# Adaptive Spatial Filtering for Co-Channel Interference Rejection in Narrowband Digital Cellular Systems

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## Abstract

The problem of adapting a spatial filter to reject co-channel interference experienced in narrowband digital cellular systems, and IS-54 in particular, is addressed. A two-part approach using blind adaptive spatial filtering and decision-directed adaptive spatial filtering is proposed to circumvent the difficulties associated with the use of training signals or schemes based on direction finding. A spatial filter, blindly adapted using the Phase SCORE algorithm, obtains an initial estimate of the symbol stream. Decision-directed adaptation then further refines this spatial filter to obtain a high-quality estimate of the desired symbol stream. Computer simulation results illustrate the successful operation of the new scheme.

## I Introduction

In cellular systems such as the US Digital Cellular standard IS-54, two motivations exist for using adaptive spatial filters to attenuate co-channel interference (CCI): (1) to maintain quality while reducing the frequency reuse distance and thus increasing overall capacity, or (2) to improve quality when propagation loss insufficiently attenuates CCI from other cells under the present scheme of frequency reuse.

Several well-known means for adapting spatial filters might be considered for this task. The IS-54 standard includes an embedded training sequence of 14 symbols in each 162-symbol TDM slot. However, the presence of CCI complicates the otherwise straightforward adaptive algorithm that minimizes the mean-squared error (MSE) between this training signal and the symbol-rate sampled spatial filter output. In particular, acquisition of symbol-clock synchronization for the desired signal, which is necessary prior to this adaptation process, is complicated by the CCI. Also, depending upon the number of antennas in the array and on the level of the CCI relative to the level of the desired signal, the 14-symbol training sequence might be insufficiently long to ensure reliable adaptation.

Another well-known class of algorithms is based on estimating the directions of arrival of the multiple received signals and then forming spatial filters for each. However, it is also well-known that these direction-finding (DF) algorithms (with the exception of the ESPRIT algorithm which needs an array having a doublet geometry) require very precise calibration data for the antenna array, which can be costly to obtain and impractical to update in the presence of component drift and aging, and incur substantial computational load in searching over this calibration data during the direction estimation process. An additional weakness of DF algorithms is their need to have accurate estimates of the number of received signals; this problem has been "solved" theoretically but the estimators can be unreliable in practice.

Yet another class of algorithms is based on Programmable Canonical Correlation Analysis (PCCA) [1, 2], of which the Spectral Coherence Restoral algorithms [3, 4] are members. These algorithms are well-suited to separating signals on the basis of the differing degrees to which they exhibit user-selectable statistical properties such as cyclostationarity, spectral support, temporal activity profiles, and so forth. However, the desired IS-54 signal and the CCI from nearby cells all exhibit these various properties to exactly the same degree, rendering almost all of the PCCA-based algorithms inapplicable.

Nonetheless, the closely related algorithm called Phase SCORE [5, 4] is able to separate uncorrelated signals of the same modulation type on the basis of their differing symbol-clock phases as explained in Section II. Since it is likely (either from purely probabilistic reasoning about randomly chosen phases or as a result of cooperative timing control among cells) that the desired signal and the CCI have different clock phases, Phase SCORE is a candidate algorithm for adapting the spatial filter. Figure 1 shows the corresponding receiver architecture. To improve this separation capability, the Phase-SCORE adaptive spa-



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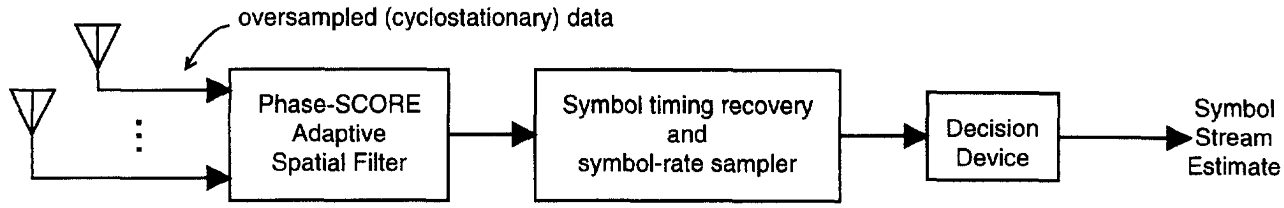


Figure 1: Block diagram of one-step blind adaptive spatial filter system.

