Adaptive Equalization of a Digital Communications Channel with a Reduced-Order Equalizer

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Abstract

Extended Kalman Filters have been suggested for adaptive equalization of digital communication channels. They are designed from a state space structure that describes the dynamics of both the transmission of the digital signals as well as the properties of the channel. When the channel is modeled as a finite impulse response filter with many taps, the Extended Kalman Filter would be unacceptable for high speed operation due to the required number of calculations. In this paper we provide a reduced-order alternative, which can require fewer calculations because of its ability to estimate states of interest, while ignoring other states and at the same time giving performance that can be almost as good as the full-order Extended Kalman Filter.

1 Introduction

This paper deals with simultaneously recovering a message and estimating the channel in the presence of both noise and intersymbol interference. Extended Kalman Filters have been suggested to perform this function [1-3]. When the channel is modeled as a finite impulse response filter, the Extended Kalman Filter (EKF) can be designed to estimate both the channel coefficients as well as a smoothed version of the transmitted signals simultaneously. The effect of intersymbol interference varies depending on the length of the channel coefficient vector and the magnitudes of the vector's individual elements. Long channel coefficient vectors can increase the dimensionality such that the EKF would be unacceptable for high speed operation due to the required number of calculations. In this paper we provide a reduced-order alternative, which is derived from a modified form of the full-order state space and measurement equation. This Adaptive Reduced Order Equalizer (AROE) requires linearization about the current estimate as in the EKF [4] and has particular advantages when the channel output is sampled at a rate higher than the baud rate and/or there are multiple independent receivers. The reduced-order formulation allows for fewer calculations because it has the ability to estimate only the states of interest while ignoring the rest. Note that a reduced-order filter cannot outperform a full-order filter in terms of bit-error rate performance but when used appropriately the reduced-order filter can give acceptable performance while requiring less calculations. The theory will be illustrated with BPSK and QAM-16 signal constellations.

2 Full-order case

The full-order EKF is derived from a state-space dynamical model that describes both the transmission of symbols and the dynamics of the channel parameters:

$$x(j+1)=f[x(j)]+\tilde{B}w(j) \tag{1}$$

where

$$\begin{aligned} f[x(j)] &= \tilde{J}x(j)=\begin{bmatrix} J_{d\times d} & 0 \\ 0 & (1-\varepsilon)I_{d\times d}\end{bmatrix}x(j) \\ \tilde{B} &= \begin{bmatrix} \begin{bmatrix} B & 0_{d\times(d-1)}\end{bmatrix} & 0_{d\times d} \\ 0_{d\times d} & \varepsilon I_{d\times d}\end{bmatrix} \end{aligned} \tag{2}$$

In the above equation, $I_{d\times d}$ is an identity matrix of dimension *d*, and *d* is the number of coefficients in the channel's impulse response, with:

$$J=\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix} \qquad B=\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

In this problem, the measurement, $y(j+1)$, contains the, noise corrupted, output of the communications channel:

$$y(j+1)=h[x(j+1)]+v(j+1) \tag{3}$$

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where when $h[x(j+1))]$ is a row vector:

$$\begin{aligned} h[x(j+1)] &= \sum_{k=1}^{d} x_k(j+1)x_{k+d}(j+1) \\ &= [H(j+1) \quad Z_d]x(j+1) \end{aligned} \quad (4)$$

and

$$\begin{aligned} H(j+1) &= [h_{a0}(j+1) \quad h_{a1}(j+1) \quad \cdots \quad h_{ad}(j+1)] \\ Z_d &= [0 \quad 0 \quad \cdots \quad 0]_{1\times d} \end{aligned} \quad (5)$$

