# Optimal Filterbanks for Signal Reconstruction from Noisy Subband Components

Anastasios Delopoulos and Stefanos Kollias
Computer Science Division
Electrical Engineering Department
National Technical University of Athens
Zografou 15773, Athens, GREECE

## Abstract

*Conventional design techniques for analysis and synthesis filters in subband processing applications guarantee perfect reconstruction of the original signal from its subband components. The resulting filters lose, however, their optimality when additive noise, due for example, to signal quantization, disturbs the subband sequences. In this paper, we propose filter design techniques that minimize the reconstruction mean squared error taking into account the second order statistics of signals and noise in the case of either stochastic or deterministic signals. A novel recursive, pseudo-adaptive algorithm is proposed for efficient design of these filters. Analysis and derivations are extended to two dimensional signals and filters using powerful Kronecker product notation. A prototype application of the proposed ideas in subband coding is presented. Simulations illustrate the superior performance of the proposed filter banks versus conventional perfect reconstruction filters.*

## 1 Introduction

Multirate signal processing has recently gained a great amount of popularity in applications such as digital audio/speech coding, progressive image coding, spectrum analysis and time-varying system identification. Extensive research has been carried out on the design of decimators/interpolators that allow perfect reconstruction [5, 1] encountering removal of both phase and amplitude distortions. The associated analysis is based on the assumption that all subband signals are available to the interpolation bank with infinite precision. Thus, perfect reconstruction filter banks can be thought as a combination of filters with all-pass and linear phase overall transfer function.

In most practical applications, though, infinite accuracy is not possible. Subband signals are quantized and/or corrupted by external disturbances. Typical examples are subband coding applications, where quantization of the subband signals is necessary due to limitations in the bandwidth of transmission channels or in the space of storage media; consequently, reconstruction is performed on the basis of subband signals that include quantization error. In addition, external disturbances of the transmission channels can be modeled as an extra subband noise term. It should also be mentioned that quantization noise may have time-varying spectral characteristics especially in variable bit-rate situations that require on-line switching between modes with different number of quantization levels.

The goal of the present work is to propose methods for designing interpolation filters that suppress the effects of additive subband noise. This is possible by adjusting the impulse responses of the interpolating filters to the particular autocovariance structures of the original signal and of the additive noise.

Our approach is developed purely in the time domain and essentially uses the Mean Squared Error (MSE) criterion as a measure of similarity between the original signal and the reconstructed version of it. Both stochastic and deterministic inputs are considered. The proposed design algorithm is time adaptive, being thus able to track signals with slowly time-varying spectral characteristics.

Extensions of the proposed filter design methods for 2-D signals and filterbanks are also provided. Effective Kronecker product notation and properties are used in order to handle the 2-D problem via 1-D optimization tools.

A real life application for very low bit-rate transmission and/or storage of speech, audio and images is considered making the assumption that quantization error can be modeled as second order stationary additive, signal independent, noise.

Simulation results that illustrate the advantages of the proposed filters versus the fixed perfect reconstruction filters that are currently used have been included by the end of the article.

## 2 One Dimensional Design

Using polyphase decomposition in the time-domain filter banks can be shown to correspond to multichannel filters (e.g., [2]),

$$\mathbf{y}(n) = \sum_{k=0}^{p} H(k)\, \mathbf{x}(n-k) = H_a\, \mathbf{x}_a(n), \quad (1)$$

and

$$\hat{\mathbf{x}}(n) = \sum_{k=0}^{q} F(k)^T\, \mathbf{y}(n-k) = F_a^T\, \mathbf{y}_a(n), \quad (2)$$



1058-6393/95 $4.00 © 1995 IEEE

where,

$$H_a \doteq [\ H(0)\ ...\ H(p)\ ]\ , \tag{3}$$

and,

$$F_a \doteq \left[\ F(0)^T\ ...\ F(q)^T\right]^T \tag{4}$$

$$\mathbf{x}_a(n) \doteq [\ \mathbf{x}(n)^T\ ...\ \mathbf{x}(n\ -\ p)^T\ ]^T$$

and

$$\mathbf{y}_a(n) \doteq [\ \mathbf{y}(n)^T\ ...\ \mathbf{y}(n-q)^T\ ]$$

Consequently, $\mathbf{y}_a(n) = \mathcal{H}\,\chi(n)$ , where the $2(q+1)\times 2(p+q+1)$ matrix $\mathcal{H}$ is formed by appropriately augmenting shifts of $H_a$

$$\mathcal{H} \doteq \begin{bmatrix} H_a & & & \cdots \\ 0_2 & H_a & & \cdots \\ 0_2 & 0_2 & H_a & \cdots \\ \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdots H_a \end{bmatrix}, \tag{5}$$

and the $2(p+q+1)$-dimensional vector $\chi(n)$ is defined as, $\chi(n) \doteq \left[\ \mathbf{x}(n)^T\ ...\ \mathbf{x}(n-p-q)^T\right]^T$.

