Using Cyclostationarity for Timing Synchronization and Blind Equalization

German Feyh

CIRRUS LOGIC
350 Interlocken Parkway, Suite 275, Broomfield, CO 80021

Abstract

Timing synchronization and blind equalization of digital communication signals can be compared to the chicken and the egg problem. Timing synchronization algorithms [1] work well on equalized signals. Blind equalization algorithms based on cyclostationary work well on perfectly time synchronized signals. The question remains what to do at the start of a signal, where we have neither perfect timing synchronization nor an estimate of the channel for equalization. We propose

1) a new timing synchronization algorithm for phase shift keying signals, based on short time cyclostationary cumulants
2) a new cyclostationary autocorrelation/cumulant slice based blind equalization algorithm.

The timing algorithm is compared to traditional timing methods using nonlinearities.

1 Introduction

Communication channels introduce severe distortion in communication signals. If there are equal amplitudes in both path of a two path channel, then spectral nulls are introduced within the bandwidth of the signal. These channels are encountered in HF-communications. The CCIR-POOR channel is such an equal amplitude channel that is widely used for HF-communication systems evaluation.

Depending on the relative delay between the two paths, timing synchronization using nonlinearities can be made an almost impossible task. The timing tone generated by the nonlinearities relies on a nonconstant envelope of the communication signal. If the delay of a two-path channel is $\frac{k+1}{2}$ times the symbol period, the envelope of the signal is nearly constant for a raised cosine signal. To generate a reliable timing tone over all ranges of the delay and at low SNR, we introduce a cyclostationary measurement of the timing tone, based on the shorttime cumulant at point 0.

Blind equalization algorithms are a very active area of research [1,2,3,4]. These algorithms trade receiver complexity for frequency economy. Training patterns waste channel time, since they are not transmitting any data. Blind equalization algorithms are traditionally used in multidrop telephone modem lines. A further area of application is signal surveillance. HF-channels introduce severe channel distortion. Reliable reception uses equalization or Viterbi detection, to combat the channel distortions. In both cases, one has to estimate the channel impulse response.

Blind channel estimators using cyclostationary second order statistics rely on the exact frequency synchronization of the transmitter and the receiver timing oscillators. Blind channel estimators are used primarely in the startup phase of the signal reception. The second order statistics are computed for a whole block of signal samples. Typically the samples corresponding to at least 256 signal symbols are collected, before one attempts to estimate the channel impulse reponse. Allowing for a symbol phase jitter of the signal of 10%, the symbol rate at the transmitter and the estimated one at the receiver should not differ more than $\frac{0.1}{256} \approx 0.4 \times 10^{-3}$. This precision are attainable in telephone applications. In signal surveillance applications, the estimation of the baudrate from the samples corresponding to 256 signal samples will hardly be that precise at low SNR.

We introduce a blind channel estimation algorithm based on the averaged, cyclostationary autocorrelation $r_a(k)$ and the averaged, cyclostationary cumulant slice $c_a(0,0,k)$. These averaged statistics can be estimated over smaller blocks, where the cyclostationary premise might hold. Then one can average over all blocks to get the input to the algorithm.

It is well known that the estimation of the fourth order cumulants is numerically more unreliable than the estimation of autocorrelation statistics alone. The proposed algorithm needs longer symbol sequences than the algorithms based on cyclostationary autocorrelations.

2 Higher Order Statistics for Oversampled Signals

For symbol synchronization and fractionally spaced equalization digital communication signals are oversampled by an integer multiple P of the symbol rate. In a recent paper by Slock [1], the samples within one symbol interval were stacked into a vector. The oversampled digital communications signal is thus represented by a vector random process, which is sampled at the symbol rate.

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The data $a(k)$ at the input of the channel be complex valued and iid. We also assume that the data is symmetric, i.e. that statistics over an uneven number of the random variable x and its conjugate complex x^* are 0. The oversampled signal is cyclostationary with the period T, the symbol duration. The vectorized signal is stationary. Thus one can compute the higher order statistics (HOS) of this vectorized signal. To illustrate this point we assume an oversampling factor $P = 4$ and a finite length channel impulse response $h_0, h_1, h_2, \ldots h_N$ for the oversampled signal. One computes the vector of cumulants $c_p(0,0,0), p = 0,1,2,3$ for the vectorized signal, e.g.

$$c(0,0,0) = \begin{pmatrix} c_0(0,0,0) \\ c_1(0,0,0) \\ c_2(0,0,0) \\ c_3(0,0,0) \end{pmatrix} = \gamma_4 \begin{pmatrix} |h_0|^4 + |h_4|^4 + \ldots \\ |h_1|^4 + |h_5|^4 + \ldots \\ |h_2|^4 + |h_6|^4 + \ldots \\ |h_3|^4 + |h_7|^4 + \ldots \end{pmatrix}.$$

