RECEIVERS WITH IMPROVED PERFORMANCE IN THE PRESENCE OF CO-CHANNEL INTERFERENCE BASED ON THE WAVELET TRANSFORM

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ABSTRACT

In this paper, we present the Co-channel Interference Mitigation in the Time-Scale domain (CIMTS) algorithm which estimates the SOI and the interfering signal from their superposition in the presence of additive noise. This method is inspired by the reconstruction of the interference from the null space of the SOI in the time-scale domain. Once the null space of the SOI is determined, the interfering signal is reconstructed via a set of linear operations. Then, the SOI is estimated by a simple subtraction of the estimated interference from the observations.

1. INTRODUCTION

The subject of co-channel interference reduction has received much attention for many years [1] - [7]. In such systems as mobile communications, radio networks and radar, the problem of co-channel interference is encountered when along with the signal of interest (SOI), one or more interfering signals are present in a common receiver. It is known that the co-channel interference often degrades the SOI more severely than the additive noise or intersymbol interference.

Usually, we are only interested in recovering the SOI, but in certain applications, such as multi-access communication systems [2] - [4], we desire to recover all the signals. The objective of this research is to recover the SOI and the interfering signal(s) from the observation of their superposition which is embedded in additive noise. The SOI and the interference(s) possess similar characteristics and share the same region of support both in time and frequency domains.

The performance of the interference reduction systems, which treat the interference as white additive noise, will degrade significantly in the presence of co-channel interference. The methodology using the matched filters is sub-optimum when the interfering signal is correlated to the SOI. In [8], the filter banks are utilized to suppress the narrow-band interference when the SOI is wideband. In this particular method, the wideband SOI is recovered after nulling the narrowband interference in each subband. The method is not suitable when the SOI and the interference are wideband. Giridhar *et al* [5] utilize nonlinear co-channel demodulation methods based on maximum-likelihood sequence estimation and maximum *a posterior* symbol detection to jointly recover the SOI and the interference. A higher performance is achieved by Giridhar's method; however, their method is computationally very expensive.

In this paper, a new method for co-channel interference mitigation is introduced and analyzed. The methodology is based on reconstructing the interference from the null space of the SOI in the time-scale domain. The Wavelet Transform (WT) has several properties which make it more attractive than time-frequency distributions for this particular problem. Two of the most important properties are the linearity and low complexity (for Discrete Wavelet Transform (DWT).)

In this paper, we present the Co-channel Interference Mitigation in the Time-Scale domain (CIMTS) algorithm which estimates the SOI and the interfering signal from their superposition in the presence of additive noise. This method is inspired by the reconstruction of the interference from the null space of the SOI in the time-scale domain. The null space of the SOI is used instead of the null space of the interference since it is easier to identify. Once the null space of the SOI is determined, the interfering signal is reconstructed via a set of linear operations. Then, the SOI is estimated by a simple subtraction of the estimated interference from the observations.

In section 2, the CIMTS algorithm and its different versions are presented. The complexity and performance of these algorithms are investigated respectively in sections 3 and 4.

2. CO-CHANNEL INTERFERENCE MITIGATION IN THE TIME-SCALE DOMAIN: THE CIMTS ALGORITHM

This work assumes that only one interfering signal is present; however, the results may easily be extended for the case of many interfering signals. The problem is formulated as

$$S(t) = S_1(t) + S_2(t) + w(t) \tag{1}$$

where $S(t)$ is the observation, $S_1(t)$ is the SOI, $S_2(t)$ is the interference, and $w(t)$ is the noise. The SOI and the interference are given respectively by,

$$S_1(t) = A_1 \sum_{k=0}^{L} \exp(j\theta_k)\chi_{[kT_1,(k+1)T_1]}(t) \tag{2}$$

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$$S_2(t) = A_2 \sum_{k=0}^{L} \exp(j(2\pi\delta f t + \phi_k))\chi_{[kT_2,(k+1)T_2]}(t) \quad (3)$$

where the signal $S_1(t)$ is baseband and the signal $S_2(t)$ has a very small unknown modulation frequency δf, $1/\delta f << T_2$. It is assumed that the symbol duration of the signals $S_1(t)$ and $S_2(t)$ are given and are respectively equal to T_1 and T_2, where $T_2 = T_1 + \delta T$ and δT is small [9].

