A Simple Design Method for Nonuniform Multirate Filter Banks

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Abstract

The theory and design of perfect-reconstruction uniform-band QMF banks have been extensively studied. For nonuniform-band filter banks, there is a lack of efficient design methods. In this paper we propose a simple design method for nonuniform integer-decimated filter banks. The approach is to transform the nonuniform filter bank design problem to a corresponding uniform filter bank design problem. Therefore, only one prototype filter needs to be designed. The analysis and synthesis filters are simple combinations of the cosine modulated versions of the prototype filter. An example is given to demonstrate the proposed method.

1 INTRODUCTION

Recently, M-channel quadrature mirror filter (QMF) banks have received considerable attention, see [1-6] for examples. They are widely used in a number of applications. In some applications, nonuniform band filter banks are used because signals are divided into unequal bands.

In perfect-reconstruction (PR) QMF banks, the amplitude distortion, phase distortion, and aliasing are completely canceled. Due to their efficient design procedure and implementation, PR cosine-modulated QMF banks are particularly attractive. However, PR QMF banks, as well as PR cosine-modulated QMF banks, having high stopband attenuation are difficult to design. Therefore, there has been some interest in relaxing the PR condition and consider near-perfect-reconstruction (NPR) QMF banks, which allow some aliasing for better stopband attenuation. NPR cosine-modulated uniform QMF banks can be desgined so that the amplitude distortion and phase distortion are canceled, while the aliasing is kept small [6].

Because nonuniform PR filter banks are much more difficult to design compared to uniform PR filter banks, not much literature addresses this problem. Some time-domain and frequency-domain design methods [7-9] have been proposed for nonuniform PR filter banks. However, most of these methods are quite complex. In this paper, we present a simple method for designing nonuniform NPR filter banks having integer decimation factors.

2 THE PROPOSED DESIGN

A K-channel nonuniform filter bank is depicted in Figure 1, where the sampling factors $\{n_0, n_1, ..., n_{K-1}\}$ are integers such that $\sum_{k=0}^{K-1}(1/n_k) = 1$. The z-transform of the output can be expressed as

$$\widehat{X}(z) = \sum_{k=0}^{K-1} F_k(z)\frac{1}{n_k}\sum_{\ell=0}^{n_k-1} X(zW_{n_k}^{\ell})H_k(zW_{n_k}^{\ell}) \quad (1)$$

where $W_L = e^{-j2\pi/L}$.

First, suppose that there is only one branch with sampling factor n', as shown in Figure 2(a). An equivalent k'-branch uniform filter bank with sampling factor $M = k'n'$, where k' is any positive integer, has been obtained in [8]. A slight modification of that structure yields an equivalent uniform filter bank as shown in Figure 2(b). We can apply such equivalent transformation to the filter bank in Figure 1. Let M be the least common multiple of the $n_0, n_1, ..., n_{K-1}$, then the K-channel nonuniform bank in Figure 1 can be redrawn as an M-channel uniform QMF bank, as shown in Figure 3.

We propose to use the analysis and synthesis filters designed for a NPR cosine-modulated uniform QMF bank to obtain the analysis and synthesis filters in a nonuniform bank. We assume that only one branch of the nonuniform filter bank, say the i-th branch, has a sampling factor different from all the other branches, as depicted in Figure 4(a). The method we present here can be extended to the cases that more than one branches have different sampling factors. Note that Figure 4(a) is a special case of Figure 1. Notice that the filter subscripts range from 0 to $M-1$, with a gap after the i-th branch. We assign the subscripts in this fashion for later convenience. Let the sampling factors be $\{n_0, n_1, ..., n_i, ..., n_{K-1}\} = \{M, M, ..., n', ..., M\}$, i.e, $n_k = M$ for $k \neq i$ and $n_i = n'$. Redrawing the i-th branch in an equivalent k'-branch form, where $k' = M/n'$, yields an equivalent uniform filter bank as shown in Figure 4(b). In this figure, the filters $H'_k(z)$ and $F'_k(z)$ are related to the filters $H_k(z)$ and $F_k(z)$ as follows:

$$H'_k(z) = H_k(z) \quad \text{and} \quad F'_k(z) = F_k(z)$$
$$\text{for } k = 0, 1, ..., i;$$
$$H'_{i+j}(z) = H'_i(z)z^{-jn'} \quad \text{and} \quad F'_{i+j}(z) = F'_i(z)z^{jn'}$$