The elements of $H(j+1)$ correspond to the time-varying coefficients of the impulse response of the digital communications channel. Time variations in the channel are studied by including a deterministic and a non-deterministic parameter in the channel model:

$$H(j+1) = (1-\varepsilon)H(j) + \varepsilon w_h(j) \quad (6)$$

where $0<\varepsilon<1$ to maintain stability and $w_h(j)$ is a vector of uniformly distributed random parameters. Note that $w(j)$ represents the jth symbol in a sequence of transmitted digital symbols: $w=\{s_1,s_2,s_3,\cdots\}$, and v is the measurement noise. The transmitted symbols are further modeled as a white sequence with covariance matrix Q, and the measurement noise as white with a covariance matrix R. Using an Extended Kalman Filter (EKF), the jth symbol and channel estimates appear in an estimate of the state vector $x(j)$. The EKF has the following form:

$$\hat{x}(j+1) = F_f(j)\hat{x}(j) + K_f(j)y(j+1) + d_f(j) \quad (7)$$

where F_f, K_f, and d_f are selected by linearizing about the current state estimate and optimizing. In this paper, the optimization was done by minimizing the following performance measure:

$$J = E\left\{[x(j+1)-\hat{x}(j+1)]^T[x(j+1)-\hat{x}(j+1)]\right\} (8)$$

3 Reduced-order case

As stated previously, the advantage of a reduced-order filter is its ability to estimate states of interest. A corresponding discrete, reduced-order filter, that performs the same function as the full-order case, can be constructed by modifying the original model and measurement equation. The new model and measurement equation is:

$$x(j+1) = f_r[x(j)] + g[z(j)] + \tilde{B}_r w(j) \quad (9)$$

$$y(j) = h_r[x(j)] + c[z(j)] + v(j) \quad (10)$$

The vector functions g and c are used to model non-linear aspects which may arise in either the system or the system's output. These functions must each have first order derivatives for this method to be applicable. Note that z is a vector which contains at the very least all the states involved in non-linearities and also all states that the user desires to estimate. The term z is created with a selection matrix L as follows:

$$z(j) = Lx(j) \quad (11)$$

A reduced-order version of the EKF, therefore, estimates the vector z rather than the state vector x. The selection matrix L is a short fat matrix. It is desirable for L to have fewer rows than columns so that the order of the filter which estimates z will be smaller than the order of the filter which estimates x. Providing L with this structure has an advantage in that it can reduce the number of corresponding computations, it also has a disadvantage because it can place an upper limit on bit error rate performance that is below that of the full-order filter. Note that in this paper, the functions f_r and h_r will have the following forms:

$$\begin{aligned} f_r[x(j)] &= \tilde{J}_r x(j) \\ h_r[x(j)] &= C_r x(j) \end{aligned} \quad (12)$$

The reduced-order version of the EKF will be designed to have the following form:

$$\hat{z}(j+1) = F_r(j)\hat{z}(j) + K_r(j)y(j) + d_r(j) \quad (13)$$

where, similar to the full-order case, F_r, K_r, and d_r are optimized subject to the constraint that $E\{z-\hat{z}\}=0$, and the performance measure is:

$$J = E\left\{[z(j+1)-\hat{z}(j+1)]^T[z(j+1)-\hat{z}(j+1)]\right\} (14)$$

Note that this is the discrete case, a continuous-time version of this filter is derived in [5]. In any case, the discrete reduced-order EKF that we will consider will be unbiased. As shown in [4], this leads to the following rank requirement and matrix constraint:

$$
\begin{aligned}
rank\begin{bmatrix} L \\ C_r \end{bmatrix} &= n = \dim(x) \\
\begin{bmatrix} F_r & K_r \end{bmatrix}\begin{bmatrix} L \\ C_r \end{bmatrix} &= \begin{bmatrix} F_r & K_r \end{bmatrix} U = L\tilde{J}_r
\end{aligned}
\tag{15}
$$

