# DF Directed Multipath Equalization

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**Abstract**

*This paper addresses the problem of equalizing a (narrowband) multipath channel using data from an array of sensors. By exploiting the structure inherent in the data, it is shown that the number of multipath rays that can be uniquely identified is on the order of the number of data samples collected, rather than the number of sensors as in most direction finding applications. Two equalization algorithms are presented, both based on a frequency domain representation of the data and the availability of array calibration data. The performance of these algorithms is compared with that of several other recently developed equalization techniques, and is shown to be superior provided the calibration data is reasonably accurate.*

## 1. Introduction

In the multi-channel blind equalization problem, an unknown signal source (often a digitally modulated communications signal) is transmitted and then received via different channels by an array of sensors. In most situations, the channels are modeled as finite impulse response (FIR) filters with unknown coefficients. Using minimal assumptions about the transmitted signal (hence the term "blind"), the goal is to remove the distortion imposed by the FIR channels, combine the array outputs in some fashion (perhaps implicitly), and recover the transmitted signal or symbol sequence.

Recently, a number of promising blind, single channel equalization techniques have been proposed [1, 2, 3, 4, 5, 6]. These techniques have the advantage of requiring only second order cyclostationary statistics, and not higher-order moments. While they have typically been developed in the context of single channel *fractionally spaced* equalization, these algorithms can be readily reinterpreted as applying in the multichannel case [7, 8]. Under this interpretation, all of the above algorithms assume an unstructured FIR filter of known (or estimated) duration separating the symbol source and the samples of each sensor output. While this model is quite general, it was shown in [9] that if some structure can be imposed upon the channel (*e.g.*, a known pulse shaping filter at the source), a significant performance and computational advantage can result.

In this paper, we achieve similar results by imposing structure on each of the channels at the receiver array rather than the transmitter. In particular, we assume the received signals are narrowband relative to the array aperture, and explicitly incorporate into the model the fact that each channel consists of a sum of multipath rays arriving from various directions, combined together according to the calibrated array response for those directions [10]. Under this model, instead of FIR filter coefficients, the channel parameters are the directions of arrival (DOAs), complex amplitudes, and relative time delays of the multipath rays. Together with the samples of the transmitted signal, these parameters can be estimated by solving a least-squares minimization problem involving the array data in the frequency domain (as proposed, for example, in [11]).

Since this approach exploits the additional information provided by the array calibration data, it can yield significantly improved performance. However, since such data is often imprecisely known, there is a point where it does more harm than good. One objective of this paper is to examine what level of array perturbation is required to reach this "break even" point. Besides the potential performance advantage of this approach, it can also lead to a substantial computational savings, especially in situations involving relatively few multipaths with long delays. In such cases, the array based model provides a much more compact parameterization than the unstructured algorithms above, where the number of estimated parameters can be quite large (number of sensors times number of FIR filter taps).

We also show in this paper that, unlike standard DOA estimation problems, the array response can be exploited even when the number of multipath signals received by the array far exceeds the number of array elements. In particular, the number of resolvable multipath rays turns out to be on the order of the number of data samples collected, rather than the number of sensors. This fact is due of course to the additional structure that is imposed on the data model, but surprisingly does not depend on the availability of the array response data.

## 2. Data Model

We will assume an antenna array of $m$ sensors, having arbitrary positions and characteristics, that receive $d$ multipath "reflections" of a signal in the far-field of the array. The vector of complex sensor outputs is denoted $\mathbf{x}(t)$, and is modeled as

$$\mathbf{x}(t) = \begin{bmatrix} h_1(t) \\ \vdots \\ h_m(t) \end{bmatrix} * s(t) + \mathbf{n}(t) \ ,$$

where $*$ represents convolution, $s(t)$ is the transmitted signal (after pulse shaping, if any), $\mathbf{n}(t)$ is noise, $h_i(t)$ is the impulse response of the multipath channel between the source and the $i^{th}$ receiver given by

$$h_i(t) = \sum_{k=1}^{d} \alpha_{ik}\delta(t - \tau_{ik}) \ ,$$

where $\alpha_{ik}$ is a complex constant, $\tau_{ik}$ is the propagation delay between the source and the $i^{th}$ sensor for the $k^{th}$ multipath, and $\delta(t)$ is the Dirac delta function.

