Vector Filter Banks and Multirate Filter Banks with Block Sampling *

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Abstract

In this paper, we study general vector filter banks where the input signals and transfer functions in conventional multirate filter banks are replaced by vector signals and transfer matrices, respectively. We show that multirate filter banks with block sampling and linear time invariant transfer functions studied by Khansari and Leon-Garcia are special vector filter banks where the transfer matrices are pseudo-circulant. We then present necessary and sufficient conditions for the alias free property, FIR systems with FIR inverses, paraunitariness for vector filter banks. We also present a necessary and sufficient condition for paraunitary multirate filter banks with block sampling.

1 Introduction

Multirate filter banks have been recently studied extensively, and found many applications in data compression, adaptive signal processing, numerical analysis and many other fields. As a natural generalization of multirate filter bank theory, multirate filter banks with block sampling were recently discussed by Khansari and Leon-Garcia [1], where the traditional uniform down/up sampling in multirate filter banks is replaced by block down/up sampling. More specifically, in traditional multirate filters, down-sampling by a factor M corresponds to taking the first sample in each block of M samples, while in multirate filter banks with block sampling, the down-sampling by a factor M (called *block down-sampling*) involves taking the first N samples of each block of MN samples. *Block up-sampling* can be defined in an analogous manner. In block sampling, there are two parameters: M, the sampling rate and N, the block size. In this paper, we will consider more general filter banks so called *vector filter banks* where the input signals are replaced by vector signals and the transfer functions are replaced by transfer matrices in conventional multirate filter banks. We will show that multirate filter banks with block sampling are a special case of vector filter banks when the transfer matrices are pseudo-circulant.

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In this paper, we focus on multirate filter banks with linear time invariant transfer functions and matrices. We show that multirate filter banks with block sampling and linear time invariant transfer functions studied by Khansari and Leon-Garcia [1] are special vector filter banks where the transfer matrices are pseudo-circulant. We present polyphase representations for vector filter banks. With the polyphase representations we present a necessary and sufficient condition for alias free vector filter banks, which is that the product of analysis and synthesis polyphase matrices is *generalized pseudo-circulant* matrix. We then study necessary and sufficient conditions for perfect reconstruction vector filter banks. With these developments we present paraunitariness for a vector filter bank for FIR paraunitary vector filter banks. We then apply the theory for vector filter banks to multirate filter banks with block sampling and present a necessary and sufficient condition for paraunitary multirate filter banks with block sampling.

2 Some Results on Vector Filter Banks

We begin with some basic definitions. Throughout this paper, lowercase letters denote discrete-time signals, lowercase boldface letters denote N dimensional vectors blocked from the corresponding lowercase letters, captial letters denote z-transforms of the corresponding lowercase letter signals, and captial boldface letters denote matrices. All transfer functions and transfer matrices are assumed as linear time invariant (LTI).

The block down/up sampling (or decimator/expander) by a factor M and a block size N is defined by

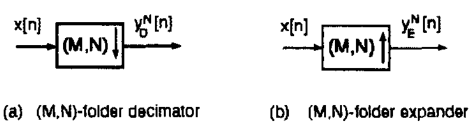

Figure 1: Block down/up sampling

$$y_D^N[n] = x[MNk + j], \tag{2.1}$$

when $n = kN + j$ for certain integers k and j with $0 \le j \le N-1$, and

$$y_E^N[n] = \begin{cases} x[kN+j], & \text{if } n = kMN + j \text{ for certain integer } k \text{ and } j \text{ with } 0 \le j \le N-1, \\ 0, & \text{otherwise.} \end{cases} \quad (2.2)$$

Their block diagrams are shown in Fig. 1. For some basic properties, we refer the reader to [1,6].

In the following, we first study general vector filter banks with vector sequences $\mathbf{x}[n]$ as input signals and arbitrary $N \times N$ transfer matrices $\mathbf{H}(z)$. We then discuss multirate filter banks with block sampling, where input signals are blocked from discrete sequences $x[n]$ and transfer matrices are pseudo-circulant.