The problem is to recover the SOI, $S_1(t)$ in the time-scale domain. This can be done by recovering the interference $S_2(t)$ first, and then subtracting it from $S(t)$. The following theorem can be utilized to identify the null space of a baseband PSK signal. This is a fundamental theorem as it serves as the basis of the new CIMTS algorithm.

Fundamental Theorem : *The Haar DWT of a baseband MPSK signal, $WT(n,b)$, is zero at scale $b = T_1/L$, and n and L any integers,*

$$WT_{s(t)}(n, T_1/L) = \int S_1(t)\psi(\frac{t}{T_1/L} - n)dt = 0 \quad (4)$$

where $S_1(t)$ is a baseband MPSK signal with symbol duration T_1.

As a result of the fundamental Theorem, we can establish the following fundamental proposition.

Fundamental Proposition: *There exist a linear relation between the sampled interfering signal and the Haar DWT of the observation at scale $b = T_1/L$,*

$$\underline{WT_{S(t)}}(T_1/L) = \mathbf{A}^{T_1/L}\underline{S}, \quad (5)$$

where T_1 is the symbol duration of the SOI, L is an arbitrary integer, $\underline{S}$ is the vector of the sampled interference signal, and

$$\underline{WT_{s(t)}}(T_1/L) = \begin{bmatrix} WT_S(t)(0, T_1/L) \\ WT_S(t)(1, T_1/L) \\ \cdots \\ WT_S(t)(N-1, T_1/L) \end{bmatrix} \quad (6)$$

The matrix $\mathbf{A}^{\frac{T_1}{L}}$ is expressed as,

$$\mathbf{A}^{\frac{T_1}{L}} = \begin{bmatrix} a_0^0 & a_0^1 & a_0^2 & \cdots & a_0^{N-1} \\ a_1^0 & a_1^1 & a_1^2 & \cdots & a_1^{N-1} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{N-1}^0 & a_{N-1}^1 & a_{N-1}^2 & \cdots & a_{N-1}^{N-1} \end{bmatrix} \quad (7)$$

where

$$a_n^k = \int \chi_{[k\frac{T_2}{L},(k+1)\frac{T_2}{L}]}(t)\psi(\frac{L}{T_1}t - n)dt. \quad (8)$$

The fundamental proposition is then used to develop the CIMTS algorithm. Different versions of the CIMTS algorithm are summarized in Table (1). The CIMTS (V1) algorithm assumes that the modulation frequency of the interference, δf, is small but unknown. This technique utilizes only one scale of the WT. Using Eq.(5) the interfering signal is reconstructed via the Singular Value Decomposition (SVD) [10]. If the modulation frequency of the interfering signal is zero then the results in the fundamental proposition are exact. We refer to this case as the CIMTS (V1.2) algorithm. However, when an approximation is made due to the *small* frequency-offset of the interfering signal, the algorithm will be denoted as CIMTS (V1.1).

The CIMTS (V2) algorithm is similar to the CIMTS (V1) algorithm except for the assumption that δf is known. The matrix $\mathbf{A}$ is calculated exactly.

In CIMTS (V3), we employ more than one scale of the WT to reconstruct the signals and we assume that the modulation frequency offset of the interference is unknown. The estimation algorithm will improve by utilizing all possible values of L. For higher modulation frequencies (higher δf), it is necessary to use values of $L > 1$, since the approximation of the signal to constant, in the interval of integration will be more accurate.

In all the above methods, once the interference is reconstructed, the SOI is recovered by a simple subtraction.

