Optimal Adaptive Equalization of Multipath Fading Channels

Venceslav Kafedziski

Department of Electrical Engineering
Arizona State University
Tempe, AZ, 85287-5706

Darryl Morrell

Department of Electrical Engineering
Arizona State University
Tempe, AZ, 85287-5706

Abstract

In this paper, we investigate optimal channel equalization techniques that incorporate a priori statistical information about multipath fading channels for mobile radio. We use a transformation from a physical ray model to a discrete-time finite impulse response model, along with statistical assumptions about the ray paths, to obtain a state-space model of the time evolution of the channel impulse response. This model is used with the Kalman filter to develop optimal channel estimators. We also implemented these estimators in a blind equalization scheme, based on MAP symbol-by-symbol detector. We present simulation results that characterize the performance of the proposed channel estimator and blind equalizer.

1 Introduction

We address the problem of estimation of multipath fading channels using a linear, time-varying channel model in which the channel is represented by a discrete-time finite impulse response filter. Currently, equalization algorithms based on this type of channel model use RLS or LMS filters as channel estimators, [1]; in the cases when a Kalman filter is used, a simple state-variable model of the channel dynamics, which does not reflect known statistical characteristics of real channels, is usually assumed. In this paper, we incorporate *a priori* knowledge of the channel statistics into a state variable model of the correlation structure and time dynamics of the channel impulse response; we use this state variable model to develop Kalman filter channel estimators. These estimators are included into a blind equalization scheme based on the MAP symbol-by-symbol detector [2], in which a bank of Kalman filters maintains channel estimates conditioned on all possible transmitted sequences. In Section 2 we describe the connection between a physical ray model of the mobile multipath fading channel, and a discrete time linear FIR channel model. In Section 3 we develop an AR model for the time evolution of the discrete channel coefficients. From this model we derive the state variable equations used in the Kalman filters. In Section 4 we briefly describe the blind equalization scheme of [2], which we use in conjunction with the state space model derived in 3. Computer simulations for both known and unknown data and conclusions follow in Sections 5 and 6, respectively.

2 Connection Between Physical Ray and Discrete Channel Model

In this section we derive the discrete model for a time varying channel, including the transmitter and receiver filter, which is then used in the MAP symbol-by-symbol detector. The transmitter filter has an impulse response $g(t)$, and the receiver filter has a fixed impulse response $g_r(t)$; the equivalent impulse response $g_e = g * g_r$ meets the first Nyquist criterion. The data signal is given by

$$I(t) = \sum_n d(n)\delta(t - nT) \qquad (1)$$

where $d(n)$ is a sequence from a complex signal set of size S. This data signal is transmitted through a dispersive medium, which is also time variant. The impulse response of the physical linear dispersive fading channel is expressed as the sum of M delayed Dirac pulses with time-varying complex path weights $c_m(t)$

$$c(\tau; t) = \sum_{m=1}^{M} c_m(t)\delta(\tau - \tau_m(t)) \qquad (2)$$

The equivalent channel impulse response is

$$h(\tau; t) = \sum_{m=1}^{M} c_m(t)g(\tau - \tau_m(t)) \qquad (3)$$

The signal at the output of the receiver filter is

$$y(t) = r(t) * g_r(t) = \sum_n d(n) h_e(\tau - nT, t) + u(t) \quad (4)$$

where $h_e = h * g_r$, and $u(t)$ is the response of the noise at the output of the receiver filter.

Next, we assume that the delays $\tau_m(t)$ corresponding to tap weights, $c_m(t)$ are short term stationary, i.e. $\tau_m(t) = \tau_m$. Following the approach in [3] we obtain an optimal MSE approximation $\hat{h}(\tau;t)$ of the time varying channel impulse response $h(\tau;t)$:

$$\hat{h}(\tau;t) = \sum_{\mu=-\infty}^{\infty} q_\mu(t) g(\tau - \mu T) \quad (5)$$

where

$$q_\mu(t) = \int_{-\infty}^{\infty} h(\tau;t) g(\tau - \mu T) d\tau = \sum_{m=1}^{M} c_m(t) \eta(\tau_m - \mu T) \quad (6)$$

and

$$\eta(\tau) = \int_{-\infty}^{\infty} g(\alpha - \tau) g(\alpha) d\alpha \quad (7)$$