In the narrowband case where the array aperture is small enough to assume that the signal envelope does not vary appreciably as the wavefront traverses the array, we may write

$$h_i(t) = \sum_{k=1}^{d} a_i(\theta_k)\alpha_k\delta(t - \tau_k) \ ,$$

where $\theta_k$ is the DOA of the $k^{th}$ multipath, $a_i(\theta_k)$ is the response of the $i^{th}$ sensor in the direction $\theta_k$, and $\tau_k$ is measured with respect to some arbitrary reference point. This leads to

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta}) \left( \begin{bmatrix} 1 \\ \alpha_2\delta(t-\tau_2) \\ \vdots \\ \alpha_d\delta(t-\tau_d) \end{bmatrix} * s(t) \right) + \mathbf{n}(t) \ , \quad (1)$$

where without loss of generality we have assumed that the first sensor is the reference sensor ($\alpha_1 = 1, \tau_1 = 0$), and each column of

$$\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_d)] \ ,$$

$$\mathbf{a}(\theta_k) = [a_1(\theta_k), \cdots, a_m(\theta_k)]^T$$

is the array response to a unit signal arriving from the direction of one of the $d$ multipaths. We will assume throughout this paper that the array is *unambiguous*, or in other words that any collection of $m$ vectors $\mathbf{a}(\theta_i), i = 1, \cdots, m$ with distinct $\theta_i$ is linearly independent.

In the above model, the "channel" is split into two parts:

- multipath channel parametrized by $\alpha_k$, $\tau_k$, which are assumed to be unknown.
- "channel" due to propagation across the array, parametrized by $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_d]$ , assumed to be unknown, and the array manifold $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_d)]$ which is assumed to be known for a given $\boldsymbol{\theta}$.

The special case of perfectly coherent multipath reflections is handled by allowing the vector $\boldsymbol{\tau}$ to have elements that are non-distinct.

In the frequency domain, the dependence of the data model on the channel parameters $\alpha_k$, $\tau_k$, and $\theta_k$ is somewhat simpler:

$$\mathbf{x}_i = \mathbf{A}(\boldsymbol{\theta})\mathbf{h}_i s_i + \mathbf{n}_i \qquad i = 1, \cdots, N \ , \quad (2)$$

where $\mathbf{x}_i$, $s_i$, $\mathbf{n}_i$ are samples of the Fourier transforms $\mathbf{x}(\omega_i)$, $s(\omega_i)$, $\mathbf{n}(\omega_i)$,

$$\mathbf{h}_i = \mathbf{h}(\omega_i, \boldsymbol{\alpha}, \boldsymbol{\tau}) = \begin{pmatrix} 1 \\ \alpha_2 e^{-j\omega_i\tau_2} \\ \vdots \\ \alpha_d e^{-j\omega_i\tau_d} \end{pmatrix} \ , \quad (3)$$

and $\boldsymbol{\alpha}$ and $\boldsymbol{\tau}$ are vectors containing the parameters $\alpha_k$, $\tau_k$, respectively. All $N$ samples may be combined together in matrix form as

$$\mathbf{X} = \mathbf{A}(\boldsymbol{\theta})\mathbf{H}(\boldsymbol{\alpha}, \boldsymbol{\tau})\mathbf{S} + \mathbf{N} \ , \quad (4)$$

where $\mathbf{X} = [\mathbf{x}_1 \cdots \mathbf{x}_N], \mathbf{N} = [\mathbf{n}_1 \cdots \mathbf{n}_N], \mathbf{H}(\boldsymbol{\alpha}, \boldsymbol{\tau}) = [\mathbf{h}_1 \cdots \mathbf{h}_N], \mathbf{S} = \mathrm{diag}\{s_1 \cdots s_N\}$, and $\mathrm{diag}\{\cdot\}$ denotes the diagonal matrix formed from the elements of its argument.