2.1. Properties of Matrix A

In order to better understand the CIMTS algorithm, the matrix $\mathbf{A}^{\frac{T_1}{L}}$ (Eq. (7)) is analyzed for $L = 1$. The $\mathbf{A}^{T_1}$ $(N \times N)$ is expressed as

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \delta T & -\delta T & & 0 & 0 & & 0 & 0 \\ 0 & 2\delta T & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & P\delta T & -P\delta T & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & (P-1)\delta T & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & -2\delta T & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & \delta T & -\delta T \end{bmatrix} \quad (9)$$

where P is an integer, such that $P\delta T \leq T_1/2 < (P+1)\delta T$ and N is the smallest possible integer such that $NT_1 = (N+1)T_2$. After some simple algebraic manipulations, the eigenvalues of the matrix $\mathbf{A}^{T_1}$ are identified as;

$$\lambda_i = \begin{cases} -(i-1)\delta T & \text{For } i = 1, 2, \cdots, P \\ -((2P+1)-i)\delta T & \text{For } i = P+1, \cdots, N. \end{cases} \quad (10)$$

The rank of the matrix is $N - 1$, therefore, the matrix contains one zero eigenvalue. It is necessary to identify the eigenvector corresponding to the zero eigenvalue, since the signal information is lost in its direction. The eigenvector corresponding to zero eigenvalue is

$$\underline{x}_0 = \frac{1}{\sqrt{N}}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \end{bmatrix}. \quad (11)$$

The following theorem is used to identify the lost direction of the signal in N dimensional signal space.

Theorem : *The CIMTS method is DC component blind in the sense that the mean of the signal is not recovered.*

Table 1. The CIMTS algorithm where L corresponds to the scale of the WT used, N is the data block size, and N_{T_1} is the number of samples in each symbol duration.

CIMTS	Version				
	V1.1	V1.2	V2	V3.1	V3.2
Frequency–offset	unknown		known	unknown	
	$\delta f \neq 0$	$\delta f = 0$	δf	$\delta f \neq 0$	$\delta f = 0$
Computation of A matrix	Approximate	Exact	Exact	Approximate	Exact
Number of scales incorporated	One		One	2 or more	
Complexity	$O((LN)^2 + N_{T_1}N)$		$O((LN)^2 + N_{T_1}N)$	$O(5N^2 + 2N_{T_1}N)$	

Proof: We have

$$\mathbf{A}^{T_1}\underline{S} = \underline{WT}. \quad (12)$$

We can write the signal $\underline{S}$ as,

$$\underline{S} = \underline{S_0} + m_{\underline{S}}\underline{a} \quad (13)$$

where $m_{\underline{S}}$ is the mean of the signal $\underline{S}$ and $\underline{a}$ is a vector of all ones.

Then,

$$\underline{WT} = \mathbf{A}^{T_1}\underline{S} = \mathbf{A}^{T_1}\left(\underline{S_0} + m_{\underline{S}}\underline{a}\right) = \mathbf{A}^{T_1}\underline{S_0} + m_{\underline{S}}\mathbf{A}^{T_1}\underline{a}. \quad (14)$$

Since $\underline{a}$ is the eigenvector corresponding to zero eigenvalue of the matrix $\mathbf{A}^{T_1}$,

$$\underline{WT} = \mathbf{A}^{T_1}\underline{S_0} \quad (15)$$

3. THE COMPUTATIONAL COMPLEXITY OF CIMTS

If the processing is done for fixed block size data, and T_1 and T_2 are time-invariant, the matrix A and its *pseudoinverse* are calculated only once. Therefore, reconstructing the N symbols of interference from the WT of the observed signal at the scale T_1/L requires $(LN)^2$ multiplications. The DWT of the signal at scale T_1/L is computed by $(N_{T_1}N)$ multiplication, where N_{T_1} is the number of samples in each symbol duration of the SOI. Therefore, the computational complexity for CIMTS (V1) and CIMTS (V2) assumes an $O((LN)^2 + N_{T_1}N)$ form, and for CIMTS (V3) is $O(5N^2 + 2N_{T_1}N)$ (Table (1).) It is clear that lower complexity can be achieved by processing the data in smaller blocks and/or utilizing coarser scales of the WT.