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$$\text{for } j = 1, ..., k' - 1; \quad (2)$$

$$H'_k(z) = H_k(z) \quad \text{and} \quad F'_k(z) = F_k(z)$$
$$\text{for } k = i + k', ..., M - 1$$

Using Figure 4(b), the z-transform of the output can be written as

$$\widehat{X}(z) = X(z)T_0(z) + \sum_{\ell=1}^{M-1} X(zW_M^\ell)T_\ell(z) \quad (3)$$

where $W_M = e^{-j2\pi/M}$ and

$$T_0(z) = \frac{1}{M}\sum_{k=0}^{M-1} H'_k(z)F'_k(z) \quad (4)$$

$$T_\ell(z) = \frac{1}{M}\sum_{k=0}^{M-1} H'_k(zW_M^\ell)F'_k(z), \ell = 1, ..., M-1 \quad (5)$$

It is clear that $T_0(z)$ is the overall distortion transfer function and $T_\ell(z)$, $\ell = 1, 2, ..., M-1$, is the aliasing transfer function corresponding to $X(zW_M^\ell)$.

Each aliasing term $T_\ell(z)$ is composed of two parts. One is the aliasing caused by adjacent bands, which will be called "first-order (FO) aliasing". The other part is the aliasing caused by non-adjacent bands, which will be called "higher-order (HO) aliasing". Compared to the higher-order aliasing, the first-order aliasing is more significant and contributes much more to the total aliasing. For PR filter banks, both the FO and HO aliasings are completely canceled. For NPR filter bank, the FO aliasings should be canceled, while the HO aliasings are kept very small.

A design method for NPR uniform pseudo-QMF banks has been proposed in [6]. The prototype filter is constrained to be a linear-phase spectral-factor of $2M$-th band filter. The stopband attenuation is minimized using the nonlinearity constrain minimization algorithm of Schittkowski. As a result, the reconstructed signal has no amplitude or phase distortions. The only error at the output is due to aliasing, which is small, comparable to the stopband attenuation of the prototype filter.

Using the method in [6], we can design an M-channel uniform filter bank with analysis filters $H''_k(z)$ and synthesis filters $F''_k(z)$, $0 \le k \le M-1$. The impulse responses of $h''_k(n)$ and $f''_k(n)$ are cosine-modulated versions of the prototype filter $h(n)$, i.e.,

$$h''_k(n) = 2h(n)\cos[\frac{(2k+1)\pi}{2M}(n - \frac{N-1}{2}) + (-1)^k\frac{\pi}{4}]$$

$$f''_k(n) = 2h(n)\cos[\frac{(2k+1)\pi}{2M}(n - \frac{N-1}{2}) - (-1)^k\frac{\pi}{4}]$$

$$\text{for} \quad 0 \le n \le N-1, 0 \le k \le M-1 \quad (6)$$

where N is the length of $h(n)$. If the prototype filter has flat passband and high stopband attenuation, then so do $H''_k(z)$ and $F''_k(z)$. Also, the FO aliasings are cancelled, ie., $H''_{j-1}(zW^j)F''_{j-1}(z) + H''_j(zW^j)F''_j(z) = 0$ for j=1,2,...,M.

For the following design to work, we assume that i is an integer multiple of k', i.e., $i = k'\nu$ for some integer ν. If i is not an multiple of k', it can be shown that no matter what kind of filter we can design, some first-order aliasing terms cannot be cancelled in the filter bank in Figure 4(a). Using the filters $H''_k(z)$ and $F''_k(z)$ for the uniform filter bank obtained in the previous paragraph, we now construct the analysis and synthesis filters for the nonuniform filter bank (Figure 4(a)) as follows:

$$H_k(z) = H''_k(z), F_k(z) = F''_k(z)$$
$$\text{for } k = 0, 1, ..., i-1;$$

$$H_i(z) = \frac{1}{\sqrt{k'}}\sum_{j=0}^{k'-1} H''_{i+j}(z), F_i(z) = \frac{1}{\sqrt{k'}}\sum_{j=0}^{k'-1} F''_{i+j}(z);$$

$$H_k(z) = H''_k(z), F_k(z) = F''_k(z)$$
$$\text{for } k = i + k', ..., M-1 \quad (7)$$

When $F''_k(z)$ and $H''_k(z)$ have flat passband and high stopband attenuation, we show in Sect. 2.1 that the resulting $H_i(z)$ and $F_i(z)$ have small passband ripples and high stopband attenuations.