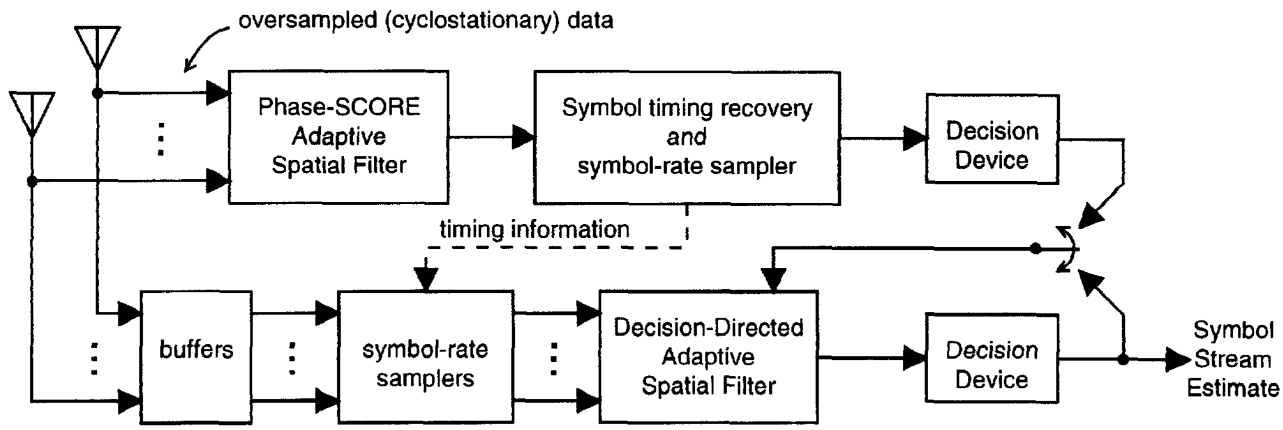


Figure 2: Block diagram of two-step blind adaptive spatial filter system. To initialize the decision-directed algorithm, the switch is placed in the upper position. For subsequent iterations, the switch is placed in the lower position. In all cases, the decision-directed algorithm adapts a spatial filter to operate on the buffered data (i.e., the same block of data processed by Phase SCORE during initialization).

tial filter is followed by a symbol-rate sampler and a decision-directed adaptive spatial filter, as shown in Figure 2. The symbol-clock timing acquisition can be done in a standard way here because the Phase-SCORE adaptive spatial filter has substantially attenuated the CCI.

Both conventional symbol-by-symbol decision direction and block decision-directed adaptation (with possibly multiple passes over the same data block) are considered in this paper. The principles of operation of the decision-directed portion of the two-step system are explained in Section III.

Finally, the performance of the one-step (no decision direction) and the two-step (decision direction) systems is illustrated by computer simulation results summarized in Section IV.

## II Phase SCORE

Although no derivation of the Phase SCORE algorithm as the solution to an optimization problem exists, its *unique ability to separate multiple co-channel signals having the same symbol rate on the basis of their differing symbol-clock phases*, without needing a training signal or array calibration data or particular array geometries, makes it a promising candidate for the first part of the two-part system described in this paper. The principles of operation are explained in this section.

Denote by $\mathbf{x}(n)$ the $M \times 1$ vector of sampled complex envelopes at the output of the antenna array,

$$\mathbf{x}(n) = \sum_{l=1}^{L} \mathbf{a}_l \, s_l(n) + \mathbf{i}(n) = \mathbf{A}\,\mathbf{s}(n) + \mathbf{i}(n),$$

where $s_1(n), \cdots, s_L(n)$ are the $L$ received signals that exhibit cyclostationarity at cycle frequency $\alpha$, $\mathbf{a}_l$ is the array response vector for signal $s_l(n)$, and $\mathbf{i}(n)$ is other interference and noise that do not exhibit cyclostationarity at cycle frequency $\alpha$. In particular, it is assumed that the cyclic autocorrelations obey the following relationships when $\alpha$ is equal to the symbol rate of the desired signal:

$$\begin{aligned}\mathbf{R}^{\alpha}_{\mathbf{ss}} &= \left\langle \mathbf{s}(n)\,\mathbf{s}^H(n)\,e^{-j2\pi\alpha n}\right\rangle \neq 0 \\ \mathbf{R}^{\alpha}_{\mathbf{ii}} &= \left\langle \mathbf{i}(n)\,\mathbf{i}^H(n)\,e^{-j2\pi\alpha n}\right\rangle = 0 \\ \mathbf{R}^{\alpha}_{\mathbf{xx}} &= \mathbf{A}\,\mathbf{R}^{\alpha}_{\mathbf{ss}}\,\mathbf{A}^H.\end{aligned}$$

