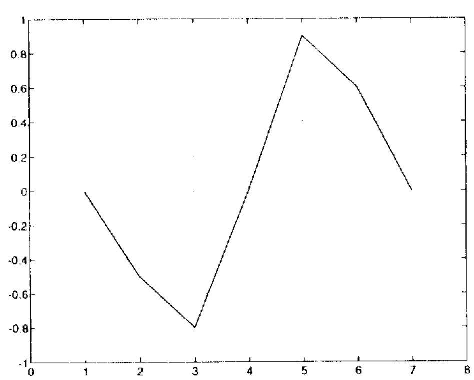

Figure 2: Impulse response of the second channel.

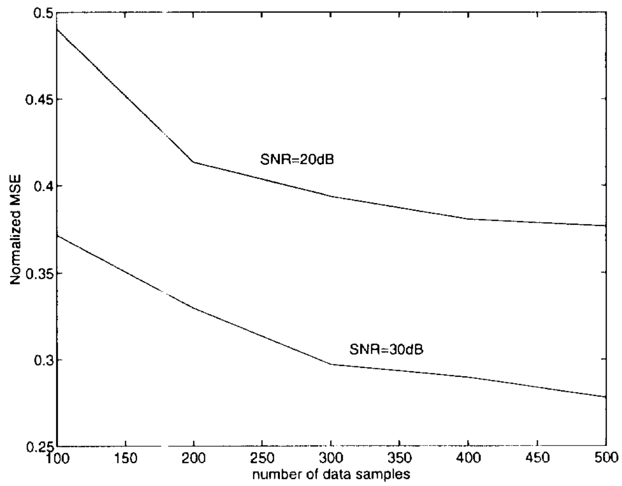

Figure 3: Normalized MSE after equalization.

5 Conclusion

We present an adaptive algorithm for the identification of SIMO linear system. We also present an SIMO equalization algorithm that can be implemented adaptively. Both algorithms are based on the length and zero condition that is essential for the identifiability of SIMO linear systems. Simulation results demonstrate satisfactory performances by both algorithms using a few hundred data samples.

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