2.1 Definitions and Connections of Multirate Filter Banks with Block Sampling and Vector Filter Banks

We start with multirate filter banks with block sampling. An M-channel filter bank with block sampling of size N is defined by the block diagram in Fig. 2, where $x[n]$ is a discrete time input signal, $H_j(z)$, $j = 0, 1, ..., M-1$, are M transfer functions of analysis filters and $G_j(z)$, $j = 0, 1, ..., M-1$, are M transfer functions of synthesis filters, and $\hat{x}[n]$ is the output signal. Notice that everything in Fig. 2 is one dimensional and *no* vectors is appeared.

An M-channel vector filter bank with vector size N is defined by the block diagram in Fig. 3, where $\mathbf{H}_j(z)$, $j = 0, 1, ..., M-1$, are $N \times N$ transfer matrices of M analysis filters and $\mathbf{G}_j(z)$ $j = 0, 1, ..., M-1$, are $N \times N$ transfer matrices of M synthesis filters. Notice that $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$ may not be pseudo-circulant in general.

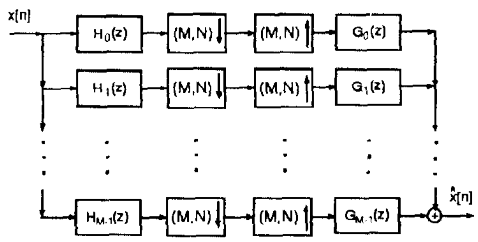

Figure 2: Multirate filter bank with block sampling.

With the above definition we next want to see their relationship. From the equivalence of the block sampling and the sampling for vectors, see [6], an M-channel filter bank with block sampling of size N in Fig. 2 can be rewritten as the one in Fig. 4, where $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$ are the N-block transfer matrices of the transfer functions $H_j(z)$ and $G_j(z)$ in Fig. 2, respectively, for $j = 0, 1, ..., M-1$. Notice that $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$, $j = 0, 1, ..., M-1$, in Fig. 4 are pseudo-circulant $N \times N$ matrices and not arbitrary $N \times N$ transfer matrices.

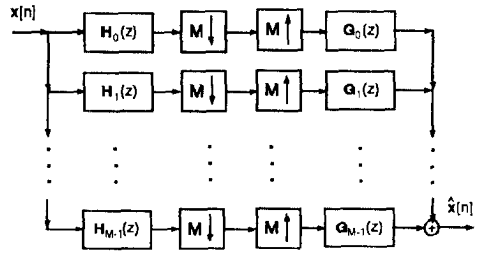

Figure 3: Vector filter bank.

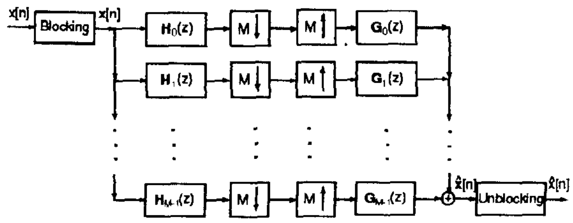

Figure 4: Vector filter bank representation of a multirate filter bank with block sampling.

Conversely, if $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$, $j = 0, 1, ..., M-1$, of an M-channel vector filter bank with vector size N in Fig. 3 are all pseudo-circulant, then the vector filter bank can be converted to an M-channel filter bank with block sampling of size N in Fig. 2, where $H_j(z)$ and $G_j(z)$ are the corresponding transfer functions of $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$, $j = 0, 1, ..., M-1$, by Proposition 1. By combining Proposition 1, we have the following theorem.

Theorem 1 *An M-channel vector filter bank of size N in Fig. 3 can be converted to an equivalent M-channel filter bank with block sampling of size N in Fig. 2 if and only if all $N \times N$ transfer matrices $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$, $j = 0, 1, ..., M-1$, in Fig. 3 are pseudo-circulant.*

Theorem 1 tells us that to consider a multirate filter bank with block sampling is equivalent to consider a vector filter bank with pseudo-circulant transfer matrices.

Remark 1: If the transfer functions $H_j(z)$ and $G_j(z)$ in Fig. 2 are linear time variant, then the result in Theorem 1 may not be true.

We next want to study general vector filter banks.

2.2 Polyphase Representations of Vector Filter Banks

Similar to the conventional multirate filter bank theory, in this section we want to re-represent the block diagram of a vector filter bank in Fig. 3 by using block polyphase matrices firstly appeared in [1].

