Transient Signal Detection Using High-Resolution Line Detection on Wavelet Transforms

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Abstract

In this paper we associate the problem of estimating the locations of signal transients with the image processing task of detecting and estimating lines and axes of symmetry in an image obtained through the wavelet transform of the signal. We investigate the application of the recently-developed SLIDE algorithm - an efficient, high-resolution algorithm for line detection and estimation, to the image processing task associated with the detection of transient signals.

1 Introduction

In a variety of signal processing tasks it is useful to be able to detect 'abrupt' changes in the behavior of a signal, *e.g.* for signal segmentation such as in speech processing. An abrupt change may for example be a discontinuity in the derivative of some order. We use the term *singularity* here to collectively refer to such changes. The wavelet transform has developed into an important tool for the analysis of signals with various types of 'time-varying' frequency content. In particular, singularities in a signal may be regarded as a local variation in the frequency content. A significant feature of the wavelet transform is the fact that the structure of many commonly encountered signals may be readily recognized by observing the two-dimensional image provided by the wavelet transform. It has remained however a challenge to automate the detection and parametrization of interesting signal structures.

In this paper, we investigate the application of an image processing technique to the problem of detecting and locating the occurrence of transients in a signal by way of the image generated by the wavelet transform of the signal. We associate the problem of estimating the locations of signal singularities with the image processing task of estimating the axes of symmetry of lines that 'point' to the locations of these singularities in the wavelet transform of the signal. High resolution and computationally efficient estimates of the desired symmetry axes are obtained by application of the recently-developed *SLIDE* algorithm [1, 2, 3] for line detection and estimation.

1.1 Continuous Wavelet Transforms

Let $\hat{f}$ denote the Fourier transform of the function f. Given a function $g \in L^2(\mathbb{R})$ satisfying the *admissibility* condition,

$$C_g = \int_{-\infty}^{\infty} |\hat{g}(\omega)|^2 \,/\, |\omega| \; d\omega, \tag{1}$$

where $\hat{g}$ denotes the Fourier transform of g, the continuous wavelet transform (CWT) $(\mathbf{W}f)(a,b)$ of any $f \in L^2(\mathbb{R})$ with respect to g is defined by

$$(\mathbf{W}f)(a,b) = \int_{\mathbb{R}} f(x)\overline{g_{a,b}(x)}dx. \tag{2}$$

where

$$g_{a,b}(x) = a^{-1/2} g\left(\frac{x-b}{a}\right) \tag{3}$$

is a translated and dilated copy of g. Any $g \in L^2(\mathbb{R})$, satisfying (1) is referred to as an *analyzing (or mother) wavelet.* Admissibility of g ensures the invertibility of the CWT by means of the formula,

$$f(x) = C_g^{-1} \int_{\mathbb{R}} \int_0^{\infty} (\mathbf{W}f)(a,b) g_{a,b}(x) a^{-2} da db. \tag{4}$$

Note that the admissibility condition (1) essentially only requires that $\hat{g}(0) = 0$ or equivalently $\int g = 0$. Hence analyzing wavelets are required to be approximate bandpass functions. Computation of the CWT is facilitated by the translation structure of the analyzing functions, which enables the use of FFT based convolutions to compute the CWT. A key property of the wavelet transform is the fact that it provides a 'time-frequency localized' analysis of signals. In particular the parameter a, which we shall refer to as the *scale parameter*, corresponds approximately to 1/frequency, while the *location parameter* b corresponds approximately to time. What is particularly important to the application we consider here is the manner in which this time-frequency localization occurs. By this we are referring to the fact that the high frequency components of the signal are analyzed via very 'narrow' functions in time since, as we may recall, $f(ax) \longleftrightarrow a^{-1}\hat{f}(\omega/a)$ form a Fourier transform pair. Hence the CWT allows us to 'zoom in' on localized singularities in the signal.