Under the assumption of a Nyquist root-raised-cosine transmitter filtering there is no ISI under flat fading conditions:

$$\eta(\tau) = \begin{cases} \frac{\beta}{2} \sin\left(\frac{\pi}{2\beta}\right) & \text{if } |\tau| = \frac{1}{2\beta} T \\ \frac{\sin(\pi\tau/T)}{\pi\tau/T} \cdot \frac{\cos(\pi\beta\tau/T)}{1-4(\beta\tau/T)^2} & \text{otherwise} \end{cases} \quad (8)$$

where β is the bandwidth expansion factor. The coefficients $q_\mu(t)$ are Rayleigh (complex Gaussian) distributed random processes since they result from a linear combination of Rayleigh distributed tap weights $c_m(t)$. Using (5) in (4) we get:

$$y(t) = \sum_n d(n) \sum_\mu q_\mu(t) g_e(t - \mu T - nT) + u(t) \quad (9)$$

By sampling this signal at the symbol rate, we get:

$$y(kT) = \sum_n d(n) \sum_\mu q_\mu(kT) g_e(kT - nT - \mu T) + u(kT) \quad (10)$$

By using the change of variables $k - n \rightarrow n$ and the Nyquist property of the equivalent filter g_e we can write

$$y(kT) = \sum_n d(k-n) q_n(kT) + u(kT) \quad (11)$$

The noise samples $u(kT)$ are uncorrelated, because of the cosine-shaped receiver filter.

Assuming that the channel coefficients $q_n(kT)$ have significant power only for values of n from $-L_-$ to L_+ and dropping the T from the formulas above, we arrive at the equivalent discrete model of the communication system with a dispersive fading channel and fixed receiver filter, matched to the transmitter filter:

$$y(k) = \sum_{n=-L_-}^{L_+} q_n(k) d(k-n) + u(k) \quad (12)$$

A similar expression is derived in [4]. By introducing a proper delay we can describe the received signal by the following equation

$$y(k) = \sum_{n=0}^{N-1} b_n(k) d(k-n) + u(k) = \mathbf{d}(k)\mathbf{b}(k) + u(k) \quad (13)$$

where $N-1 = L_- + L_+$, $b_n(k) = q_{n-L_-}(k)$, $\mathbf{b}(k) = [b_0(k), b_1(k), ..., b_{N-1}(k)]'$ is a column vector of complex channel coefficients, and $\mathbf{d}(k) = [d(k), d(k-1), ..., d(k-N+1)]$ is a row vector of data.

3 AR Model for Channel Impulse Response Variation

In this section we derive a state space model for the variation of the channel coefficients $\mathbf{b}(k)$, which can be used by the Kalman filters for the purpose of channel estimation in the optimum MAP detector. The statistical behavior of the physical channel $c(\tau;t)$ is known from measurements or widely accepted specifications. In particular, we assume that ρ_m, the average path power, and τ_m, the path delay, are known for the M paths. Short-term channel variations may differ for each ray and are characterized by the WSSUS (wide-sense stationary uncorrelated scattering) scattering function

$$S(\tau; f_d) = \sum_{m=1}^{M} S_m(f_d) \delta(\tau - \tau_m) \quad (14)$$

i.e., the collection of Doppler spectra of individual path fading processes $c_m(t)$. The Fourier transform of $S(\tau; f_d)$ with respect to f_d yields the autocorrelation function

$$R(\tau;\Delta t) = E[c(\tau; t+\Delta t) c^*(\tau;t)] = \sum_{m=1}^{M} R_m(\Delta t) \delta(\tau - \tau_m(t)) \quad (15)$$

where $R_m(\Delta t) = E[c_m(t+\Delta t)c_m^*(t)]$ is the tap autocorrelation function, and $\rho_m = R_m(0)$ is the average tap power. For the mobile communication channel, the spectra $S_m(f_d)$ are modelled by the so-called Jakes spectrum

$$S(f_d) = \frac{1}{\pi f_{d_{max}} \sqrt{1-\left(\frac{f_d}{f_{d_{max}}}\right)^2}} \quad (16)$$