Substituting Eq.(7) into Eq.(2) yields the following expressions for $H'_k(z)$ and $F'_k(z)$ in terms of $H''_k(z)$ and $F''_k(z)$

$$H'_k(z) = H''_k(z) \ , \ F'_k(z) = F''_k(z)$$
$$\text{for } k = 0, 1, ..., i-1;$$

$$H'_i(z) = \frac{1}{\sqrt{k'}}\sum_{j=0}^{k'-1} H''_{i+j}(z), F'_i(z) = \frac{1}{\sqrt{k'}}\sum_{j=0}^{k'-1} F''_{i+j}(z);$$

$$H'_{i+j}(z) = H'_i(z)z^{-jn'}, F'_{i+j}(z) = F'_i(z)z^{jn'}$$
$$\text{for } j = 1, ..., k'-1;$$

$$H'_k(z) = H''_k(z), F'_k(z) = F''_k(z)$$
$$\text{for } k = i + k', ..., M-1 \quad (8)$$

Using Eq.(8) in Eq.(3), we obtain an expression for $\widehat{X}(z)$ in terms of $H''_k(z)$ and $F''_k(z)$.

In the following subsections, we show that the filters $H_k(z)$ and $F_k(z)$ have good characteristics and that the resulting nonuniform filter bank has small distortion and alising.

2.1 Frequence Responses of the Analysis and Synthesis Filters

For the i-th channel (with sampling factor n') in the nonuniform filter bank in Figure 4(a), the analysis and synthesis filters are $H_i(z)$ and $F_i(z)$. We now show that $H_i(z)$ and $F_i(z)$ are passband filters with small passband ripples and high stopband attenuation. Recall that $i = k'\nu$, $H_{k'\nu}(z) = \frac{1}{\sqrt{k'}}\sum_{j=0}^{k'-1} H''_{k'\nu+j}(z)$ and $F_{k'\nu}(z) = \frac{1}{\sqrt{k'}}\sum_{j=0}^{k'-1} F''_{k'\nu+j}(z)$.

First, we show that the sum of two adjacent filters have flat passband and high stopband attenuation. To this end, we write

$$H''_\xi(z) + H''_{\xi+1}(z) =$$
$$a_\xi c_\xi H(zW_{2M}^{(\xi+1/2)}) + a_\xi^* c_\xi^* H(zW_{2M}^{-(\xi+1/2)})$$
$$+a_{\xi+1}c_{\xi+1}H(zW_{2M}^{(\xi+3/2)}) - a_{\xi+1}^* c_{\xi+1}^* H(zW_{2M}^{-(\xi+3/2)})$$
$$= \{a_\xi c_\xi H(zW_{2M}^{(\xi+1/2)}) + a_{\xi+1}c_{\xi+1}H(zW_{2M}^{(\xi+3/2)})\}$$
$$+\{a_\xi^* c_\xi^* H(zW_{2M}^{-(\xi+1/2)}) + a_{\xi+1}^* c_{\xi+1}^* H(zW_{2M}^{-(\xi+3/2)})\} \quad (9)$$

where $a_k = e^{j\theta_k}$, $c_k = W_{2M}^{(k+\frac{1}{2})(\frac{N-1}{2})}$ and $\theta_k = (-1)^k \frac{\pi}{4}$. Because both $H''_\xi(z)$ and $H''_{\xi+1}(z)$ are symmetric in frequency, we only need to consider the positive frequency part: $a_\xi c_\xi H(zW_{2M}^{(\xi+1/2)}) + a_{\xi+1}c_{\xi+1}H(zW_{2M}^{(\xi+3/2)})$. Since $h(n)$ is real and symmetric, it can be shown that the magnitude is

$$|a_\xi c_\xi H(zW_{2M}^{(\xi+1/2)}) + a_{\xi+1}c_{\xi+1}H(zW_{2M}^{(\xi+3/2)})|$$
$$= |W_{2M}^{\frac{N-1}{4}} H(zW_{2M}^{(\xi+3/2)}) \pm jW_{2M}^{-\frac{N-1}{4}} H(zW_{2M}^{(\xi+1/2)})|$$
$$= |W_{2M}^{\frac{N-1}{4}} H(zW_{2M}^{(\xi+3/2)}) - jW_{2M}^{-\frac{N-1}{4}} H(zW_{2M}^{(\xi+1/2)})| \quad (10)$$