For an $N \times N$ transfer matrix $\mathbf{H}(z)$ let

$$\mathbf{H}(z) = \sum_n \mathbf{H}[n] z^{-n}, \qquad (2.3)$$

where $\mathbf{H}[n]$ are the coefficient $N \times N$ constant matrices in $\mathbf{H}(z)$. Then, the lth forward block polyphase component of $\mathbf{H}(z)$ with sampling factor M is

$$\mathbf{E}_l[n] \triangleq \mathbf{H}[Mn + l], \quad 0 \leq l \leq M-1. \qquad (2.4)$$

Let $\mathbf{E}_l(z)$ be its z-transform, i.e.,

$$\mathbf{E}_l(z) = \sum_n \mathbf{H}[Mn + l] z^{-n}, \qquad (2.5)$$

which is called the lth *block polyphase component* of $\mathbf{H}(z)$ with sampling factor M. Then,

$$\mathbf{H}(z) = \sum_{l=0}^{M-1} z^{-l} \mathbf{E}_l(z^M), \qquad (2.6)$$

and

$$\mathbf{E}_l(z^M) = \frac{1}{M} z^l \sum_{k=0}^{M-1} W_M^{kl} \mathbf{H}(W_M^k z), \quad l = 0, 1, ..., M-1, \qquad (2.7)$$

where $W_M^k = \exp(i2\pi k/M)$ and $i = \sqrt{-1}$. Similarly, one can define backward block polyphase components. For each j with $0 \leq j \leq M-1$, let $\mathbf{E}_{j,l}(z)$ and $\mathbf{R}_{j,l}(z)$ be the lth forward and backward block polyphase components of the $N \times N$ transfer matrices $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$, respectively. Let $\mathcal{E}(z) \triangleq (\mathbf{E}_{j,l}(z))_{0 \leq j,l \leq M-1}$ be the $MN \times MN$ matrix with $\mathbf{E}_{j,l}(z)$ as its block submatrix at the jth row and the lth column. i.e.,

$$\mathcal{E}(z) = \begin{bmatrix} \mathbf{E}_{0,0}(z) & \cdots & \mathbf{E}_{0,M-1}(z) \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \mathbf{E}_{M-1,0}(z) & \cdots & \mathbf{E}_{M-1,M-1}(z) \end{bmatrix}. \qquad (2.8)$$

Similarly, we define $\mathcal{R}(z) \triangleq (\mathbf{R}_{l,j}(z))_{0 \leq l,j \leq M-1}$, the $MN \times MN$ matrix with $\mathbf{R}_{j,l}(z)$ as its block submatrix at the lth row and the jth column. The $MN \times MN$ matrices $\mathcal{E}(z)$ and $\mathcal{R}(z)$ are called the *block polyphase analysis* and *synthesis matrices* of a vector filter bank in Fig. 3, respectively. Then, by the Noble identities for vectors in Fig. 5, a vector filter bank in Fig. 3 can be represented by its block polyphase matrices $\mathcal{E}(z)$ and $\mathcal{R}(z)$ as shown in Fig. 5, which is similar to the traditional polyphase representations for multirate filter banks.

Remark 2: Since $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$ are arbitrary $N \times N$ matrices, $\mathcal{E}(z)$ and $\mathcal{R}(z)$ are also arbitrary $MN \times MN$ matrices. Since a multirate filter bank with block sampling in Fig. 2 is a vector filter bank, we also have its polyphase representation as the one in Fig. 5. We call it the *first kind polyphase representation* of a multirate filter bank with block sampling, shown in Fig. 6.

Remark 3: The polyphase matrices $\mathcal{E}(z)$ and $\mathcal{R}(z)$ are not arbitrary $MN \times MN$ matrices because of the factor that $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$ in Fig. 4 are $N \times N$ pseudo-circulant and not arbitrary like in Fig. 3.

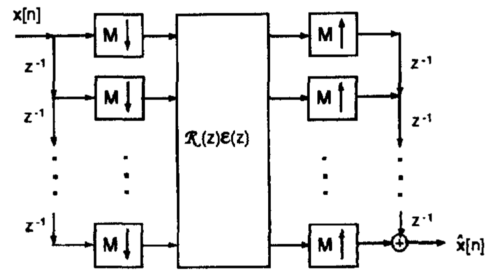

Figure 5: Polyphase representation of a vector filter bank.