## 3. Parameter Identifiability

The frequency domain model of (4) is said to be *identifiable* if a given set of noise free data $\mathbf{X}$ corresponds to a unique set of parameters $\boldsymbol{\theta}, \boldsymbol{\alpha}, \boldsymbol{\tau}, s_1, \cdots, s_N$. In this section, we present necessary and (generically) sufficient conditions for identifiability, and determine the maximum number of multipath signals that can be resolved. We note that, strictly speaking, a finite length Fourier transform of

$\mathbf{x}(t)$ will not yield a set of frequency domain data $\mathbf{X}$ that exactly satisfies (4), even if aliasing is eliminated. However, we will neglect such finite sample effects by implicitly assuming that $N$ is large enough to ensure

$$NT_s \gg \max|\tau_i - \tau_k| \quad i,k = 1,\cdots,d ,$$

where $T_s$ is the time period between array samples.

The issue of parameter identifiability for sensor array data involving *unstructured* signals has been addressed by a number of authors, perhaps most generally in [12] where the following conditions were derived:

- $d < (m+d')/2$ guarantees identifiability for *every* batch of data $\mathbf{X}$,
- $d < 2d'm/(2d'+1)$ guarantees identifiability for *almost every* $\mathbf{X}$ (or, in other words, with probability 1),

where $d'$ represents the rank of the sample covariance of the signals, and an unambiguous array has been assumed. In either case, the maximum number of resolvable signals is always less than the number of sensors $m$. A slightly weaker set of conditions was derived in [13] for the special case of a uniform linear array.

For the equalization problem considered herein, the identifiability of the channel parameters $\boldsymbol{\theta}, \boldsymbol{\alpha}$, and $\boldsymbol{\tau}$ is of much less importance than that of the signal samples $\mathbf{s} = [s_1, \cdots, s_N]$. As presented below, the signal samples can be identified to within an arbitrary complex scaling under much more general conditions than those given above. As a simple example of this idea, suppose in the model of (4) that $\tau_2 = \cdots = \tau_d = 0$ (in other words, the multipath signals are all perfectly coherent), so that $\mathbf{X}$ is rank one. Clearly, for any rank one factorization of the form $\mathbf{X} = \mathbf{u}\mathbf{v}^*$, we have $\mathbf{s} = \beta\mathbf{v}$ for some complex scalar $\beta$, regardless of $d$. However, from [12, 13], we know that $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$ can only be uniquely identified with probability 1 from $\mathbf{u}$ when $d < 2m/3$ (in such cases, $\mathbf{s}$ can be found exactly).

The purpose of this section is to show that $\mathbf{s}$ can be uniquely identified up to a complex scaling when $d > m$ for the more general case where some or all of the elements of $\boldsymbol{\tau}$ are distinct. Before addressing this problem in more detail, it is helpful to write the model of (4) in a slightly different form. Let $\bar{\boldsymbol{\tau}} = [\bar{\tau}_1, \cdots, \bar{\tau}_{d'}]$ be a vector that contains the $d' \leq d$ distinct elements of the vector $[1 \;\; \boldsymbol{\tau}]$. Then (4) may be rewritten (without noise) as

$$\mathbf{X} = \bar{\mathbf{A}}(\boldsymbol{\theta}, \boldsymbol{\alpha})\mathbf{V}(\bar{\boldsymbol{\tau}})\mathbf{S} , \tag{5}$$

where

$$\mathbf{V}(\bar{\boldsymbol{\tau}}) = \begin{bmatrix} e^{-j\omega_1\bar{\tau}_1} & \cdots & e^{-j\omega_N\bar{\tau}_1} \\ \vdots & \vdots & \vdots \\ e^{-j\omega_1\bar{\tau}_{d'}} & \cdots & e^{-j\omega_N\bar{\tau}_{d'}} \end{bmatrix} \tag{6}$$

$$\bar{\mathbf{A}}(\boldsymbol{\theta}, \boldsymbol{\alpha}) = [\bar{\mathbf{a}}_1, \cdots, \bar{\mathbf{a}}_{d'}] \tag{7}$$

$$\bar{\mathbf{a}}_k = \sum_{i=1}^{p_k} \alpha_{k_i}\mathbf{a}(\theta_{k_i}) , \tag{8}$$

and $p_k$ represents the number of elements of $\boldsymbol{\tau}$ that equal $\bar{\tau}_k$.