Now, the expression for $H''_0(z)$ is

$$H''_0(z) = a_0 c_0 H(zW_{2M}^{1/2}) + a_0^* c_0^* H(zW_{2M}^{-1/2})$$
$$= e^{j\frac{\pi}{4}}[W_{2M}^{\frac{N-1}{4}} H(zW_{2M}^{1/2}) - jW_{2M}^{-\frac{N-1}{4}} H(zW_{2M}^{-1/2})] \quad (11)$$

Then we obtain the magnitude of $H''_0(z)$ as follows

$$|H''_0(z)| = |W_{2M}^{\frac{N-1}{4}} H(zW_{2M}^{1/2}) - jW_{2M}^{-\frac{N-1}{4}} H(zW_{2M}^{-1/2})| \quad (12)$$

Compare Eq.(10) and Eq.(12), we notice that the magnitude of $a_\xi c_\xi H(zW_{2M}^{(\xi+1/2)}) + a_{\xi+1}c_{\xi+1}H(zW_{2M}^{(\xi+3/2)})$ is $|H''_0(z)|$ shifted by $W_{2M}^{\xi+1}$. Therefore, $H''_\xi(z) + H''_{\xi+1}(z)$ has flat passband and high stopband attenuation.

Since the prototype filter has sharp transition band and high stopband attenuation, non-adjacent filters do not overlap in frequency, except adding small ripples to each other's passband. Therefore, the sum of more than two filters, $\sum_{i=0}^{P-1} H''_{\xi+i}(z)$, has small passband ripples. Hence, we conclude that $H'_i(z) = \frac{1}{\sqrt{k'}} \sum_{j=0}^{k'-1} H''_{i+j}(z)$ has good filter characteristics. In the same way, we can show $F'_i(z)$ have good filter shapes too.

2.2 Distortion Function

With Eq.(8), the distortion transfer function given by Eq(4) can be written as

$$T_0(z) = \frac{1}{M}\left\{\sum_{k=0}^{M-1} H''_k(z)F''_k(z) + \frac{1}{k'}\sum_{j=0}^{k'-1}\sum_{\substack{m=0\\ m\neq j}}^{k'-1} H''_{i+j}(z)F''_{i+m}(z)\right\} \quad (13)$$

Since the prototype filter $H(z)$ is a spectral factor of a $2M$-th band filter, it has been shown [6] that $\sum_{k=0}^{M-1} H''_k(z)F''_k(z) = z^{-(N-1)}$. Therefore, we obtain

$$T_0(z) = \frac{1}{M}\left\{z^{-(N-1)} + D\right\} \quad (14)$$

where D represents the distortion, given by

$$D = \frac{1}{k'}\sum_{j=0}^{k'-1}\sum_{\substack{m=0\\ m\neq j}}^{k'-1} H''_{i+j}(z)F''_{i+m}(z) \quad (15)$$

In the same way as we define the FO aliasing and HO aliasing, we define FO distortion and HO distortion as the distortion caused by adjacent bands and distortion caused by non-adjacent bands, respectively. Since the prototype filter has sharp transition band and high stopband attenuation, higher-order aliasing is small and comparable to the stopband ripples. Then D can be expressed as

$$D = \frac{1}{k'}\sum_{j=0}^{k'-2}\{H''_{k'\nu+j}(z)F''_{k'\nu+j+1}(z)$$
$$+H''_{k'\nu+j+1}(z)F''_{k'\nu+j}(z)\} + \text{HO distortion} \quad (16)$$

According to Eq.(6), we obtain the following equations

$$H''_k(z) = a_k c_k H(zW_{2M}^{k+1/2}) + a_k^* c_k^* H(zW_{2M}^{-(k+1/2)})$$
$$F''_k(z) = a_k^* c_k H(zW_{2M}^{k+1/2}) + a_k c_k^* H(zW_{2M}^{-(k+1/2)}) \quad (17)$$