Then the tap autocorrelation function is given by $R_m(\Delta t) = \rho_m J_0(2\pi f_{d_{max}}\Delta t)$, where f_d is the maximum frequency in the Doppler spectrum, obtained as $f_{d_{max}} = v_{max}/cf_c$, f_c being the carrier frequency, v_{max} the maximum speed of the vehicle and c the speed of light. For the discrete model, the sampled autocorrelation is $R_m(l) = \rho_m J_0(2\pi l f_n)$ where $f_n = f_{d_{max}}/f_s$ is the maximum Doppler frequency normalized by the symbol rate f_s. The temporal cross correlation between any two processes $b_\mu(k)$ and $b_\nu(k)$ is

$$R_{\mu,\nu}(l) = E\{b_\mu(k+l)b_\nu^*(k)\} =$$
$$\sum_{m=1}^{M} R_m(l)\eta(\tau_m - (\mu - L_-)T)\eta(\tau_m - (\nu - L_-)T) \quad (17)$$

and these are the entries in the correlation matrix $\mathbf{R}(l) = E[\mathbf{b}(k+l)\mathbf{b}^H(k)]$. Notice that the matrices $\mathbf{R}(l)$ are scaled versions of $\mathbf{R}(0)$ with a scaling factor equal to $J_0(2\pi l f_n)$. The matrix $\mathbf{R}(0)$ depends only on the ρ_m and τ_m, $1 \le m \le M$, obtained from measurements.

We assume that the channel coefficients evolve according to the following vector p-th order autoregressive model

$$\mathbf{b}(k) = \sum_{i=1}^{p} \Phi_i \mathbf{b}(k-i) + \mathbf{w}(k) \quad (18)$$

where $\mathbf{w}(k)$ is the process noise with covariance matrix $\mathbf{Q} = E[\mathbf{w}(k)\mathbf{w}^H(k)]$. The matrices $\Phi_1,\ldots,\Phi_p$ can be determined from $\mathbf{R}(0),\mathbf{R}(1),\ldots,\mathbf{R}(p)$ by solving the system of matrix Yule-Walker equations [6]:

$$\sum_{i=1}^{p} \mathbf{R}(l-i)\Phi_i^H = \mathbf{R}(l), \quad \text{for} \quad l = 1,2,\ldots,p. \quad (19)$$

Once the Φ_i are determined, $\mathbf{Q}$ can be obtained as

$$\mathbf{Q} = \mathbf{R}(0) - \sum_{i=1}^{p} \mathbf{R}(-i)\Phi_i^H. \quad (20)$$

Next we transform the $p-$th order autoregressive model into a state space model

$$\mathbf{B}(k+1) = \mathbf{F}\mathbf{B}(k) + \mathbf{G}\mathbf{w}(k) \quad (21)$$

where the state vector is $\mathbf{B}(k) = [\mathbf{b}(k)', \ldots, \mathbf{b}(k-p)']'$. It can be shown that $Np \times Np$ matrix $\mathbf{F}$ and $Np \times N$ matrix $\mathbf{G}$ are given by

$$\mathbf{F} = \begin{bmatrix} \Phi_1 & \Phi_2 & . & . & \Phi_{p-1} & \Phi_p \\ \mathbf{I} & \mathbf{0} & . & . & \mathbf{0} & \mathbf{0} \\ . & . & . & . & . & . \\ \mathbf{0} & \mathbf{0} & . & . & \mathbf{I} & \mathbf{0} \end{bmatrix} \quad (22)$$

and $\mathbf{G} = (\mathbf{I}, \mathbf{0}, \ldots, \mathbf{0})'$, $\mathbf{I}$ and $\mathbf{0}$ being $N \times N$ matrices. The measurement equation now becomes

$$y(k) = \mathbf{H}_p(k)\mathbf{B}(k) + u(k) \quad (23)$$

where $\mathbf{H}_p(k) = [\mathbf{d}(k), 0, \ldots, 0]$ is $1 \times Np$ row vector. For the purpose of the fading simulator and Kalman filters we can decorrelate the processes $\mathbf{b}(k)$, using a unitary transformation $\mathbf{U}$, such that

$$\mathbf{R}(l) = E[\mathbf{b}(k+l)\mathbf{b}^H(k)] = \mathbf{U}\Lambda(l)\mathbf{U}' \quad (24)$$