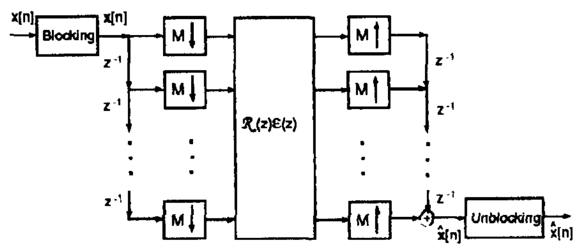

Figure 6: The first kind polyphase representation of a multirate filter bank with block sampling, $\mathcal{E}(z)$ and $\mathcal{R}(z)$ are polyphase matrices of pseudo-circulant matrices.

2.3 Alias Free and Perfect Reconstruction Vector Filter Banks

In this section, we focus on the polyphase representations in Fig. 5 and Fig. 6. We first consider a general vector filter bank in Fig. 5. A block $MN \times MN$ matrix $\mathcal{P}(z) = (\mathbf{P}_{l,j}(z))_{0 \leq l,j \leq M-1}$ with block size N is called *generalized pseudo-circulant* if and only if

$$\mathbf{P}_{l,j}(z) = \begin{cases} \mathbf{P}_{0,j-l}(z), & \text{if } 0 \leq l \leq j, \\ z^{-1}\mathbf{P}_{0,M+j-l}(z), & \text{if } l > j \geq 0, \end{cases} \qquad (2.9)$$

where $\mathbf{P}_{l,j}(z)$ are $N \times N$ matrices. As an example, a generalized pseudo-circulant matrix $\mathcal{P}(z)$ with $M = 3$ is of the form

$$\mathcal{P}(z) = \begin{bmatrix} \mathbf{P}_0(z) & \mathbf{P}_1(z) & \mathbf{P}_2(z) \\ z^{-1}\mathbf{P}_2(z) & \mathbf{P}_0(z) & \mathbf{P}_1(z) \\ z^{-1}\mathbf{P}_1(z) & z^{-1}\mathbf{P}_2(z) & \mathbf{P}_0(z) \end{bmatrix}.$$

It is clear that, when $N = 1$, the generalized pseudo-circulant property is back to the conventional pseudo-circulant property [5]. By the same approach as in [5] we are able to prove the following theorem on alias free vector filter banks.

Theorem 2 *(Necessary and Sufficient Condition for Alias Cancelation): An M-channel vector filter bank with vector size N in Fig. 5 is free from aliasing if and only if its $MN \times MN$ matrix $\mathcal{P}(z) \triangleq \mathcal{R}(z)\mathcal{E}(z)$ is generalized pseudo-circulant. With this condition $\hat{\mathbf{X}}^N(z) = \mathbf{T}(z)\mathbf{X}^N(z)$, and the distortion matrix $\mathbf{T}(z)$ can be expressed as*

$$\mathbf{T}(z) = z^{-M+1}(\mathbf{P}_0(z^M)+z^{-1}\mathbf{P}_1(z^M)+\cdots+z^{-M+1}\mathbf{P}_{M-1}(z^M)), \quad (2.10)$$

where $\mathbf{P}_m(z)$ are the block $N \times N$ submatrices of the 0th row of $\mathcal{P}(z)$.

The proof is completely similar to the one for alias free multirate filter banks in [5] except replacing functions $P_{s,l}(z)$ by $N \times N$ matrices $\mathbf{P}_{s,l}(z)$. Details are omitted. □

Similar to M-channel filter banks in [5], we also have the following perfect reconstruction property.

Theorem 3 *(Necessary and Sufficient Condition for Perfect Reconstruction): An M-channel vector filter bank with vector size N in Fig. 5 has perfect reconstruction, i.e., $\hat{\mathbf{x}}[n] = c\mathbf{x}[n-n_0]$ for certain non-zero constant c and certain integer n_0, if and only if*

$$\mathcal{R}(z)\mathcal{E}(z) = cz^{-n_0}\begin{bmatrix} \mathbf{0} & \mathbf{I}_{M-r} \\ z^{-1}\mathbf{I}_r & \mathbf{0} \end{bmatrix} \otimes \mathbf{I}_N, \quad (2.11)$$

for some integer r with $0 \le r \le M$.