4. ALGORITHMIC ASSESSMENT

In this section, the Bit Error Probability (BEP) of the algorithm is estimated for various test cases. In all the reported cases, the BEP and its variance were calculated via 500 Monte-Carlo processing the data in blocks,

$$\text{BEP} = \frac{\#\text{ of bit errors}}{\#\text{ of bits}} \quad (16)$$

$$\sigma^2_{BEP} = \frac{1}{500}\sum_{n=1}^{500}\left(\frac{\#\text{ of bit errors in one block}}{\#\text{ of bits in one block}}\right)^2 - \text{BEP}^2. \quad (17)$$

The symbol durations of the SOI and the interference were equal to $T_1 = 32$ and $T_2 = 33$ respectively. The observed signal, $S(t)$ was embedded in additive White Gaussian Noise.

In the development of the CIMTS (V1) algorithm, it was assumed that the unknown interference frequency–offset, δf, is very small. Here, we examine the effect of the interference frequency–offset. The algorithm was studied using the WT at the scales T_1/L with $L = 1, 2, 3, 4$, where T_1 is the symbol duration of the SOI. In Figs. (1 - 2), the BEP of the algorithm is presented for $\delta f = 0$ and 0.01, respectively. From these figures, it is clear that for baseband signals the estimate degrades as L increases (i.e., as the finer scales of the WT are used). However, for $\delta f = 0.01$, a better estimate is obtained by using the finer of the WT.

In many practical cases, we are unable to bring the SOI exactly to baseband, therefore a small modulation frequency is present. In this section, the performance of the CIMTS algorithm is examined when the SOI is not exactly at baseband. The algorithm was studied using the WT at the scales T_1/L with $L = 1, 2, 3, 4$, where T_1 is the symbol duration of the SOI. Fig. (3) presents the BEP of the algorithm for the case of a SOI with a small frequency–offset. In this case the null space of the WT of the SOI is only an approximation. It is clear that the performance of the algorithm degrades as the frequency–offset increases, but for small modulation frequency the effect is minimal.

Another practical situation to consider is the effect of pulse shaping. Pulse shaping is caused by the low pass filtering of the channel or the receiver. Fig. (4) presents the BEP of the algorithm when the observed signal is filtered by an ideal low pass filter of stopband at normalized frequency of 0.1. It is clear that the performance of the algorithm is not effected by the filtering. The algorithm was studied using the WT at the scales T_1/L with $L = 1, 2, 3, 4$ where T_1 is the symbol duration of the SOI.

Since the complexity of the algorithms is a function of data block size, their effect on performance is examined more closely. The CIMTS (V1) algorithm was studied using the WT at the scales T_1. In Fig. (5), the BEP of the algorithm is presented, for processing data in blocks of 4, 8, 16, and 32 symbols. It is shown that in low SNR, the performance improves when the data is processed in larger blocks.

The performance of the CIMTS algorithm for MPSK signal where $M = 2, 4, 8$, and 16 is examined and reported in

Fig. (6). For higher M, it is clear that the performance is maintained when the noise power is low. The algorithm was studied using the WT at the scale T_1, where T_1 is the symbol duration of the SOI. The data is processed in blocks of 32 symbols. The MPSK signal for $M = 4$ is represented in the complex domain.

So far we have only considered the CIMTS (V 1.0) algorithm, where only one scale of the WT is utilized to estimate the signals. As a motivation for future research, we present the BEP of the CIMTS (V3) algorithm where two scale of the WT are employed. Fig. (7) presents the results when the algorithm utilizes the WT transform at scale T_1 (CIMTS V3). Thus, by using more scales in the time-scale domain we have improved the performance of the algorithm.