If $\omega_{i+1} - \omega_i$ is constant for all $i$ (*e.g.*, if $\omega_1, \cdots, \omega_N$ are DFT frequencies), then $\mathbf{V}$ is Vandermonde, and the result of [13] can be interpreted as follows by reversing the roles of the signals and array response: When $\mathbf{S} = \mathbf{I}$, the elements of $\bar{\mathbf{A}}$ and $\bar{\boldsymbol{\tau}}$ in (5) can be uniquely determined provided

$$d' \leq \frac{N + m' - 1}{2} ,$$

where $m' = \text{rank}\{\bar{\mathbf{A}}\} \leq \min\{d', m\}$. The following theorem can be thought of generalizing this result to guarantee the identifiability of an arbitrary diagonal $\mathbf{S}$ as well:

**Theorem** – Assume the model of (5) holds with an unambiguous array of $m \geq 2$ sensors and evenly spaced frequency samples. If

$$d' < \frac{N + m' - 1}{2} , \tag{9}$$

then with probability 1,

- $\bar{\boldsymbol{\tau}}$ can be uniquely determined and $\mathbf{s}$ can be uniquely determined to within a complex scaling, and
- $\theta_i$ and $\alpha_i$ can be uniquely determined provided that no more than $2m/3$ signals share the same delay $\tau_i$.

**Proof** – The proof is too lengthy to include here, but may be found in [14]. ■

The implications of the above theorem are many, the most obvious being that since typically $N \gg m$, the samples of the signal source and the DOAs of its multipaths can be determined even if the number of signals received by the array far exceeds the number of its sensors. Secondly, it is clear that several multipaths can share the same DOA, provided that the rank of $\bar{\mathbf{A}}$ is at least two. As an example, for the case where the elements of $\boldsymbol{\tau}$ are all distinct, up to $d-1$ of the multipath signals may arrive from the same direction and still be "resolved". As a final point, we note that since the array response $\bar{\mathbf{A}}$ is itself identifiable from the data, the availability of array calibration data is only needed for estimation of $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$, and not the transmitted signal.

## 4. Equalization Algorithms

For the algorithm presented below, it is assumed that the number of multipaths $d$ is known. When $m > d$, standard techniques could be used to estimate $d$ based on the rank of $\mathbf{X}$, but such techniques would not apply when $d > m$. However, we note that, as with unstructured FIR channel models, it is not critical to know the precise value of the model order provided it is overestimated. In such cases, the resulting extra filter coefficient or channel gain ($\alpha_k$) estimates would be zero (or very small in the case of noise).

### 4.1. A Least Squares Approach

In this approach, the parameters are estimated to achieve the best least squares (LS) fit between the data and the model of (2):

$$\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\tau}}, \hat{s}_i = \arg \min_{\theta,\alpha,\tau,s_i} \sum_{i=1}^{N} \|\mathbf{x}_i - \mathbf{A}(\boldsymbol{\theta})\mathbf{h}_i s_i\|_F^2 . \quad (10)$$

The estimate of $s_i$ is separable from the other parameters

$$s_i = (\mathbf{A}\mathbf{h}_i)^{\dagger}\mathbf{x}_i = (\mathbf{h}_i^*\mathbf{A}^*\mathbf{A}\mathbf{h}_i)^{-1}\mathbf{h}_i^*\mathbf{A}^*\mathbf{x}_i , \quad (11)$$

and substituting (11) into (10) yields the concentrated criterion function

$$\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\tau}} = \arg \min_{\theta,\alpha,\tau} \sum_{i=1}^{N} \mathbf{x}_i^*\mathbf{P}_{Ah_i}^{\perp}\mathbf{x}_i , \quad (12)$$

where $\mathbf{P}_{Ah_i}^{\perp} = \mathbf{I} - \mathbf{A}\mathbf{h}_i\mathbf{h}_i^*\mathbf{A}^*/(\mathbf{h}_i^*\mathbf{A}^*\mathbf{A}\mathbf{h}_i)$.