Since the CWT $(\mathbf{W}f)(a,b)$, of a one-dimensional signal f, is a function on $\mathbb{R}^+ \times \mathbb{R}$, the CWT may also be equivalently treated as an image. We will use the term CWT to refer to both the function $(\mathbf{W}f)(a,b)$ and to the corresponding image. As mentioned earlier, a number of interesting signal structures may be readily observed (visually) in the wavelet transform image. In this paper we concentrate on exploiting the following fact, (see *e.g.* [4, 5, 6] for more precise statements):

> *Singularities in a signal generate lines of local extrema in the wavelet transform. Moreover, these lines converge to the location of the singularities at fine scales*[1] *(small values of the scale parameter a).*

To illustrate the specific characteristics of the CWT that we exploit in the present paper, consider a signal with a transient sinusoidal component as shown at the top of Figure 1. The CWT image (shown at the bottom of Figure 1) clearly reveals the structure of the signal. The horizontal band at the top of the image corresponds to the frequency of the sinusoid and also shows its localized nature. The singularities in this signal correspond to the start and stop epochs of the sinusoid, where the first derivative of the signal is discontinuous. As seen from the CWT image, the singularities in the signal do in fact give rise to lines of local extrema of the CWT that converge to the location of the singularities at fine scales. A second fact that is important in the application we consider here is that the lines of extrema pointing to the singularities are readily observable even when there is noise added to the signal as shown in Figure 2, where once again the signal contains a transient sinusoid. Hence the problem of locating singularities in a signal can be identified with the problem of estimating lines pointing to the singularities in the CWT of the signal. In the case where the analyzing wavelet is symmetric, the location of singularities can be determined by estimating the axes of symmetry of the 'cones' of extrema lines in the CWT image emanating from the singularity locations. The objective of this paper is to investigate

[1] It has been shown by Mallat [5] that it is in fact possible to provide a very detailed characterization of the types of singularities using information derived from local maxima of the wavelet transform.

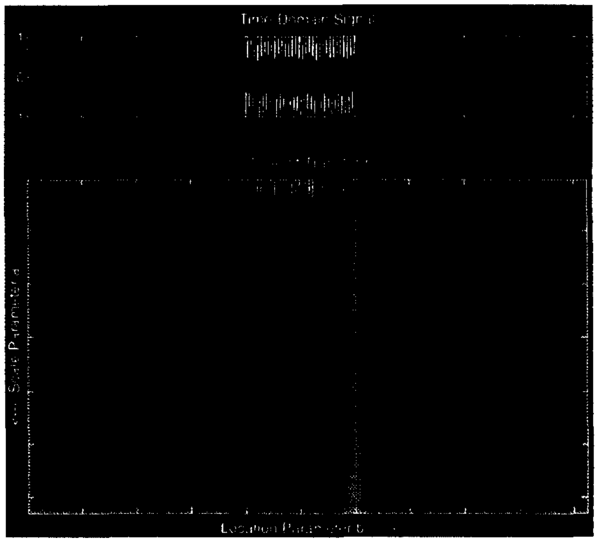

Figure 1: Top: Signal with a transient sinusoidal component. Bottom: Continuous wavelet transform of the signal using the second derivative of a Gaussian as an analyzing wavelet.

the application of a recently-developed efficient image processing algorithm, which is described in the next section, to estimate the above described axes of symmetry and thereby estimate locations of singularities in signals.

2 Subspace-Based Line Detection

The *SLIDE* (*Subspace-based Line Detection*) algorithm [1, 2, 3] is a recently-developed computationally efficient technique for estimating parameters of multiple straight lines in an image.

SLIDE reformulates the line parameter estimation problem into a subspace fitting framework by applying a *propagation* (phase-preserving projection) scheme on the image pixels. By doing so, *SLIDE* introduces a perfect mathematical analogy between the problem of estimating angles of straight lines in an image and the problem of determining the directions of arrival of planar electromagnetic waves impinging on an antenna array. In the standard setting of the *SLIDE* algorithm, *i.e.* the multiple straight line case, another analogy is made between the problem of estimating the offsets of the lines and the problem of time series harmonic retrieval. The direction-of-arrival estimation and the harmonic retrieval problems have been extensively studied, *e.g.* for radar and sonar applications and time series analysis, and in particular, in the last decade several so-called super-resolution algo-

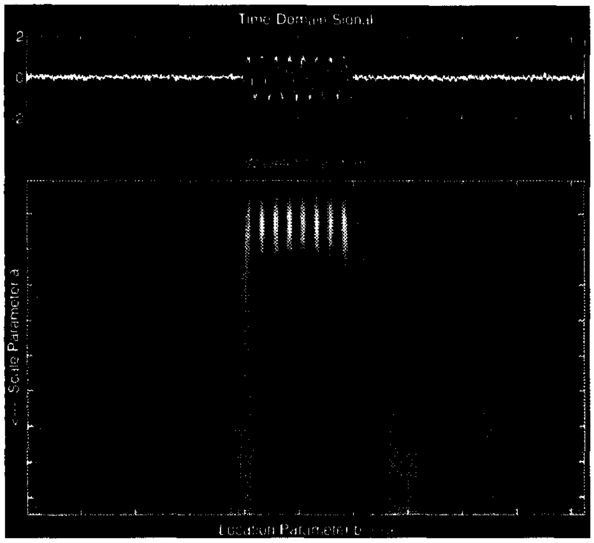

Figure 2: Top: Signal with a transient sinusoidal component plus noise. Signal-to-noise ratio in this case is about 20dB. Bottom: Continuous wavelet transform of the signal using the second derivative of a Gaussian as an analyzing wavelet.