5. CONCLUDING REMARKS

It was shown that there exist a linear relation between the interference and the WT of the observed signal. This relation is defined by a matrix A where its elements are dependent on the wavelet basis used. The stability of the matrix A may improve by choosing the proper set of wavelet basis. This will be addressed in the proposed future research.

In summary, three versions of the CIMTS were introduced and are illustrated in Table (1). In the future studies, we will develop a robust, adaptive, and higher performance algorithm that will utilize the entire null space of the SOI in the time-scale domain via a linear operation.

6. ACKNOWLEDGMENTS

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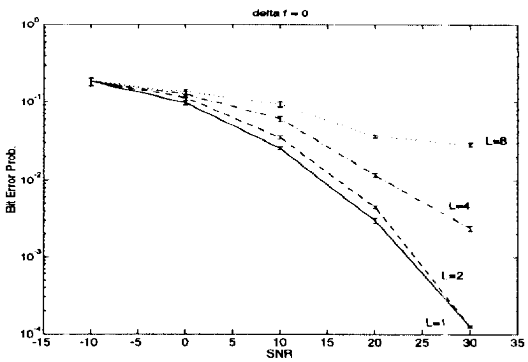

Figure 1. The BEP of the algorithm vs SNR utilizing the WT at scales equal to T_1/L for $L = 1$ (-), 2 (- -), 4 (-.), 8 (..). For $\delta f = 0$, a better estimate is obtained by using the coarser scales of the WT.

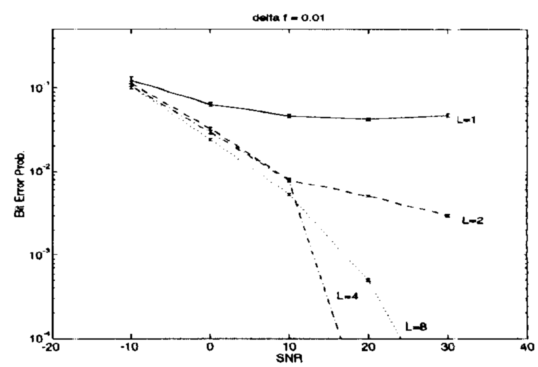

Figure 2. **Effect of the Interference Frequency-offset:** The BEP of the algorithm vs SNR utilizing the WT at scales equal to T_1/L for $L = 1$ (-), 2 (- -), 4 (-.), 8 (..). For $\delta f = 0.01$, a better estimate is obtained by using the finer scales of the WT.

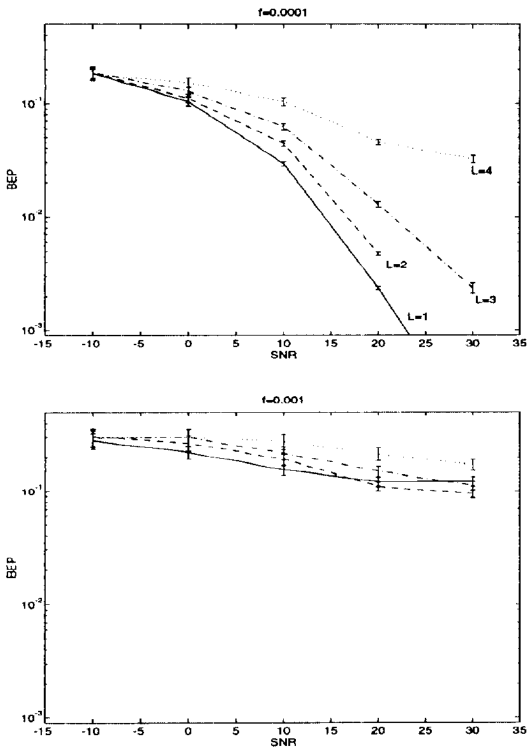

Figure 3. **Effect of the SOI Frequency–offset:** The performance of the CIMTS algorithm is examined when the SOI is not exactly at baseband. The BEP of the algorithm utilizing the WT at scales equal to T_1/L for $L = 1$ (-), 2 (- -), 4 (-.), 8 (..).