Then, for any integer $0 \le \xi < M-1$, we have

$$H''_\xi(z)F''_{\xi+1}(z) + H''_{\xi+1}(z)F''_\xi(z)$$
$$= (a_\xi a_{\xi+1}^* + a_{\xi+1}a_\xi^*)c_\xi c_{\xi+1}H(zW_{2M}^{\xi+1/2})H(zW_{2M}^{\xi+3/2})$$
$$+(a_\xi^* a_{\xi+1} + a_{\xi+1}^* a_\xi)c_\xi^* c_{\xi+1}^* H(zW_{2M}^{-(\xi+1/2)})$$
$$\cdot H(zW_{2M}^{-(\xi+3/2)}) + \text{HO distortion} \quad (18)$$

Since $a_\xi = e^{i\theta_\xi} = e^{j(-1)^\xi\frac{\pi}{4}}$ and $a_{\xi+1} = e^{i\theta_{\xi+1}} = e^{j(-1)^{\xi+1}\frac{\pi}{4}}$, we have $a_\xi a_{\xi+1}^* + a_{\xi+1}a_\xi^* = 0$. Consequently,

$$H''_\xi(z)F''_{\xi+1}(z) + H''_{\xi+1}(z)F''_\xi(z) = \text{HO distortion} \quad (19)$$

Substituting Eq.(19) into Eq.(16), we obtain D = HO distortion. If the prototype filter has a sharp transition band and a deep stopband, we can obtain very small D and conclude that the amplitude and phase distortions can be made very small.

2.3 Aliasing Functions

We now show that the aliasing terms $T_\ell(z)$, $\ell = 1, ..., M-1$, given by Eq.(5) are small.

(1) When $\ell < i$, i.e, $\ell < k'\nu$, we obtain that $T_\ell(z)=$ FO aliasing caused by band $(\ell - 1)$ + FO aliasing caused by band ℓ +HO aliasing. Referring to Eq.(8), note that $H'_\ell(z) = H_\ell(z) = H''_\ell(z)$ and $F'_\ell(z) = F_\ell(z) = F''_\ell(z)$ for $\ell < k'\nu$. So the FO aliasing caused by bands $(\ell - 1)$ and ℓ are the same as FO aliasing caused by bands $(\ell - 1)$ and ℓ in the designed uniform filter bank and thus they cancel each other. Therefore, $T_\ell(z)$ consists of only HO aliasing, which is small.

(2) When $\ell = i = k'\nu$, we obtain

$$\begin{aligned} T_\ell(z) &= \text{FO aliasing caused by band}(k'\nu - 1) \\ &\quad + k' H_{k'\nu}(zW^{k'\nu}) F_{k'\nu}(z) + \text{HO aliasing} \\ &= \text{FO aliasing caused by band}(k'\nu - 1) \\ &\quad + k' \frac{1}{\sqrt{k'}} \Big(\sum_{\xi_1=0}^{k'-1} H''_{k'\nu+\xi_1}(zW^{k'\nu})\Big) \frac{1}{\sqrt{k'}} \Big(\sum_{\xi_2=0}^{k'-1} F''_{k'\nu+\xi_2}(z)\Big) \\ &\quad + \text{HO aliasing} \\ &= \text{FO aliasing caused by band}(k'\nu - 1) \\ &\quad + H''_{k'\nu}(zW^{k'\nu}) F''_{k'\nu}(z) + \text{HO aliasing} \\ &= H''_{k'\nu-1}(zW^{k'\nu}) F''_{k'\nu-1}(z) + H''_{k'\nu}(zW^{k'\nu}) F''_{k'\nu}(z) \\ &\quad + \text{HO aliasing} \end{aligned} \tag{20}$$

Note that for the designed uniform filter, the FO aliasing is zero, which means that we have $H''_{k'\nu-1}(zW^{k'\nu})F''_{k'\nu-1}(z) + H''_{k'\nu}(zW^{k'\nu})F''_{k'\nu}(z) = 0$. Therefore, $T_\ell(z)$ has only HO aliasing.

(3) When $k'\nu < \ell < k'\nu + k'$, we have

$$\begin{aligned} T_\ell(z) &= \sum_{k=i}^{i+k'-1} H'_k(zW^\ell) F'_k(z) + \text{HO aliasing} \\ &= H_i(zW^\ell) F_i(z) \frac{1 - e^{-2\pi j \ell}}{1 - e^{-2\pi \ell / k'}} + \text{HO aliasing} \\ &= 0 + \text{HO aliasing} \end{aligned} \tag{21}$$

Therefore, $T_\ell(z)$ is small.