Since the elements of the vector are stationary, one can estimate the moments of the signal elements by their time averages. In this example the first cumulant element $c_0(0,0,0)$ of the vectorized random process is estimated using the autocorrelation $r_0(0)$ and the fourth order moment $m_0(0,0,0)$ of the record of data $x(k)$ with length K,

$$\begin{aligned} r_0(0) &= \frac{4}{K} \sum_{k=1}^{K/4} x(4k)x^*(4k), \\ m_0(0,0,0) &= \frac{4}{K} \sum_{k=1}^{K/4} x(4k)x^*(4k)x(4k)x^*(4k), \\ c_0(0,0,0) &= m_0(0,0,0) - r_0^2(0). \end{aligned}$$

The averaged cumulant $c_a(0,0,0)$ is defined as the sum of the vectorized cumulants.

$$c_a(0,0,0) = \sum_{p=0}^{P-1} c_p(0,0,0) = \gamma_4 \sum_{n=0}^{N} |h_n|^4.$$

Please notice, that the averaged cumulant for an iid data driven, oversampled digital communication signal is similar to the cumulant of a communications signal, that was not oversampled. This example can be extended to other orders of the statistics and other indici.

3 Bit Synchronization

Bit synchronization can either be done in a feedforward fashion using a nonlinearity or as feedback, using errors based on the decisions. Here we concentrate on the feedforward approach.

Most feedforward bit synchronization methods using an even nonlinearity [5,6] rely on the modulation of the envelope of the complex signal. These methods are the absolute value, the square, the fourth power of the signal, etc.. It is sometimes also deemed beneficial, to delay one arm of the square by one half of the symbol duration.

For some pulse shaping filters and two-path channel impulse response, the symbol rate does not show up in the modulation of the signal envelope. Consider for instance the case of the raised cosine signal, roll off factor 1/2, and a two path, equal amplitude channel with a path delay of $\frac{k+1}{2}$ times the symbol period.

Band Edge Timing Recovery (BETR) [7] shifts the complex signal (carrier frequency =0) up and down by half the baudrate, lowpass filters around the bandedges, which are now at 0 frequency and crosscorrelates these signals.

In todays synchronization problems, the deviations in frequency between the oscillator in the sender and in the receiver will be less than one percent of the symbol rate. The cycle of the difference frequency is in excess of 100s of symbols. This work is motivated by the observation that the cumulants of a digital communication signal is quite different if the signal is sampled at an integer multiple of the symbol rate or if it is sampled at some arbitrary frequency. If the sampling frequency deviates from an integer multiple of the symbol rate, the cumulants change from the vectorized, stationary cumulants to the averaged cumulants.

Let estimated random variables be denoted by a $\hat{c}$. Here we introduce as a nonlinearity the vectorized, shorttime cumulant $\hat{c}(0,0,0)(k)$ at the time k. The shorttime cumulant is based on the time average measurements of the autocorrelation $\hat{r}(0)(k)$ and the fourth order moment $\hat{m}(0,0,0)(k)$ over the averaging time K at the time k. If there is perfect timing synchronization, then the shorttime cumulant approximates the formula for $c(0,0,0)$ (see above). If there is some slippage between the timing of the transmitter and the receiver, the elements of the vectorized cumulant will cycle. Given the shorttime, cyclostationary cumulant

$$\hat{c}(0,0,0)(k) = \begin{pmatrix} \hat{c}_0(0,0,0)(k) \\ \hat{c}_1(0,0,0)(k) \\ \vdots \\ \hat{c}_{P-1}(0,0,0)(k) \end{pmatrix}$$

for the data signal oversampled by the factor P over the symbol rate, we compute the spectral line for the timing tone by:

$$y(k) = \sum_{p=0}^{P-1} c_p(0,0,0)(k) e^{j\frac{2\pi}{P}p}$$

This timing tone indicates the difference between the sender and receiver timing oscillator. If there is too much deviation between the two frequencies, the cumulants will be the averaged cumulants. Then it is impossible to estimate the frequency difference.

For $T < \frac{1}{f_S - f_R}$ there will be some cycling in the elements of the vectorized, shorttime cumulant $\hat{c}(0,0,0)$. Given the maximum deviation of the sender and the receiver oscillator frequency, one can give the maximum allowable averaging time T. The method is robust over variations of the averaging time T from $\frac{1}{16}$ to $\frac{1}{2}$ of the maximal avaraging time $T_{max} = \frac{1}{f_S - f_R}$.