In the application of interest in this paper, the vector $\mathbf{s}(n)$ contains the desired signal and the CCI from other users. All of these signals are $\pi/4$-shifted QPSK with the same same symbol rate and the same pulse shapes. Consequently, they all share the same cycle frequencies, one of which is the symbol rate. In the following it is assumed that $\alpha$ is set equal to the symbol rate.

The Phase SCORE algorithm finds $L$ different spatial filters by computing the $L$ dominant eigenvectors of

$$\mathbf{R}_{\mathbf{xx}}^{-1}\,\mathbf{R}_{\mathbf{xx}}^{\alpha}\,\mathbf{w}_l = \lambda_l\,\mathbf{w}_l, \qquad l = 1, \ldots, L,$$

where $|\lambda_1| \geq \cdots \geq |\lambda_L|$. This can be re-expressed as

$$\mathbf{W}_{mmse}\,\mathbf{R}_{\mathbf{ss}}^{-1}\,\mathbf{R}_{\mathbf{ss}}^{\alpha}\,\mathbf{A}^H\,\mathbf{w}_l = \lambda_l\,\mathbf{w}_l,$$

where $\mathbf{W}_{mmse} = \mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{A}\mathbf{R}_{\mathbf{ss}}$ is the collection of minimum mean-squared error (MMSE) spatial filters for the signals $\mathbf{s}(n)$. This shows that the Phase SCORE spatial filters, being linear combinations of the MMSE spatial filters, reject $\mathbf{i}(n)$ to the same degree. What remains is to specify the conditions under which they can separate the desired signal and CCI in $\mathbf{s}(n)$. To this end, premultiply both sides by $\mathbf{A}^H$ to obtain

$$\mathbf{G}_{mmse}\,\mathbf{R}_{\mathbf{ss}}^{-1}\,\mathbf{R}_{\mathbf{ss}}^{\alpha}\,\mathbf{G} = \mathbf{G}\Lambda,$$

where $\mathbf{G} = \mathbf{A}^H\mathbf{W}$ and $\mathbf{G}_{mmse} = \mathbf{A}^H\mathbf{W}_{mmse}$ are matrices of gains in each signal direction (indexed by row) obtained from each spatial filter (indexed by column). For meaningful performance by Phase SCORE, we must assume that the MMSE performance is good, in the sense that $\mathbf{G}_{mmse}$ is nearly diagonal. For simplicity of analysis, we first assume that $\mathbf{G}_{mmse}$ is exactly diagonal, and we then back off from this assumption using eigenvector perturbation theorems.

Assume that the signals are uncorrelated, such that $\mathbf{R}_{\mathbf{ss}}^{-1}\,\mathbf{R}_{\mathbf{ss}}^{\alpha}$ is diagonal. Assume further that $\mathbf{R}_{\mathbf{ss}}^{-1}\,\mathbf{R}_{\mathbf{ss}}^{\alpha}$ has distinct elements on the diagonal. These assumptions are met if $\mathbf{s}(n)$ contains co-channel IS-54 signals, from different users, having different symbol-clock phases. Under these assumptions and $\mathbf{G}_{mmse}$ being diagonal, then $\mathbf{G}_{mmse}\,\mathbf{R}_{\mathbf{ss}}^{-1}\,\mathbf{R}_{\mathbf{ss}}^{\alpha}$ is also diagonal with distinct diagonal elements. Consequently, each of its eigenvectors contains one non-zero element and $L-1$ zero elements, corresponding to perfect separation of one of the $L$ signals. Thus, each of the eigenvectors $\mathbf{W}$ of the original system separates a different signal.