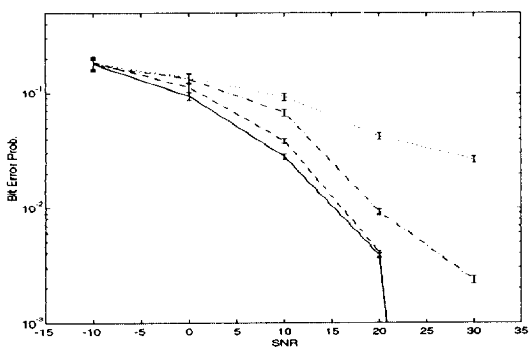

Figure 4. **Effect of Pulse Shaping:** The BEP of the algorithm vs SNR utilizing the WT at scales equal to T_1/L for $L = 1$ (-), 2 (- -), 4 (-.), and 8 (..). The observed signal is low pass filtered before processing.

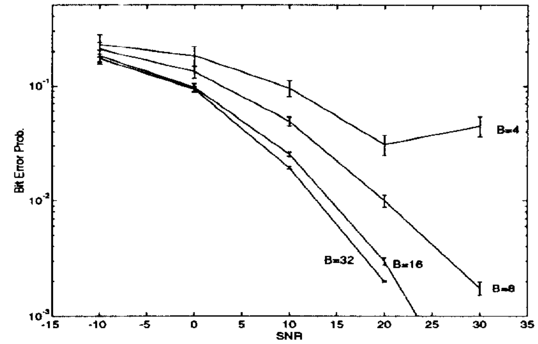

Figure 5. **Effect of data block size:** The BEP of the algorithm vs SNR utilizing the WT at scales equal to T_1; and for data processed in block of 4, 8, 16, and 32 symbols.

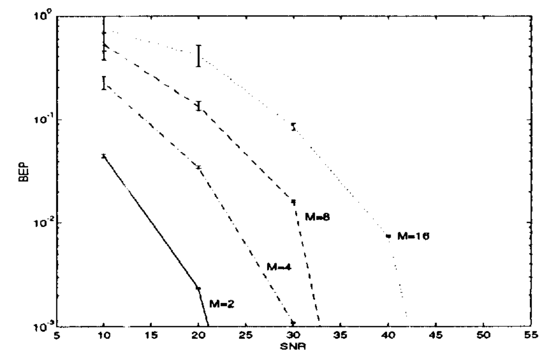

Figure 6. **M–ary Signals:** The BEP of the algorithm vs SNR utilizing the wavelet transform at scale equal to T_1 for $M = 2, 4, 8, 16$. It is clear that the performance is maintained for higher M at higher SNR.

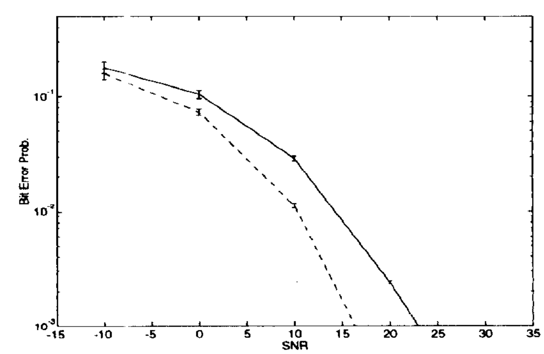

Figure 7. **Including Several Scales:** The BEP of the algorithm vs SNR utilizing the WT. The solid line corresponds to the CIMTS (V1) algorithm utilizing the scale T_1. The dashed line corresponds to the algorithms CIMTS (V3). By using more scales in the time–scale domain we have improved the performance of the algorithm.