The decorrelated processes are $\mathbf{p}(k) = \mathbf{U}'\mathbf{b}(k)$, and each $p_m(k)$ has power λ_m, where $\Lambda(0) = \text{diag}[\lambda_1, \ldots \lambda_N]$. We have shown that matrices Φ_i and $\mathbf{Q}$ can be diagonalized in the same fashion

$$\Phi_i = \mathbf{U}\Phi_{i_d}\mathbf{U}' \qquad \mathbf{Q} = \mathbf{U}\mathbf{Q}_d\mathbf{U}' \quad (25)$$

where $\Phi_{i_d} = a_i\mathbf{I}$ and a_i is the ith coefficient in the scalar AR-p model for the processes $p_m(k)$. $\mathbf{Q}_d = \rho\,\text{diag}[\lambda_1, \ldots \lambda_N]$, where ρ is the noise variance for the scalar AR-p process with unit power. $a_i, i = 1, \ldots, p$ and ρ are obtained from $J_0(2\pi l f_n)$, for $l = 0, 1, \ldots, p$.

Now, we transform the state equation (21) as

$$\mathbf{X}(k+1) = \mathbf{F}_d\mathbf{X}(k) + \mathbf{v}(k) \quad (26)$$

where $\mathbf{v}(k) = \mathbf{U}_p'\mathbf{G}\mathbf{w}(k)$, $\mathbf{X}(k) = \mathbf{U}_p'\mathbf{B}(k)$, and $\mathbf{F}_d$ is obtained from $\mathbf{F}$ by setting all Φ_i to Φ_{i_d}. $\mathbf{U}_p$ is a block diagonal matrix with the matrices $\mathbf{U}$ on the diagonal and is a unitary matrix. The covariance matrix of $\mathbf{v}(k)$ is a $Np \times Np$ block matrix with the only non-zero block in the left upper corner equal to $\mathbf{Q}_d$. We denote this matrix by $\mathbf{S}$. The measurement equation (23) becomes

$$y(k) = \mathbf{H}(k)\mathbf{X}(k) + u(k) \quad (27)$$

with $\mathbf{H}(k) = \mathbf{H}_p(k)\mathbf{U}_p = [\mathbf{d}(k)\mathbf{U}, 0, \ldots, 0]$.

4 Blind Equalization Algorithm

In this section, we briefly describe the blind deconvolution algorithm proposed by Iltis et al [2], where we also include the model of the multipath fading channel described in the previous sections. The cumulative measurements y^k represent the samples $\{y(k), y(k-1), \ldots, y(0)\}$. Denote the i-th of S^N possible data subsequences comprising the data symbols associated with the channel coefficient vector $\mathbf{b}(k)$ by $d_i^{k,N-1} = \{d_i(k), d_i(k-1), \ldots, d_i(k-N+1)\}$. Following the approach in [2], the conditional probability density function of the transformed channel coefficient vector, $\mathbf{X}(k)$ is approximated as complex Gaussian

$$p(\mathbf{X}(k)|d_i^{k,N-1}, y^{k-1}) = \mathcal{N}(\mathbf{X}(k); \hat{\mathbf{X}}_i(k|k-1), \mathbf{P}_i(k|k-1)) \tag{28}$$

where $\mathcal{N}(x, \hat{x}, P)$ represents a circular multivariate Gaussian density with mean vector $\hat{x}$ and covariance matrix P. The conditional pdf of the subsequence $d_i^{k,N-1}$ given the measurements, is given by:

$$p(d_i^{k,N-1}|y^k) = \frac{1}{c} p(y(k)|d_i^{k,N-1}, y^{k-1}) \sum_{j:d_j^{k-1,N-1} \in d_i^{k,N-1}} p(d_j^{k-1,N-1}|y^{k-1}) \tag{29}$$

where c is a normalization constant. The notation $d_j^{k-1,N-1} \in d_i^{k,N-1}$ means that the first $N-1$ symbols in subsequence $d_j^{k-1,N-1}$ equal the last $N-1$ symbols in $d_i^{k,N-1}$. Furthermore, under the approximation (28), the likelihood $p(y(k)|d_i^{k,N-1}, y^{k-1})$ is Gaussian:

$$p(y(k)|d_i^{k,N-1}, y^{k-1}) = \mathcal{N}(y(k); \hat{s}_i(k), \sigma_i^2(k|k-1)) \tag{30}$$