(4) When $\ell = k'\nu + k'$, similarly to the case for $\ell = k'\nu$, we can show that $T_\ell(z) =$ HO aliasing, which is small.

(5) When $k'\nu + k' < \ell < M$, similarly to the case for $\ell < k'\nu$, we can show that $T_\ell(z) =$ HO aliasing, which is also small.

From the above discussion , we conclude that the aliasing of the designed nonuniform filter bank is small.

3 Design Example

To demonstrate results of the proposed method, we design a three-channel nonuniform filter bank using the proposed method. The sampling factors of the filter bank are $\{4, 4, 2\}$, so that $n_0 = n_1 = 4$, $n_2 = n' = 2$, $k' = 2, i = 2$, and $\nu = 1$. The length of the prototype filter $h(n)$ is chosen to be $N = 64$. The magnitude responses of the optimized prototype filter $H(z)$, the corresponding analysis filters $H_k(z)$, the overall distortion transfer function $T_0(z)$, and the aliasing transfer function $T_\ell(z)$, $1 \le \ell \le M-1$, are plotted in Figure 5(a)-(d). We can see that the passband of the analysis filters are flat and that the distortion and aliasings are small.

4 CONCLUSION

In this paper, we propose a new design of nonuniform filter banks. The prototype filter is a linear-phase spectral factor of $2M$th band filter. The analysis and synthesis filters are the combinations of the cosine-modulated versions of the prototype filter. As a result, the overall distortion and aliasing are small, comparable to the stopband attenuation of the prototype filter.

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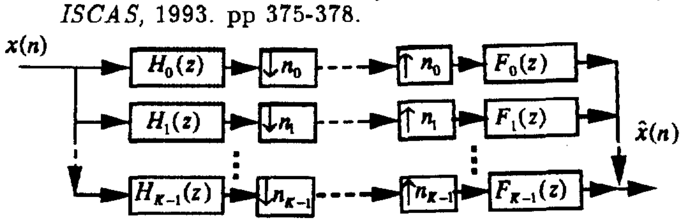
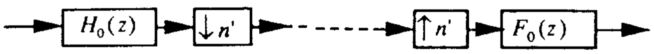
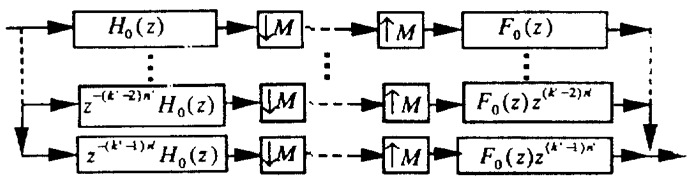