When a vector filter bank in Fig. 3 or Fig. 5 is FIR, i.e., $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$ for $j = 0, 1, ..., M-1$ are FIR, then we have the following corollary.

Corollary 1 *An FIR M-channel vector filter bank with vector size N in Fig. 5 has perfect reconstruction if and only if the determinant of $\mathcal{E}(z)$ is cz^{-m_0} for a constant $c \neq 0$ and an integer m_0.*

Notice that the condition in Corollary 1 is exactly the same as the one obtained in [1], i.e., the condition obtained in [1] is a necessary and sufficient conditon for FIR vector filter banks. However, the proof here is straightforward by making use of the conventional multirate filter bank theory and simplier than the one in [1].

Applying Theorems 2 and 4, and Corollary 1 to a multirate filter bank with block sampling, we have the following results.

Corollary 2 *An M-channel filter bank with block sampling of size N in Fig. 6 (or 9) has perfect reconstruction if and only if all the transfer matrices of analysis and synthesis filters $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$, $j = 0, 1, ..., M-1$, are pseudo-circulant and their block polyphase matrices $\mathcal{E}(z)$ and $\mathcal{R}(z)$ satisfy (2.11). In FIR case, a necessary and sufficient condition for the perfect reconstruction is that all the transfer matrices $\mathbf{H}_j(z)$ and $\mathbf{G}_j(z)$, $j = 0, 1, ..., M-1$, are pseudo-circulant and the determinant of the block polyphase matrix $\mathcal{E}(z)$ of the analysis filters is cz^{-m_0} for a nonzero constant c and an integer m_0, where the inverse matrix $\mathcal{E}^{-1}(z)$ is the block polyphase matrix of synthesis filters $\mathbf{G}_j(z)$, $j = 0, 1, ..., M-1$.*

Corollary 2 tells us that the condition that $\mathcal{E}(z)$ in Fig. 6 is FIR and has FIR inverse is not enough to ensure the multirate filter bank with block sampling in Fig. 6 (or 9) has perfect reconstruction because the inverse matrix $\mathcal{E}^{-1}(z)$ may no be a block polyphase matrix for M pseudo-circulant transfer matrices of M synthesis filters $\mathbf{G}_j(z)$. This also points out that the condition obtained in [1] is not sufficient for the perfect reconstruction of the system in Fig. 4 in [1] with LTI transfer functions. The example of synthesis filters $\mathbf{G}_0(z) = \mathbf{G}^2(z)$ and $\mathbf{G}_1(z) = \tilde{\mathbf{G}}^2(z)$ presented in (38) in [1] are *not* pseudo-circulant as mentioned in [1]. However, if we are allowed to modify the synthesis bank part in Fig. 2 to a synthesis bank in an arbitrary vector filter bank in Fig. 3, i.e., ignoring the pseudo-circulantness, then the modified system shown in Fig. 7 may have perfect reconstruction. In another words, we have the following corollary.

Corollary 3 *A necessary and sufficient condition for the perfect reconstruction of an FIR vector filter bank in Fig. 7 is that the determinant of the block polyphase matrix $\mathcal{E}(z)$ of the N-block transfer matrices $\mathbf{H}_j(z)$ of the transfer functions $H_j(z)$, $j = 0, 1, ..., M-1$, in Fig. 7 is equal to cz^{-m_0} for some nonzero constant c and an integer m_0.*

An example which is not perfect reconstruction in the conventional multirate filter bank theory but perfect reconstruction in vector filter bank theory as stated in Corollary 3 was given in [1]. This also tells us that an FIR analysis bank which does not have any FIR synthese bank in the conventional multirate filter bank theory with uniform sampling may have an FIR synthesis bank with proper block sampling.

2.4 Paraunitary multirate filter banks with block sampling

The paraunitariness for vector filter banks is similar to the conventional multirate filter banks, i.e., a vector filter bank in Fig. 3 is called paraunitary if and only if its block polyphase matrix is paraunitary.