In practice, $\mathbf{G}_{mmse}$ has small off-diagonal elements, and estimated correlation matrices are used. Then we have

$$\left[\mathbf{G}_{mmse}\,\mathbf{R}_{\mathbf{ss}}^{-1}\,\mathbf{R}_{\mathbf{ss}}^{\alpha} + \Delta\mathbf{R}\right]\,\check{\mathbf{G}} = \check{\mathbf{G}}\check{\Lambda},$$

where the two effects are subsumed into $\Delta\mathbf{R}$. Drawing upon results in [6], it can be shown that if $\Delta\mathbf{R}$ is small relative to $\mathbf{G}_{mmse}\,\mathbf{R}_{\mathbf{ss}}^{-1}\,\mathbf{R}_{\mathbf{ss}}^{\alpha}$, then the following approximation holds well:

$$\check{\mathbf{g}}_i \approx \mathbf{g}_i + \sum_{\substack{k=1\\k\neq i}}^{L} \mathbf{g}_k\,c_{ki},$$

where $c_{ki} = \mathbf{q}_k^T\,\Delta\mathbf{R}\,\mathbf{g}_i/(\lambda_i - \lambda_k)$ and $\mathbf{q}_k$ is the $k$th right eigenvector of $\mathbf{G}_{mmse}\,\mathbf{R}_{\mathbf{ss}}^{-1}\,\mathbf{R}_{\mathbf{ss}}^{\alpha}$ (i.e., $(\mathbf{G}_{mmse}\,\mathbf{R}_{\mathbf{ss}}^{-1}\,\mathbf{R}_{\mathbf{ss}}^{\alpha})^T\mathbf{q}_k = \lambda_k\mathbf{q}_k$).

Under the assumptions made above, it can be shown that the eigenvalues $\lambda_1, \cdots, \lambda_L$ are equal to the cyclic correlation coefficients $R_{s_k s_k}^{\alpha}/R_{s_k s_k}$, $k = 1, \ldots, L$ of the signals. With all $s_k(n)$ being IS-54 signals with identical pulse shapes, these eigenvalues have the same magnitude but distinct phases, provided that the corresponding signals have different symbol-clock phases. Thus, the perturbation formula above implies that the convergence time of Phase SCORE is larger if the symbol-clock phases are similar (because this reduces the smallest distance between eigenvalues) than it is if they are dissimilar.

In conclusion, with infinite time-averaging and some conditions on the statistics of the IS-54 signals, Phase SCORE provides the same performance as the optimal MMSE spatial filters. With finite time-averaging, the perturbation results argue for the plausibility of Phase SCORE. More definitive performance results from computer simulations are presented in Section 4.

## II.A An Improved Version

In the preceding development of Phase SCORE, the autocorrelation matrix $\mathbf{R}_{\mathbf{xx}}$ is assumed to be well-conditioned. In practice, the eigenvalue spread of the estimated $\mathbf{R}_{\mathbf{xx}}$ can be large enough that perturbations due to noise and finite time-averaging can degrade substantially the quality of the inverse. In these cases, it is well-known that if $\mathbf{R}_{\mathbf{ii}}$ is diagonal then the quality of $\mathbf{W}_{mmse}$ can be improved by replacing $\mathbf{R}_{\mathbf{xx}}^{-1}$ with its reduced-rank pseudo-inverse. From the perspective of principal components analysis in multivariate statistics, only the $L$ principal components (dominant eigenvectors) should be used. The corresponding version of Phase SCORE was first proposed by Biedka [7], referred to here as Principal Components Phase SCORE (PCP-SCORE), and is implemented as

$$\mathbf{U}_L\Sigma_L^{-1}\mathbf{U}_L^H\,\mathbf{R}_{\mathbf{xx}}^{\alpha}\,\mathbf{w}_l = \lambda_l\,\mathbf{w}_l,$$

where $\mathbf{U}_L$ and $\Sigma_L$ are constructed from the $L$ most dominant eigenvectors and eigenvalues, respectively, of $\mathbf{R}_{\mathbf{xx}}$.

## II.B Selecting the Desired Signal

Although Phase SCORE finds $L$ spatial filters, one for each signal, in practice we would be interested in selecting only one of these signals, namely the one originating in the correct cell. This selection could be performed easily by using the prior knowledge of the 14-symbol training sequence that is unique to each cell, although this necessitates the addition of $L-1$ parallel branches (to perform symbol timing recovery and symbol-rate sampling), followed by this selector, to the architecture in Figure 1.