The mean and innovations covariance, $\hat{s}_i(k)$ and $\sigma_i^2(k|k-1)$, are determined as follows:

$$\hat{s}_i(k) = \mathbf{H}_i \hat{\mathbf{X}}(k|k-1) \tag{31}$$

$$\sigma_i^2(k|k-1) = \mathbf{H}_i(k)\mathbf{P}_i(k|k-1)\mathbf{H}_i^H(k) + \sigma_u^2 \tag{32}$$

where

$$\mathbf{H}_i(k) = [\mathbf{d}_i(k)\mathbf{U}, 0, \ldots, 0] \tag{33}$$

and

$$\mathbf{d}_i(k) = [d_i(k), d_i(k-1), \ldots, d_i(k-N+1)] \tag{34}$$

At this point, one step in the recursion (29) can be completed. However, the approximate predicted mean $\hat{\mathbf{X}}_i(k+1|k)$ and covariance $\mathbf{P}_i(k+1|k)$ must be computed for the next iteration. The predicted mean and covariance are completely determined by the mean vector $\hat{\mathbf{X}}_i(k|k)$ and covariance matrix $\mathbf{P}_i(k|k)$. These quantities are given by the conventional Kalman filter filtering equations. The new predicted estimate, $\hat{\mathbf{X}}_i(k+1|k)$, can be computed in terms of the measurement update as follows:

$$\hat{\mathbf{X}}_i(k+1|k) = \sum_{j:d_j^{k,N-1} \in d_i^{k+1,N-1}} \mathbf{F}_d \hat{\mathbf{X}}_j(k|k) \times \frac{p(d_j^{k,N-1}|y^k)}{\sum_{m:d_m^{k,N-1} \in d_i^{k+1,N-1}} p(d_m^{k,N-1}|y^k)} \tag{35}$$

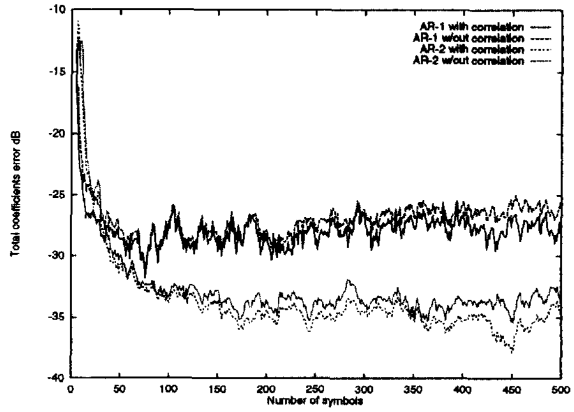

Figure 1: Total coefficient error with known data (SNR=15dB)

The blind equalizer structure consists of a bank of S^N conditional channel estimators. The symbol decisions are made using the MAP probability metrics:

$$\hat{d}(k-N+1) = arg \max_{d(k-N+1)} \sum_{d(k-N+2)} \cdots \sum_{d(k)} p(d_j^{k,N-1}|y^k) \tag{36}$$

5 Simulation Results

We have simulated the performance of this approach using data for a typical urban channel for the GSM mobile communication system. This channel is represented by 12 discrete but densely spaced physical rays. We start from the measured values of the power ρ_m and delay τ_m of these coefficients, and then compute the correlation matrix $\mathbf{R}(0)$, from which we determine the matrices $\Lambda(0)$ and $\mathbf{U}$. The time evolution is described by the matrices $\mathbf{\Phi}_{i_d}$ and $\mathbf{Q}_d$, which can be obtained as described in Section 3. The values of parameters are $\beta = 0.5$, $f_{d_{max}}/f_s = 0.001$.

The resulting FIR impulse response has four coefficients. The modulation procedure employed is QPSK. We simulated estimators created using both first and second order auto-regressive models of the channel dynamics. A Kalman filter was used as the estimator. Known and unknown data sequences were considered. Figure 1 shows the total magnitude squared error in the estimation of the channel coefficients for the estimators based on first and second order vector autoregressive models assuming that the channel coefficients are correlated or uncorrelated. The coefficient error plots were