Let $H_j(z)$, $j = 0, 1, ..., M-1$, be M analysis filters in Fig. 2. For each j, let $H_{j,l}(z)$, $l = 0, 1, ..., N-1$, be the forward polyphase components of $H_j(z)$ with sampling factor N. For each j and l, let $H_{j,l,k}(z)$, $k = 0, 1, ..., M-1$, be the forward polyphase components of $H_{j,l}(z)$ with sampling factor M. Then, for $j = 0, 1, ..., M-1$,

$$H_j(z) = \sum_{l=0}^{N-1} z^{-k} \sum_{k=0}^{M-1} z^{-kN} H_{j,l,k}(z^{MN}). \quad (2.12)$$

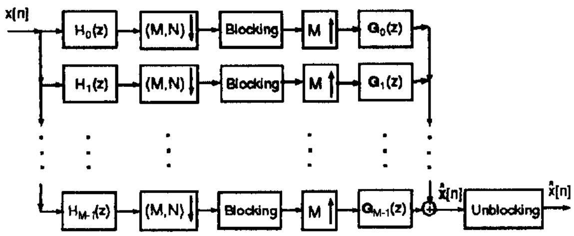

Figure 7: Modified multirate filter bank with block sampling, where $\mathbf{G}_j(z)$ are $N \times N$ transfer matrices and may not be pseudo-circulant in general.

For $l = 0, 1, ..., N-1$, let
$\mathbf{F}_l(z) =$

$$\begin{bmatrix} H_{0,l,0}(z) & H_{0,l,1}(z) & \cdots & H_{0,l,M-1}(z) \\ H_{1,l,0}(z) & H_{1,l,1}(z) & \cdots & H_{1,l,M-1}(z) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ H_{M-1,l,0}(z) & H_{M-1,l,1}(z) & \cdots & H_{M-1,l,M-1}(z) \end{bmatrix}. \tag{2.13}$$

Notice that $\mathbf{F}_l(z)$, $l = 0, 1, ..., N-1$, can be arbitrary $M \times M$ matrices since $H_j(z)$, $j = 0, 1, ..., M-1$, can be arbitrary and therefore their polyphase components can be arbitrary. We have the following theorem for the paraunitariness for a multirate filter bank with block sampling.

Theorem 4 *The block polyphase matrix $\mathcal{E}(z)$ of a multirate filter bank with block sampling in Fig. 6 (or 2) is paraunitary if and only if the following $MN \times MN$ matrix $\mathcal{F}(z)$ is paraunitary*

$$\begin{bmatrix} \mathbf{F}_0(z) & \mathbf{F}_{N-1}(z)\mathbf{D}_M(z) & \cdots & \mathbf{F}_1(z)\mathbf{D}_M(z) \\ \mathbf{F}_1(z) & \mathbf{F}_0(z) & \cdots & \mathbf{F}_2(z)\mathbf{D}_M(z) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mathbf{F}_{N-2}(z) & \mathbf{F}_{N-3}(z) & \cdots & \mathbf{F}_{N-1}(z)\mathbf{D}_M(z) \\ \mathbf{F}_{N-1}(z) & \mathbf{F}_{N-2}(z) & \cdots & \mathbf{F}_0(z) \end{bmatrix} \tag{2.14}$$

where $\mathbf{D}_M(z) = (\mathbf{C}_M(z))^T$ and $\mathbf{C}_M(z)$ is defined by

$$\mathbf{C}_M(z) = \begin{bmatrix} 0 & 0 & \cdots & 0 & z^{-1} \\ 1 & 0 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}_{M \times M}.$$

For examples that are paraunitary with block sampling but not paraunitary with conventional sampling, see [8]. In [8], we also classified all paraunitary delay chain systems with block sampling.

3 Conclusion

In this paper, we studied vector filter banks and multirate filter banks with block sampling. We then presented their polyphase representations. With these polyphase representations, we obtained necessary and sufficient conditions for alias free, perfect reconstruction and paraunitary vector filter banks and multirate filter banks with block sampling. The theory developed in this paper for vector filter banks can be easily generalized to matrix filter banks, where input and output signals are of matrix forms, which might be useful in two dimensional transform theory. Vector filter bank theory has found applications in multiwavelet transforms and vector-valued wavelet transforms [7]. Vector filter banks are also useful in vector transforms [2], and vector subband coding at low bit rates [8].

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