At this point, n independent rows are selected from the matrix U and are called H_1, the corresponding columns of $[F_r \quad K_r]$ are called G_1. The remaining rows of U are then designated as H_2 and their corresponding columns from $[F_r \quad K_r]$ are called G_2. Following this, the matrix constraint can be written as:

$$
G_1 H_1 + G_2 H_2 = L\tilde{J}_r \tag{16}
$$

Since H_1 has been chosen to be invertible, it is possible to solve for G_1:

$$
G_1 = \left(L\tilde{J}_r - G_2 H_2\right) H_1^{-1} \tag{17}
$$

Now the matrices F_r and K_r can be obtained from G_1 and G_2 by simple selection matrices which pick out the proper columns. With this, F_r and K_r may now be expressed as:

$$
\begin{aligned}
F_r &= G_1 M_1 + G_2 M_2 \\
K_r &= G_1 N_1 + G_2 N_2
\end{aligned}
\tag{18}
$$

Matrices M_i and N_i are selection matrices of appropriate size which consist of 1's and 0's. Substituting equation (17) into (18), we finally express F_r and K_r in terms of the matrix G_2 of parameters free to be optimized:

$$
\begin{aligned}
F_r &= G_2 \Gamma_2 + \Gamma_1 \\
K_r &= G_2 \Omega_2 + \Omega_1
\end{aligned}
\tag{19}
$$

where Γ_i and Ω_i can be expressed as:

$$
\begin{aligned}
\Gamma_1 &= L\tilde{J}_r H_1^{-1} M_1 \\
\Gamma_2 &= M_2 - H_2 H_1^{-1} M_1 \\
\Omega_1 &= L\tilde{J}_r H_1^{-1} N_1 \\
\Omega_2 &= N_2 - H_2 H_1^{-1} N_1
\end{aligned}
\tag{20}
$$

Following this setup process, the algorithm for the AROE with the optimal G_2 is:

$$
\begin{aligned}
\hat{z}(0) &= LE\{x(0)\} \\
P(0) &= L\,\mathrm{var}\{x(0)\}L^T \\
R &= \mathrm{var}\{v\} \\
Q &= \mathrm{var}\{w\} \\
\Theta(j) &= \left.\frac{\partial c[z(j)]}{\partial z(j)}\right|_{z(j)=\hat{z}(j)} \\
\Phi(j) &= \left.\frac{\partial g[z(j)]}{\partial z(j)}\right|_{z(j)=\hat{z}(j)} \\
S_1(j) &= S_2(j)P(j)S_2^T(j) + \Omega_2 R \Omega_2^T \\
S_2(j) &= \Gamma_2 - \Omega_2\Theta(j) \\
S_3(j) &= \Omega_1\Theta(j) - L\Phi(j) - \Gamma_1 \\
S_4(j) &= \Omega_1 R \Omega_2^T \\
G_2(j) &= \left[S_3(j)P(j)S_2^T(j) - S_4(j)\right]S_1^{-1}(j) \\
F_r(j) &= G_2(j)\Gamma_2 + \Gamma_1 \\
K_r(j) &= G_2(j)\Omega_2 + \Omega_1 \\
P(j+1) &= \tilde{F}(j)P(j)\tilde{F}^T(j) + K_r(j)RK_r^T(j) + LBQB^T L^T \\
\tilde{F}(j) &= F_r(j) + L\Phi(j) - K_r(j)\Theta(j) \\
\hat{z}(j+1) &= F_r(j)\hat{z}(j) + Lg[\hat{z}(j)] + K_r(j)\{y(j) - c[\hat{z}(j)]\}
\end{aligned}
$$

The key to the operation of the above filter is calculation of the optimal gains by way of G_2. One notable difference between the AROE and a purely linear filter is the fact that the values for G_2 cannot be computed off line and then later used during the filtering process. This is due to the fact that both Θ and Φ must be continuously revaluated with each estimate of z.