rithms (see [7, 8, 9, 10]) have been developed. In the line fitting problem, it is shown that the conditions for using a certain computationally efficient (ESPRIT) algorithm are met. The mentioned analogies are made by adopting two signal generation scenarios from the image pixels that are called the ***constant-μ propagation*** for line angle estimation, and the ***variable-μ propagation*** for line offset estimation. Detailed derivation and discussion of *SLIDE* can be found in [1, 2, 3].

2.1 Estimation of Symmetry Axes

In the so-called ***constant-μ propagation*** scenario [1], each pixel of the image in a row contributes to a received ***signal*** at a hypothetical sensor located in front of the corresponding row of the image matrix. This contribution is in effect equivalent to a phase delay if the pixel is regarded to be a source of electromagnetic wave. More formally, let us assume that all pixels in the image start to propagate narrowband electromagnetic waves with zero initial phase. Furthermore, assume that the waves emanated from pixels in a given row of the image matrix are confined to travel only along that row towards the corresponding sensor. In this propagation environment, for example, each straight line in the image will be in effect equivalent to a wavefront of a traveling planar wave, because the waves of pixels on a line retain constant relative phase along paths parallel to that line. As we will see later, this propagation scenario creates measurements at the sensors similar to the measurements obtained in real antenna array processing.

Starting with a simplified case of fitting a straight line to a binary image with a set of collinear pixels, let us imagine that there is an array of sensors in front of the vertical axis of the image. A simple sketch of such arrangement is shown in Figure 3. If we now consider the straight line to be the wavefront of a propagating wave, the measurements received at the sensors will have the form:

$$z(y) = e^{-j\mu x} = e^{-j\mu x_0} e^{j\mu y \tan\theta} \tag{5}$$

where μ is a constant parameter, and can be interpreted as the speed of propagation.

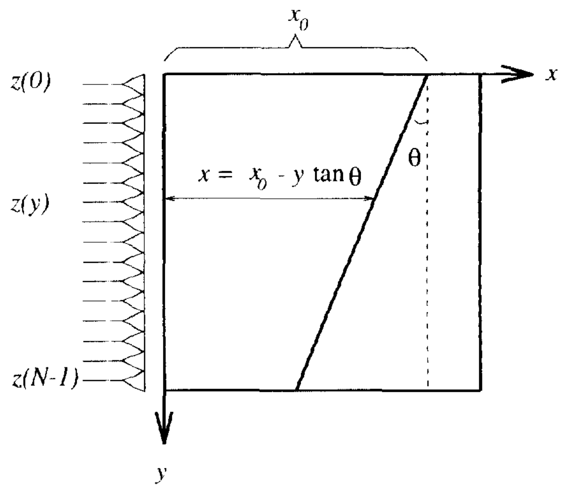

Figure 3: Image matrix and *hypothetical* sensors.

In our model, μ is a free parameter, and its choice gives us a handle to develop different applications. The measurements $z(y)$ have the form of a complex sinusoid with a frequency related to the line angle θ; the line offset has been separated and encoded in a constant complex number. The varying part of the measurements, *i.e.* the part related to θ, can be lumped into a term $a_y(\theta) = e^{j\mu y \tan\theta}$, and be called the array response vector. The above formulation readily generalizes to the multiple line case,

$$z(y) = \sum_{k=1}^{d} e^{j\mu y \tan\theta_k} e^{-j\mu x_{0_k}} + n(y) \tag{6}$$

$$= \sum_{k=1}^{d} a_y(\theta_k) s_k + n(y) \tag{7}$$

This equation is the starting point of extensive research in the last decade on the so-called subspace-based high resolution direction finding (and signal copy) algorithms (known as MUSIC, ESPRIT, WSF, etc. [7, 8, 9]). In these methods, a sample covariance matrix is computed in a certain way from the measurements, and its eigendecomposition is examined. The basic concept of subspace fitting is that the d dominant eigenvectors of this covariance matrix span the same subspace that is spanned by the array response vectors for the desired angles. *SLIDE* uses efficient numerical methods for solving this problem, and yields high resolution estimates for the line angles. There is no search procedure involved in the implementation of *SLIDE* , and its computational complexity is an order of magnitude less than that of the traditional and search-based techniques such as the Hough transform method.