# III Decision-Directed Adaptation

Provided that a high-quality, estimated symbol stream exists, decision-directed adaptation (DDA) can be used to improve the spatial filter found by Phase-SCORE or PCP-SCORE. Since DDA is a well-known technique in communication systems, only a summary description is provided here.

Let $\mathbf{y}(k)$ denote the symbol-rate sampled received signals, assuming that the symbol-timing recovery has been successful. This is reasonable in the two-part scheme of this paper when the output of Phase-SCORE or PCP-SCORE is high-quality, in which case conventional data-independent symbol-timing recovery methods can be used; alternatively, the output of the subsequent decision device can be used to solve a straightforward least-squares optimization problem to determine the proper symbol-clock phase.

Based on the discussion in Section II.B, we assume that the selection process is successful at determining which of the $L$ weight vectors found by Phase SCORE (or PCP-SCORE) corresponds to the desired signal, denoted by $s_1(k)$.

To maximize performance, multiple passes over the same data block can be made. Denote by $\tilde{s}_1^0(k)$ the symbol stream output of the decision device in the upper branch of Figure 2. After the $i$th DDA iteration, denote by $\tilde{s}_1^i(k)$ the symbol stream output of the decision device in the lower branch of Figure 2.

For the block DDA method, denote by $\mathbf{c}^i$ the vector of spatial-filter weights that are found after the $i$th DDA iteration to obtain the pre-decision estimate of the symbol stream defined by $\hat{s}_1^i(k) = \left(\mathbf{c}^i\right)^H \mathbf{y}(k)$. Then the $i$th DDA iteration attempts to minimize $\left\langle \left|e^i(k)\right|^2 \right\rangle$ where $e^i(k) = \hat{s}_1^i(k) - \tilde{s}_1^{i-1}(k)$, by computing

$$\mathbf{c}^i = \mathbf{R}_{\mathbf{yy}}^{-1} \mathbf{R}_{\mathbf{y}\tilde{s}_1^{i-1}}.$$

For the LMS-style (steepest descent, noisy gradient) method, the spatial filter $\mathbf{c}^i(k)$ is also indexed by time $k$, and is found by computing

$$\mathbf{c}^i(k+1) = \mathbf{c}^i(k) + \mu\, \mathbf{y}(k) \left(e^i(k)\right)^*.$$

The corresponding pre-decision symbol stream used to compute $e^i(k)$ and $\tilde{s}_1^i(k)$ is given by $\hat{s}_1^i(k) = \left(\mathbf{c}^i(k)\right)^H \mathbf{y}(k)$.

The performances obtained by the block DDA and LMS DDA are compared in Section IV.

# IV Results of Computer Simulations

In the simulations, two IS-54 signals ($\pi/4$-shifted DQPSK with square-root Nyquist-shaped pulses with 35% rolloff) arrive from -10 degrees and 30 degrees, respectively, in the presence of spatio-temporally white noise at a four-element uniform linear array having half-wavelength antenna spacing. The signals are oversampled at four samples per symbol, and the Phase SCORE and PCP-SCORE algorithms operate using $\alpha = 1/4$. The PCP Phase SCORE algorithm is provided with prior knowledge that two signals are present. The relative delay between the symbol clocks of the two signals is one sample. As will be seen in the simulation results, this is sufficient phase separation for Phase SCORE and PCP-SCORE to distinguish between the signals.

In the first set of simulations, the inband SNR for each of the two signals is 15 dB, comparable to a typical inband SNR for IS-54 applications, and all 160 symbols of the IS-54 TDM slot are collected for use by the processors. One hundred independent trials were performed, and the number of decision errors after each iteration was recorded for each of the following three combinations: Phase SCORE with block DDA, Phase SCORE with LMS DDA, and PCP-SCORE with block DDA. These results are shown in Figures 3-5. It can be seen that 92% of the trials of Phase SCORE with block DDA converge to zero bit errors (out of 320 bits) after one iteration, 98% within 3 iterations, and 2% failed to converge after 10 iterations. In contrast to the authors' expectations, the LMS DDA outperformed the block DDA, with 96% of the trials converging to zero bit errors after one iteration, and 100% within two iterations. In stark contrast to Phase SCORE, PCP-SCORE did not require any DDA to achieve zero bit errors in 100% of the trials.