Figure 1 A nonuniform K-channel filter bank

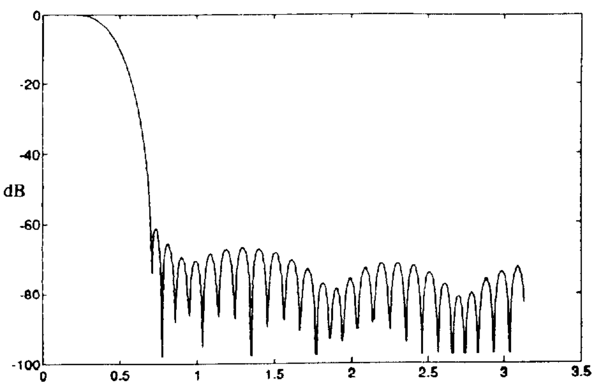


(a) single branch

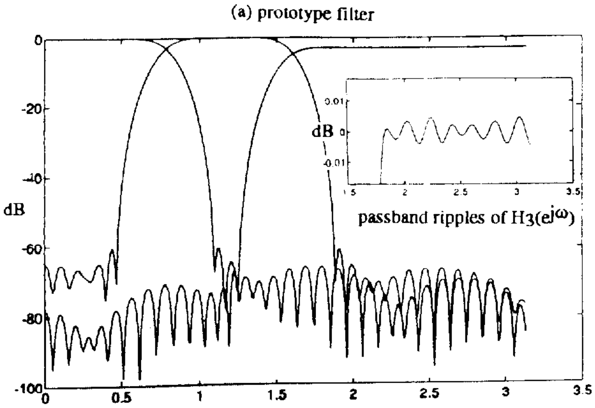


(b) equivalent branches

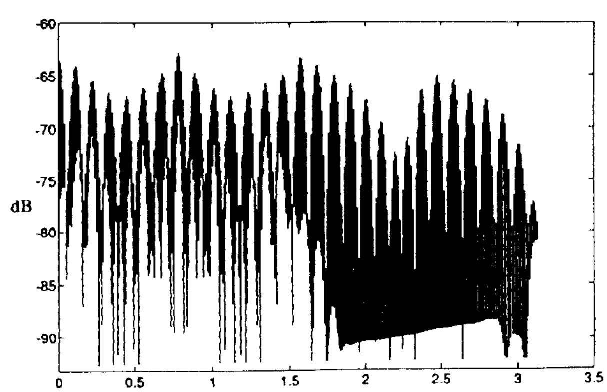
Figure 2 A single-branch and an equivalent filter bank with M=k'n'



(a) prototype filter



(b) analysis filters



(d) aliasing

Figure 5 Various plots for the example.

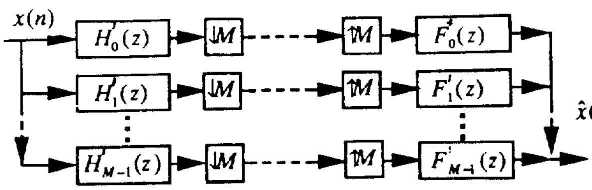
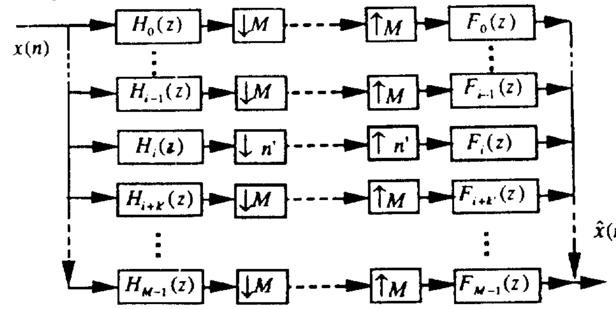
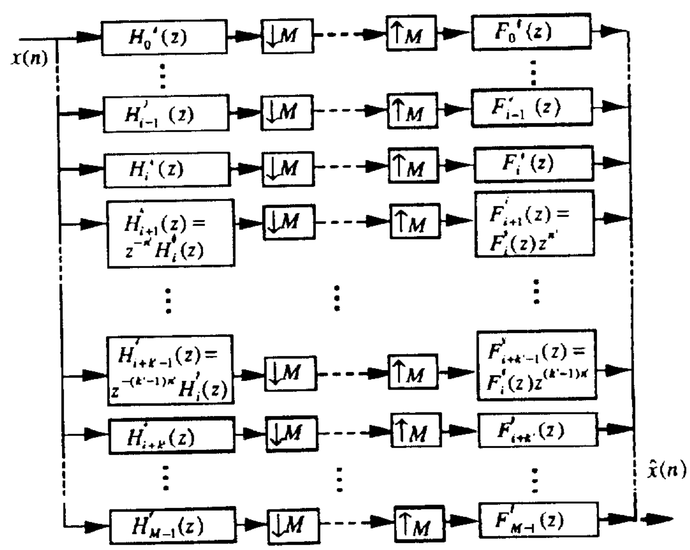


Figure 3 An equivalent M-channel uniform filter bank for Figure 1

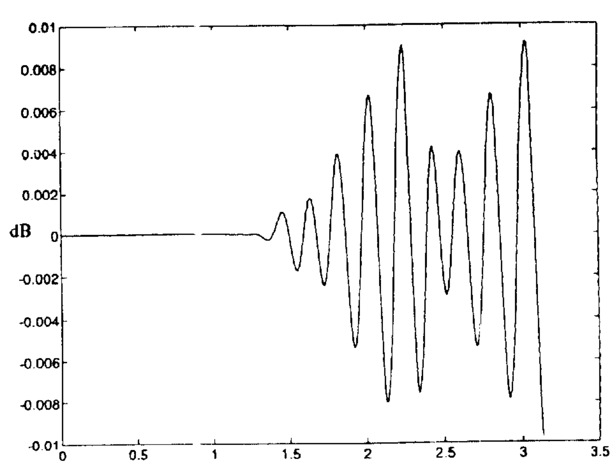


(a) nonuniform filter bank



(b) equivalent uniform filter bank

Figure 4 A nonuniform filter bank and an equivalent uniform filter bank



(c) distortion