The terms Θ and Φ are the result of the linearization process during derivation of the optimal G_2. They were required for Taylor series expansions of both g and c:

$$g[z(j)] = g[\hat{z}(j)] + \Phi[z(j) - \hat{z}(j)] + H.O.T$$
$$c[z(j)] = c[\hat{z}(j)] + \Theta[z(j) - \hat{z}(j)] + H.O.T \quad (21)$$

In this implementation, these expansions are truncated for all terms higher than first order. Using g and c to contain not only the states to be estimated but also the non-linearities is a convenient way to enable expansion of these functions of z about the current estimate of z. If the EKF model and measurement from equations (1) and (3) were used in deriving the AROE, then the linearizations would be problematic because they would require the expansion of functions of x while the only available estimates (which would be z) would be subsets of x.

Also, although not required in the derivation, during simulation the parameters of the dynamical model will be structured similar to those in equation (2):

$$\tilde{J}_r = \begin{bmatrix} J_{ds\times ds} & 0 \\ 0 & (1-\varepsilon)I_{dp\times dp} \end{bmatrix}$$
$$\tilde{B}_r = \begin{bmatrix} B_{ds\times 1} & 0 \\ 0 & \varepsilon\, I_{dp\times dp} \end{bmatrix} \quad (22)$$

Finally, it important to note that, aside from choosing states involved in non-linearities, the remaining states that are selected for estimation must meet two important conditions. The first and most important condition is that the rank requirement in equation (15) is met. The second condition is that even when the rank requirement is met, enough states must be estimated such that there are enough parameters left in G_2 to provide the desired performance.

4 Simulations

Simulations of the above full-order and reduced-order filters were carried out using both BPSK and QAM-16 signal constellations. In these simulations, the channel itself was arbitrarily selected to be an FIR filter with three coefficients and each filter samples the channel output at a rate which is equal to the symbol transmission rate. Further, in all cases, the channel varied in accordance with equation (6).

After traveling through the channel, the symbols are then corrupted with additive white gaussian noise. The Signal to Noise Ratio (*SNR*) is defined as:

$$SNR = 20\log_{10}\left(\sqrt{\frac{trace(HH^H)}{\sigma^2 N_r}}\right) \quad (23)$$

Where σ^2 is the covariance of the noise (the noise power) and N_r is the number of rows in the channel matrix.

The equalizers were initialized as follows. The process noise covariance matrix Q, was initialized to reflect both the covariance of the appropriate signal constellation, Q_s, and also the covariance of the random channel parameter variations, Q_c. During the simulations, the values of Q_s remained fixed, for BPSK, $Q_s = 1$, and for QAM-16, $Q_s = 10$. The measurement noise covariance, R, was simply initialized as the covariance of the noise power, which was determined after selecting the SNR. Also, in both cases, the mean square error matrix P was initialized such that the element associated with the first signal state $P_{1,1}$ had a relatively high value (i.e. $P_{1,1} = 1$), since its value could not be predicted, and the elements of P associated with random channel variation states had values corresponding to the variance of the non-deterministic channel components.

In the full-order case, $d=3$, so $n=6$. In the reduced-order case $ds=5$, $dp=1$, and so n=6. To get 5 signal states in the reduced-order case C_r contained shifted versions of the channels impulse response. Furthermore, in the reduced-order case, the first three signal states and the varying channel parameter state were chosen to be estimated. The corresponding L and C_r matrices were:

$$C_r = \begin{bmatrix} 1 & .1 & .8 & 0 & 0 \\ 0 & 1 & .1 & .8 & 0 \\ 0 & 0 & 1 & .1 & .8 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Results for the QAM-16 simulations are shown in Figures 1, 2, and 3. In all three cases, 1000 symbols were randomly generated, SNR=30dB and one channel coefficient was varied with $\varepsilon = .001$ and its additive random component had a variance of 1. Figure 4 shows results using the BPSK constellation with SNR=20dB. Figure 5 shows an example of the reduced-order filter tracking the time varying channel parameter.