4 Timing Synchronization Evaluation

The analytical computation of the SNR of the timing tones for the different methods seems intractable. Therefore the timing schemes were evaluated by computer simulation.

The input to the synchronization schemes was a QPSK signal. Its pulse was shaped as a raised cosine impulse with roll off $=\frac{1}{2}$ at some oversampling rate. The channel filter was a FIR with the impulses 1 and -1 separated by the spezified delay. Noise was added at the output of the channel filter. The noise bandwidth for the SNR measurement was the symbol rate. The gain of the channel filter was included in the SNR measurements.

For differences between the sender and receiver symbol rate of greater than 5%, the input signal was appropriatelly oversampled. For differences of the rates less than 5% the noisy, four times oversampled data sequence was resampled to the receiver sampling rate using Rick Behren's cubic spline [10]. Oversampling vs interpolation did not change the results.

The nonlinearities square, square and delay by half the symbol period $T_S/2$ and fourth power of the input signal were implemented without the bandpass in front of the nonlinearity. This bandpass could have destroyed the amplitude balance of the two-path channel. Then the worst case timing tone channel filter would have been difficult to identify.

The BETR was implemented for a carrier frequency 0. One arm shifted down, the other up by half the baudrate.

The cyclostationary shorttime cumulant was computed for $\hat{c}(0,0,0)$ and $\hat{c}(0,P/2,P/2)$, where P stands for the oversampling factor.

The timing tones corresponding to the different methods were modulated when necessary to give the difference timing tone. To simulate a sender/receiver rate difference of 0.1%, a record of 10^5 transmitter symbols and ten Hamming windowed FFTs of blocklength 9990 receiver symbols were computed. The ten FFT blocks were averaged by a median filter within $\pm$ twice the sender/receiver rate difference. The SNR of the timing tone was the difference in dB between the timing tone and the median average.

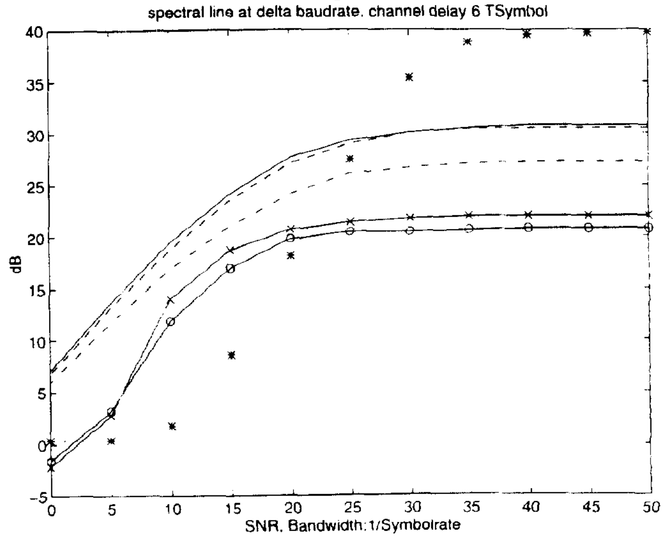

Figure 1: Timing Tone, Path Delay: 6 Symbols

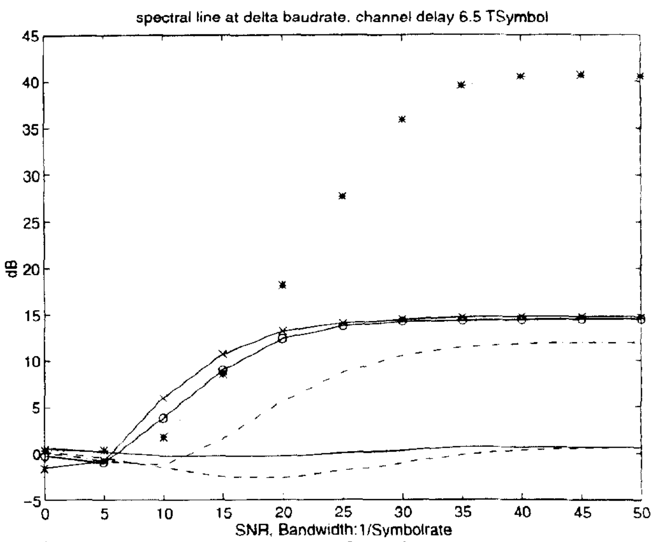

Figure 2: Timing Tone, Path Delay: 6.5 Symbols

Figure 1 gives the relationship of the timing tones for the easy timing tone generation, when the path delay is an integer (here 6) times the symbol period. For SNR less than 20dB the square (solid line), square delay (dashed line) and the fourth power (dash-dot line) outperform the shorttime cumulant $\hat{c}(0,1,1)$ (solid line, x-marked) and the shorttime cumulant $\hat{c}(0,$

0,0) (solid line, circled). The BETR (dotted line) posses the best SNR for signal SNRs in excess 25dB, while the ordering of the other methods stays the same.