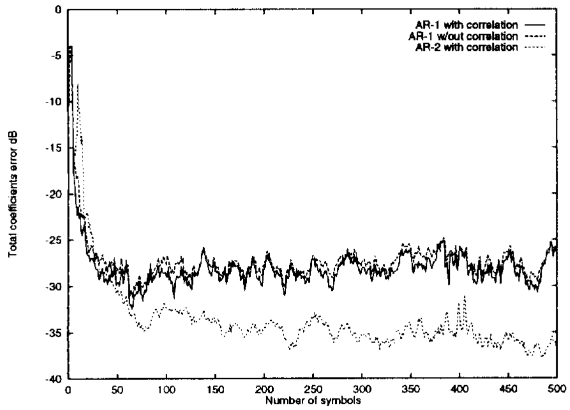

Figure 2: Total coefficient error with unknown data (SNR=15dB)

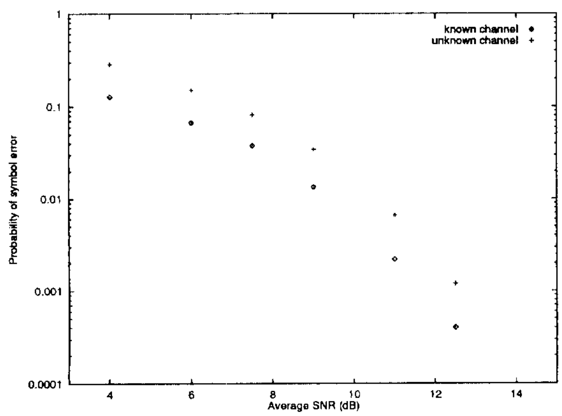

Figure 3: Probability of symbol error for AR-1 MAP detector

obtained by averaging over 10 independent runs. It can be seen that in the steady state, the estimator based on the second order model performs better than the estimator based on the first order model. Also, the estimator obtained using the correlation between coefficients in the channel impulse response shows better performance than the estimator based on the assumption of uncorrelated channel coefficients. We present the performance of the blind equalization scheme in Figure 2, in terms of the total error in the channel coefficients, and in terms of the probability of error in Figure 3, where AR-1 model is used. Because of the phase ambiguity for the blind equalization case, we used differentially encoded QPSK. Notice the difference of factor 2 due to differential encoding.

6 Conclusions

The channel estimator performance is improved by incorporating *a priori* statistical information about the channel coefficients into the estimator structure. Proper orthogonalization of the Kalman filter equations enables use of the correlation between coefficients for improved performance. Using a second order autoregressive model for the time evolution of the channel coefficients improves the steady state performance compared to the first order model. Proposed estimators can be incorporated in the blind equalization scheme based on the optimal MAP detector. Due to high complexity, we suggest the use of AR-1 model. We became aware of a related work [7, 8] during the course of our investigations.

References

[1] J.G. Proakis, "Adaptive Equalization for TDMA Digital Mobile Radio", *IEEE Transactions on Vecihular Technology*, Vol.40, No.2, pp. 333-341, 1991.

[2] R.Iltis, J.Shynk and K.Giridhar, "Bayesian Algorithms for Blind Equalization Using Parallel Adaptive Filtering" *IEEE Transactions on Communications*, Vol.42, No.2/3/4, pp. 1017-1032, 1994.

[3] S.Fechtel, "A Novel Approach to Modeling and Efficient Simulation of Frequency-Selective Fading Radio Channels", *IEEE Journal on Selected Areas in Communications*, Vol.11, No.3, pp. 422-431, 1993.

[4] P. Hoeher, "A Statistical Discrete-Time Model for the WSSUS Multipath Channel",*IEEE Trans. on Vecihular Technology*, Vol.41, pp. 461-468, 1992.

[5] R.A.Zigler and J.M.Cioffi, "Estimation of Time-Varying Digital Radio Channels", *IEEE Trans. on Vecihular Technology* Vol.41, pp.134-151, 1992.

[6] G. C. Reinsel, *Elements of Multivariate Time Series Analysis*, Springer-Verlag Publishers, 1993.

[7] Youjirou Ara and Haruo Ogiwara, "Adaptive Equalization of Selective Fading Channel Based on AR-Model of Channel-Impulse-Response Fluctuation", *Electronics and Communication in Japan*, Part 1, Vol.74, No.7, pp.76-86, 1991.

[8] Qingyuan Dai and Ed Shwedyk, "Detection of Bandlimited Signals Over Frequency Selective Rayleigh Fading Channels",*IEEE Transactions on Communications*, Vol.42, No.2/3/4, pp. 941-950, 1994.