The next step is to estimate the line offsets. It can be shown that by modifying the propagation scenario, new measurements are obtained that contain chirp (quadratic frequency) contributions from the angles, and linear contributions from the offsets. In other words, the offsets will be encoded as frequencies of complex sinusoids. Dividing the *chirped* measurements $z(y)$ by the array response $a_y(\theta)$ results in new *dechirped* measurements $w(y)$

$$w(y) = \frac{z(y)}{a_y(\theta)} = \sum_{k=1}^{d} e^{-j\alpha y x_{0_k}} + n'(y) \qquad (8)$$

on which fast high resolution spectral estimation methods ('dual' to the previously mentioned array processing methods) can be applied to obtain estimates of the line offsets. The above formulation also generalizes to gray-scale images by assigning the value of the gradient at each pixel to the amplitude of the wave emanating from it. More details on the implementation of the *SLIDE* algorithm can be found in [2, 3].

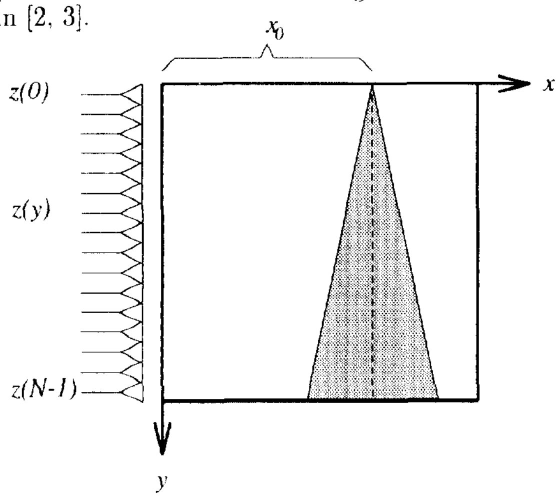

Figure 4: Image with symmetric pattern.

Now assume that an image contains a pattern with a vertical axis of symmetry, as is the case with CWT images of signals with singularities. Let the axis of symmetry be located at position x_0. We can model the intensity of the pattern at the point (x, y) by $f(x - x_0, y)$. The measured signal at the y-th row will now be

$$z(y) = \sum_x e^{j\alpha x y} f(x - x_0, y) \qquad (9)$$

$$= e^{j\alpha x_0 y} \sum_x e^{j\alpha x y} f(x, y) \qquad (10)$$

$$= e^{j\alpha x_0 y} F_x(\alpha y, y) \qquad (11)$$

where $F_x(\alpha y, y)$ is the one dimensional Fourier transform of the pattern $f(x, y)$ with respect to the x-variable, evaluated at the frequency point αy. In the above equation the measurement vector possesses a modulated structure with central carrier frequency αx_0. The contribution of the shape of the symmetric pattern in the image appears as a modulating function in the measurements that has a symmetric equivalent in the frequency domain. Therefore, the distribution of energy in the frequency domain is symmetric around the carrier frequency. Applying the ESPRIT algorithm to the sample covariance matrix computed from the measurements results in an estimate of the location of the symmetry axis x_0 [11].

3 Simulation Results

Figure 5 shows a transient signal contaminated with noise. Note that due to the continuity of the noise-free signal at the transition points, and the large amount of noise, gradient techniques cannot be readily used to estimate the epochs. To estimate the epochs we apply the procedure described earlier, *i.e.* we compute the wavelet transform of the signal and then estimate the axes of symmetry of the lines pointing to the singularities.

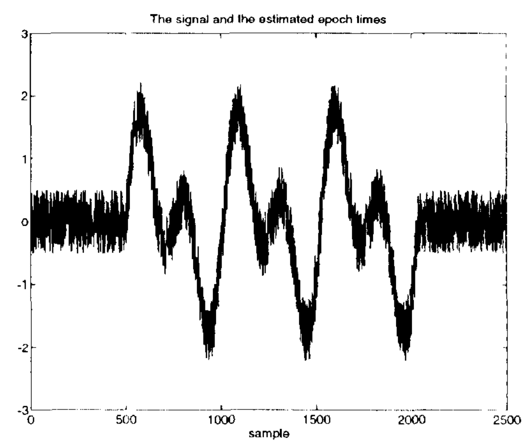

Figure 5: A transient signal in noise. Dotted lines indicate the estimated epoch times.