In the second set of simulations, the inband SNR is reduced to only 10 dB, and the data collection time is reduced to only 40 symbols, such as might be needed to accommodate very quickly varying channels, and 400 trials are performed. Due to space limitations, the results are only stated, not depicted. PCP-SCORE with no iteration achieves zero bit errors in 94% of the trials, and one iteration of LMS DDA improves that to 100% of the trials. Similar results were obtained with PCP-SCORE using block DDA. In contrast Phase SCORE with either block DDA or LMS DDA fails to yield acceptable results.

# V Conclusions

In this paper we have demonstrated two alternative receiver architectures that use blind adaptive spatial filtering to reject CCI in an IS-54 system. Significantly, the simulation results imply that the substantially simpler non-iterative architecture of Figure 1 could be used instead of the iterative one of Figure 2 for collection intervals of 160 symbols. With only 40 symbols, only one pass of the DDA is needed when PCP-SCORE is used. However, the authors would like to emphasize that these simulation results are preliminary, particularly in light of the unexplained superiority of the LMS DDA over the block DDA; a more complete evaluation of the performances of Phase SCORE assisted by DDA and PCP-SCORE (perhaps without DDA), is needed to draw firmer conclusions.

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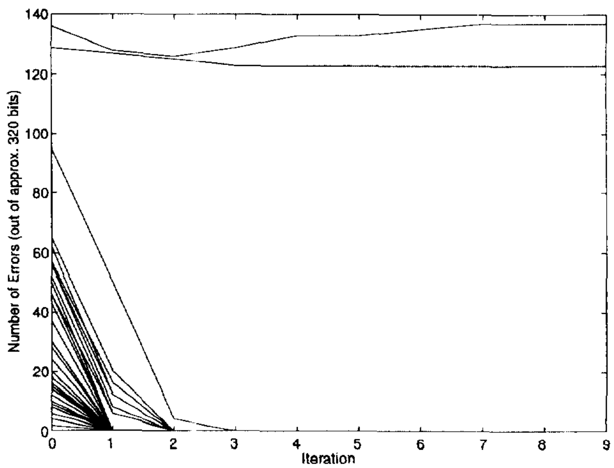


Figure 3: Family of curves, one per trial, showing the number of bit errors obtained after each block DDA iteration using Phase SCORE to initialize the symbol stream used in iteration 1.

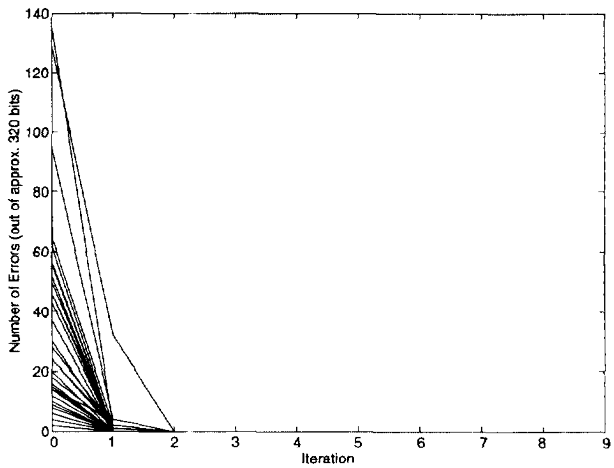


Figure 4: As in Fig. 3, but using LMS DDA.

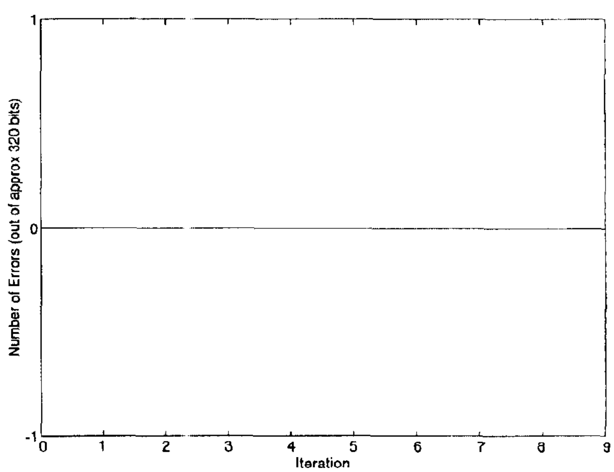


Figure 5: As in Fig. 3, but using PCP-SCORE to initialize. No bit errors were detected.