Figure 2 gives the SNR for the timing tones for the difficult timing tone generation. The path delay is 6.5 times the symbol period. The square and squaredelay method (solid and dashed line) never give a timing tone, the fourth power gives some timing tone above 20 dB signal SNR.

For low SNR the cyclostationary methods are comparable to the BETR, which outshines all other methods for high SNR.

5 Blind Channel Estimation

A blind channel estimation algorithm using cyclostationary autocorrelation measurements and a subspace approach was introduced by Tong et. al. [2]. The limitations of the subspace approach was overcome by the authors of [1], [3] and [4]. The approach here uses the averaged autocorrelation $r_a(k)$ and the averaged cumulant slice $c_a(0,0,k)$ and their respective Z-transform

$$\begin{aligned} R(z) &= \sum_{k=-N}^{N} r_a(k) z^{-k}, \\ C(z) &= \sum_{k=-N}^{N} c_a(0,0,k) z^{-k}. \end{aligned}$$

In the noiseless case, these polynomials are connected

$$\begin{aligned} H(z) &= \sum_{n=0}^{N} h(n) z^{-n} \\ H_3(z) &= \sum_{n=0}^{N} h(n)|h(n)|^2 z^{-n} \\ R(z) &= H(z)H(z^{-1}) \\ C(z) &= H(z)H_3(z^{-1}) \end{aligned}$$

The two polynomials $R(z)$ and $C(z)$ enjoy the common polynomial $H(z)$. This is the channel impulse response.

The problem reduces to finding the common roots of $R(z)$ and $C(z)$. The Euclidean algorithm [9] is one fast way of doing that. Unfortunately the Euclidean algorithm is not very robust against errors in the estimation of $R(z)$ and $C(z)$. We propose an algorithm to find the common roots based on the Bezoutian [8].

The Sylvester matrix $S(R,C)$ has full rank iff the polynomials $R(z)$ and $C(z)$ are coprime [9]. The Bezoutian $B(R,C)$ has the same rank as the Sylvester matrix $S(R,C)$.

Let the $2N \times 2N$ upper triangular matrices R_U and C_U as well as the $2N \times 2N$ lower triangular matrices R_L and C_L, as well as the flip matrix $J = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$,

$$\begin{aligned} R_U &= \begin{pmatrix} r_{-N} & \dots & r_{N-1} \\ 0 & \ddots & \vdots \\ 0 & 0 & r_{-N} \end{pmatrix}, \\ R_L &= \begin{pmatrix} r_N & 0 & 0 \\ \vdots & \ddots & 0 \\ r_{-N+1} & \dots & r_N \end{pmatrix}, \\ C_U &= \begin{pmatrix} c_{-N} & \dots & c_{N-1} \\ 0 & \ddots & \vdots \\ 0 & 0 & c_{-N} \end{pmatrix}, \\ C_L &= \begin{pmatrix} c_N & 0 & 0 \\ \vdots & \ddots & 0 \\ c_{-N+1} & \dots & c_N \end{pmatrix}. \end{aligned}$$

Then the Sylvester and the Bezoutian matrix are connected by these triangular matrices.

$$\begin{aligned} S(R,C) &= \begin{pmatrix} R_U & R_L \\ C_U & C_L \end{pmatrix}, \\ B(R,C) &= J\left[C_L R_U - R_L C_U\right]. \end{aligned}$$

The dimension of the Bezoutian $B(R,C)$ is $2N$, the dimension of the Sylvester matrix $S(R,C)$ is $4N$.

If the polynomials $R(z)$ and $C(z)$ have a common factor $H(z)$ then there exists a Toeplitz matrix $D(H)$

$$D(H) = \begin{pmatrix} h(0) & \dots & h(N-1) & h(N) & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & h(0) & h(1) & \dots & h(N) \end{pmatrix},$$

$$\text{such that} \quad \mathrm{B(R,C) = D^*(H)\hat{B}D(H)}.$$

The dimension of $\hat{B}$ is N.