The minimization of (12) is highly non-linear, and clearly there exists no analytical solution. In general, (12) must be solved using some type of multidimensional search. When $m > d$, initial conditions for the non-linear search can be obtained using the algorithm described below.

### 4.2. A Suboptimal Approach

If we replace $\mathbf{h}_i s_i$ in (10) with an unstructured vector of parameters $\mathbf{u}_i$, then the following problem may be considered:

$$\hat{\boldsymbol{\theta}}, \hat{\mathbf{u}}_i = \arg \min_{\theta,u_i} \sum_{i=1}^{N} \|\mathbf{x}_i - \mathbf{A}(\boldsymbol{\theta})\mathbf{u}_i\|_F^2 .$$

The solution to this problem is standard, and is given by

$$\hat{\boldsymbol{\theta}} = \arg \min_{\theta} \mathrm{Tr}\left(\mathbf{P}_A^{\perp}(\boldsymbol{\theta})\hat{\mathbf{R}}_{xx}\right) , \quad (13)$$

$$\hat{\mathbf{u}}_i = \mathbf{A}^{\dagger}(\hat{\boldsymbol{\theta}})\mathbf{x}_i = \left[\mathbf{A}(\hat{\boldsymbol{\theta}})^*\mathbf{A}(\hat{\boldsymbol{\theta}})\right]^{-1}\mathbf{A}(\hat{\boldsymbol{\theta}})^*\mathbf{x}_i , \quad (14)$$

where $\mathbf{P}_A^{\perp}(\boldsymbol{\theta}) = \mathbf{I} - \mathbf{A}(\boldsymbol{\theta})\mathbf{A}^{\dagger}(\boldsymbol{\theta})$ and

$$\hat{\mathbf{R}}_{xx} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i\mathbf{x}_i^*$$

is the sample covariance matrix of the frequency domain data.

The estimate $\hat{\mathbf{u}}_i$ will of course be different from its true value $\mathbf{u}_i = \mathbf{h}_i\mathbf{s}_i$, but a second LS fit can be used to obtain estimate of $s_i$, $\alpha_k$, and $\tau_k$:

$$\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\tau}} = \arg \min_{\alpha,\tau} \sum_{i=1}^{N} \|\hat{\mathbf{u}}_i - \mathbf{h}_i s_i\|^2 = \arg \min_{\alpha,\tau} \sum_{i=1}^{N} \hat{\mathbf{u}}_i^*\mathbf{P}_{h_i}^{\perp}\hat{\mathbf{u}}_i \quad (15)$$

where $\mathbf{P}_{h_i}^{\perp} = \mathbf{I} - \mathbf{h}_i\mathbf{h}_i^*/(\mathbf{h}_i^*\mathbf{h}_i)$ and the estimate of $s_i$ is given by

$$\hat{s}_i = \hat{\mathbf{h}}_i^{\dagger}\hat{\mathbf{u}}_i = \hat{\mathbf{h}}_i^*\hat{\mathbf{u}}_i/(\hat{\mathbf{h}}_i^*\hat{\mathbf{h}}_i) , \quad (16)$$

where $\hat{\mathbf{h}}_i$ is obtained by substituting $\hat{\alpha}_k$, $\hat{\tau}_k$ into (3).

## 5. Simulation Examples

In this section, we include several simulation examples to demonstrate the advantage of using array calibration in equalization. In all examples, the multipath amplitudes were normalized at each trial to have a combined norm of unity (*i.e.*, $\sum|\alpha_i| = 1$). The source signals were assumed to have unity power.