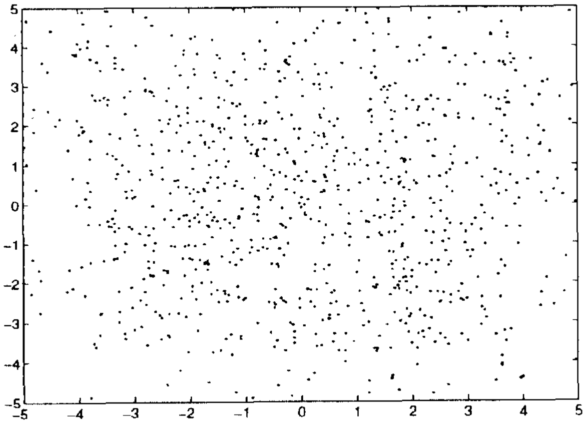

Figure 1: Unequalized channel output.

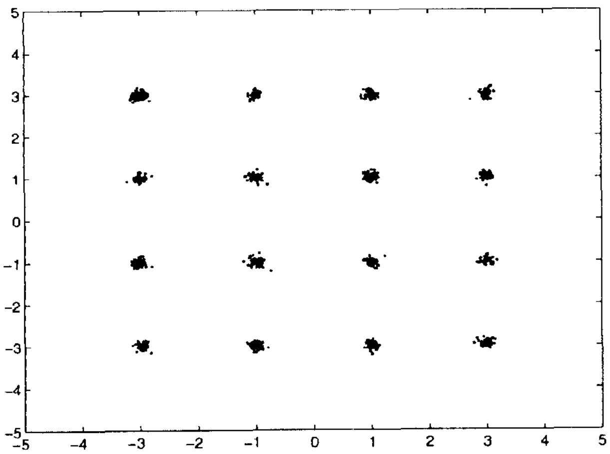

Figure 2: Full-order equalized output.

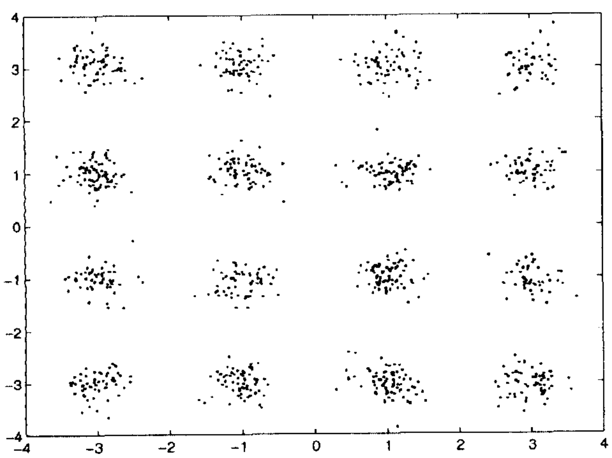

Figure 3: Reduced-order equalized output.

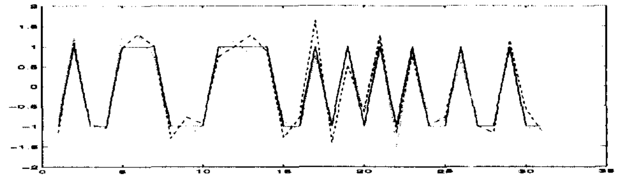

Figure 4: Binary signals with SNR=20dB, dashed=full order, dotted=reduced order, solid=true sequence.

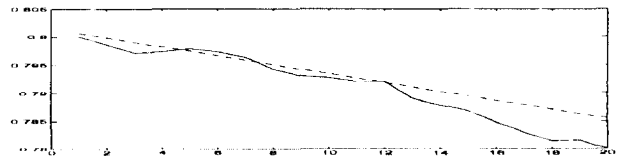

Figure 5: Channel coefficient tracking.

5 Conclusions

Both the full-order and reduced-order filters were able to track and equalize a time-varying channel when that channel consisted of both a deterministic and a non-deterministic time-varying component. Although the reduced-order implementation required a different formulation, reduced-order filters, in general, require less calculations than full-order filters. The penalty associated with reducing the order, however, is bit error rate performance degradation.

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