To compute the common factor of $R(z)$ and $C(z)$ we calculate a singular value decomposition of the Hermitian Bezoutian matrix $B(R,C) = U\ \Sigma U^*$. The nullspace U_N of the Bezoutian has to be orthogonal to the Toeplitz matrix $D(H)$. This gives $N \times N$ equations in the $N+1$ unknowns $h(n)$. One solution is to minimize the Frobenius norm of the matrix $||D(H)U_N||_F$. Since there are only $N+1$ unknowns, one can reformulate this problem into a minimal eigenvalue search involving the tensor V. One computes the tensor V from $U_N * U_N^*$ and the smoothing operator S. The smoothing operator $S(A,M)$ adds N numbers along the diagonals in $U_N * U_N^*$ starting at the position, where the tensor is to be computed.

Since the solution of the eigenvalue problem does not give the gain, one has to compute the gain of the channel impulse response $H(z)$ from the cumulant $c_a(0,0,0)$.

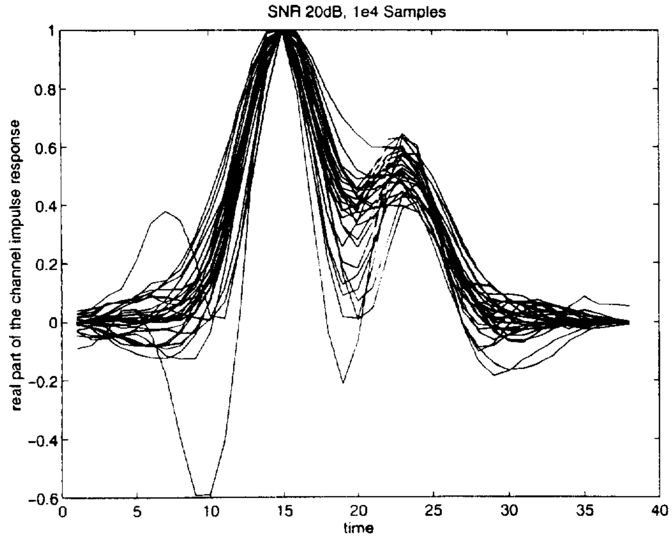

Figure 3: Real Part of the Estimated CIR

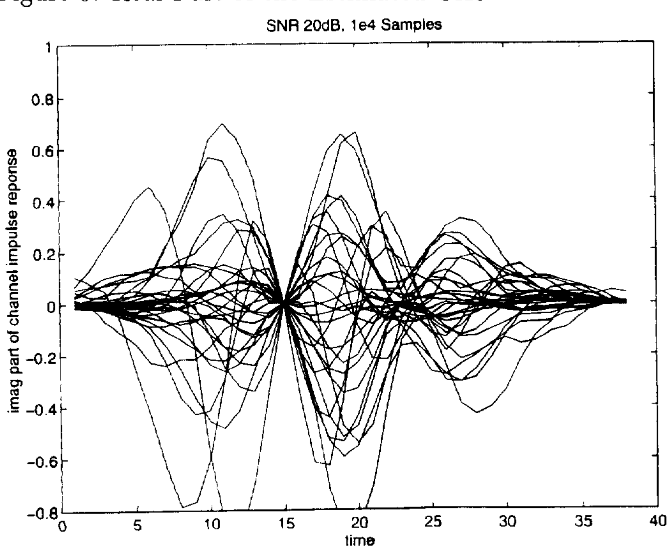

Figure 4: Imag.Part of the Estimated CIR

6 Channel Estimation Evaluation

To evaluate the porposed channel estimation method, the raised cosine (roll off $\frac{1}{2}$) four times oversampled QPSK signal was filtered by a two path channel with amplitudes 1 and $\frac{1}{2}$ and a path delay of two symbol periods T_S. The SNR of the signal was 20dB at the output of the channel filter. For the comparison the maximum of the estimated channel impulse response was scaled to 1 and shifted in time to sample number 15.

30 estimation trials were plotted on top of each other for a symbol block length of 2.5×10^3 in Figure 3 and 4. This algorithm will not give the absolute time delay within a symbol period. Thus the estimated channel impulse responses can shift by ± half a sample period. The real part of the estimated channel impulse response in Figure 3 shows two outliers from the target. The target of the imaginary part of the impulse responses is the x-axis. The imaginary part of the estimated channel impulse reponses fluctuate around the x-axis.

7 Conclusion

The cyclostationarity of oversampled digital communication signals was exploited to give new timing synchronization and equalization algorithms. The timing synchronization algorithm was compared to nonlinearities and BETR. In low SNR and possible equal amplitude two-path channels, the new synchronization algorithm offers an advantage.

A new blind channel identification algorithm based on the autocorrelation $r_a(k)$ and the averaged, cyclostationary cumulant slice $c_a(0,0,k)$ was introduced. A robust algorithm for finding the common roots in two polynomials was derived from the Bezoutian. First results of the algorithm for two-path channels were given.

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