The wavelet transform of the signal is presented in

Figure 6. The horizontal axis in this figure represents sampled time, and the vertical axis represents scale. The symmetric structure of the lines around the transition points is observable in this figure. Application of the *SLIDE* algorithm to this image resulted in estimates of the two axes of symmetry, as shown superimposed on the image with white lines. The dotted lines in Figure 5 were also plotted using these estimates.

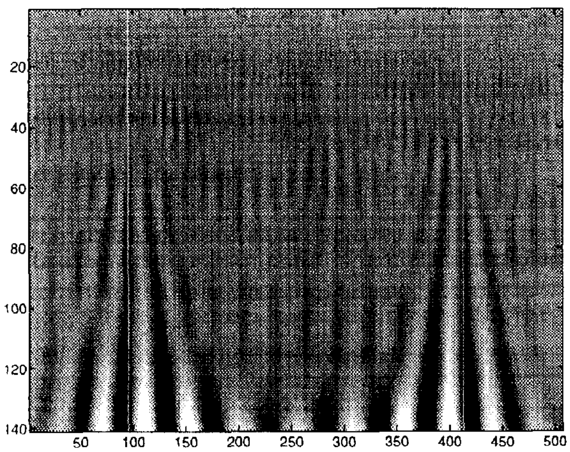

Figure 6: Two-dimensional wavelet-transform representation of the transient signal. Estimated axes of symmetry are superimposed on the image.

4 Conclusions

An image processing framework has been proposed for estimating the locations of transients in signals. The proposed technique consists of (1) computing the wavelet transform of the signal using a symmetric analyzing wavelet, and (2) estimating the symmetry axes of the lines in the wavelet transform image that point to the singularities in the signal, *i.e.* the beginning and the end of the transient. An algorithm has been developed for estimating the location of axes of symmetry in images based on subspace fitting concepts of sensor array processing. The proposed technique has potential applications in segmenting speech signals and fault detection.

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References

[1] H. K. Aghajan and T. Kailath, *SLIDE*: subspace-based line detection, In *Proc. of IEEE ICASSP*, pages V: 89–92, Minneapolis, MN, April 1993.

[2] H. K. Aghajan and T. Kailath, Sensor array processing techniques for super resolution multi-line fitting and straight edge detection, *IEEE Trans. on Image Processing*, **2**(4):454–465, Oct. 1993.

[3] H. K. Aghajan and T. Kailath, *SLIDE*: subspace-based line detection, *IEEE Trans. on Pattern Analysis and Machine Intelligence*, **16**(11), Nov. 1994.

[4] S. G. Mallat and S. Zhong, A theory for multiresolution signal decomposition: The wavelet representation, *IEEE Transactions On Pattern Analysis and Machine Intelligence*, **14**(7):710–732, July 1992.

[5] S. G. Mallat, Complete signal representation with multiscale edges, Courant Institute of Mathematical Sciences, Technical Report 483, December 1989.

[6] Ph. Guillemain and R. Kronland-Martinet, Horizontal and vertical ridges associated to the continuous wavelet, In *Proceedings IEEE International Symposium on Time-Frequency and TIme-Scale Analysis*, pages 63–66, Vancouver, Canada, October 1992.

[7] R. O. Schmidt, Multiple emitter location and signal parameter estimation, *IEEE Trans. on Antennas and Propagation*, **34**(3):276–280, March 1986. First presented at RADC Spectrum Estimation Workshop, 1979. Griffiss AFB, NY.

[8] A. Paulraj, R. Roy, and T. Kailath, Estimation of signal parameters by rotational invariance techniques (ESPRIT), In *Proc. of 19th Asilomar Conference on Circuits, Systems and Comp.*, pages 564–567, 1985.

[9] M. Viberg, B. Ottersten, and T. Kailath, Detection and estimation in sensor arrays using weighted subspace fitting, *IEEE Trans. on SP*, **39**(11):2436–2449, Nov. 1991.

[10] R. Roy and T. Kailath, ESPRIT: Estimation of signal parameters via rotational invariance techniques, *IEEE Trans. on ASSP*, **37**(7):984–995, July 1989.

[11] H. K. Aghajan, *Subspace Techniques for Image Understanding and Computer Vision*, PhD thesis, Stanford University, Stanford, CA 94305, 1994.