In the first example, a miscalibrated four element $\lambda/2$ spaced uniform linear array (ULA) and a three-ray multipath channel were simulated. The channel input was BPSK with a random bit stream and 20dB SNR relative to the noise at the array, the DOAs of the received multipaths were 0°, 30°, 60°, and their respective delays 0, 1, and 2 symbols. The complex gains of each multipath were randomly varied from trial to trial. The signal ws generated with Nyquist pulse shaping and 35% excess bandwidth. A total of 64 snapshots (one sample per symbol) were used in each trial to perform the equalization, and Figure 1 gives a plot of the resulting root MSE for several algorithms versus the standard deviation of the random perturbation made to the nominal ULA response. A total of 100 trials were performed. The methods simulated were the algorithm described above (both the optimal and the suboptimal methods) ("Array method"), and

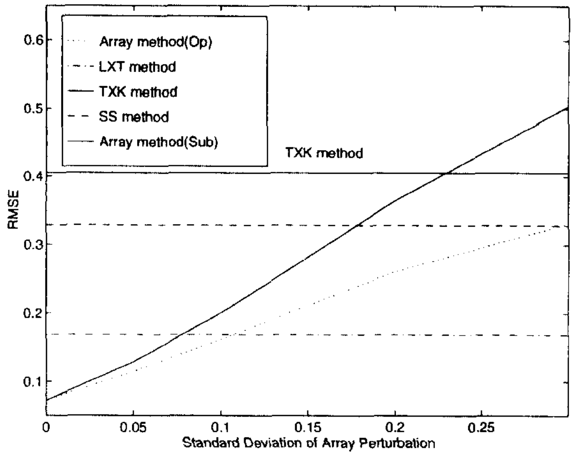


*Figure 1:* RMSE vs. Standard Deviation of Array Perturbation

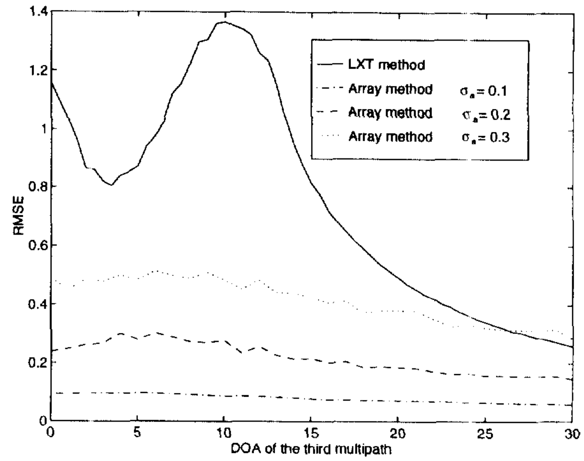


*Figure 2:* RMSE vs. DOA of Third Multipath

the algorithms of Tong *et al.* [7] ("TXK method"), Liu *et al.* [3] ("LXT method"), and Schell and Smith [4] ("SS method", $K = L = 0$). The array-based optimal method significantly outperforms the SS and TXK approaches, and was better than LXT provided the standard deviation of the array error ($\sigma_a$) was below 0.1, which corresponds to an array response with roughly a 10% error in gain and 6° in phase. The array-based suboptimal method performs close to the optimal method for $\sigma_a < 0.1$, but quickly degrades as $\sigma_a$ increases. This is because the suboptimal approach estimates the DOAs first, and the quality of the DOA estimates is sensitive to the inaccurate array calibration data.

The parameters of the second example were identical to the first, except that the DOA of the third multipath was varied from 0° to 30°, and the multipath amplitudes were fixed at 1, 0.4, and 0.9. Figure 2 gives a plot of the resulting root MSE of the LXT and array-based methods based on 500 trials. The array-based method significantly outperforms LXT method within this range of DOAs, even when the standard deviation of array calibration error $\sigma_a$ was as high as 0.3 (30% gain errors, 18° phase errors). Unlike the LXT method, the array-based method is insensitive to DOA separation, even when two multipaths arrive from the same direction.

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