Incorporating, now, the effect of additive subband noise $\mathbf{v}(n)$ we get an expression of the total effect of decimation/interpolation,

$$\begin{aligned} \hat{\mathbf{x}}(n) &= F_a^T \mathbf{z}_a(n) \\ &= F_a^T \mathcal{H}\chi(n) + F_a^T \mathbf{v}_a(n)\,. \end{aligned} \tag{6}$$

where $\mathbf{z}_a(n) = \mathbf{y}_a(n) + \mathbf{v}_a(n)$.

If $\{\mathrm{H_i(z), F_i(z)}\}$ consist an analysis / synthesis structure and due to linearity, the reconstruction error will be, $\epsilon(n) \doteq \mathbf{x}(n) - \hat{\mathbf{x}}(n) = \mathbf{x}(n) - F_a^T\mathcal{H}\chi(n) - F_a^T\mathbf{v}_a(n)$.

For a given analysis bank $\{\ \mathrm{H_i(z)}\ \}_{\mathrm{i=0,\ 1}}$ the best synthesis bank $\{\ \mathrm{F_i(z)}\ \}_{\mathrm{i=0,\ 1}}$, i.e., the one that minimizes the power of the error signal $\epsilon(n)$ is obtained setting the derivative, w.r.t. $F_a$, of the mean squared error to zero; this yields the following system of normal equations,

$$(\ \mathcal{H}\ R\,\mathcal{H}^T\ +\ Q\ )\ F_a\ =\ \mathcal{H}\,\mathbf{r}\,,$$

where $R \doteq \bar{E}\ \{\ \chi(\mathrm{n})\ \chi(\mathrm{n})^{\mathrm{T}}\ \}$ is the autocovariance matrix of the original input, $Q \doteq \bar{E}\ \{\ \mathbf{v}_\mathrm{a}(\mathrm{n})\ \mathbf{v}_\mathrm{a}(\mathrm{n})^{\mathrm{T}}\ \}$ the autocovariance matrix of the additive noise vector $\mathbf{v}_a(n)$, and $\mathbf{r} \doteq E\{\ \chi(\mathrm{n})\ \mathbf{x}(\mathrm{n})^{\mathrm{T}}\ \}$

The above expression for the computation of optimal reconstruction filters include the autocorrelation matrices of the input signal and the additive noise. Hence, estimation of the necessary autocorrelation lags is necessary in some stage of the filter design procedure.

# 3 Recursive Implementation

The normal equations 7 that yield optimal interpolators involve the theoretical values of the covariance structures of the signals. In practice, we replace the expectations by summations over the available data record; Consistency of the resulting parameter estimators can be established along the lines of [3] Given a data record of length $N$, estimates of $R$ and $Q$ can be obtained in a batch mode and then 7 is solved by replacing $R$ and $Q$ by their estimates.

Time recursive algorithms can be employed for solving these equations in a computationally efficient manner; moreover, by inserting an appropriate forgetting factor in the recursive definition of the auto- (cross-) covariance estimators, we make the resulting parameter estimators capable to track slow variations of the signals' spectral characteristics. In the sequel, we concentrate to the multichannel formulation and correspondingly to eq. (7), in order to illustrate one of the possible recursive schemes that can be used.

$$\Phi \doteq \mathcal{H}\ R\mathcal{H}^T + Q = \bar{E}\{\mathbf{z}_\mathrm{a}(\mathrm{n})\mathbf{z}_\mathrm{a}(\mathrm{n})^{\mathrm{T}}\}\,,$$

where we used eq. (6) and the fact that $\mathbf{y}_a(n)$, $\mathbf{v}_a(n)$ are mutually uncorrelated. Similarly, let

$$\phi \doteq \mathcal{H}r = \bar{E}\{\mathbf{z}_\mathrm{a}(\mathrm{n})\mathbf{x}(\mathrm{n})^{\mathrm{T}}\}\,.$$

The corresponding estimators are,

$$\Phi_N = \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{z}_a(n)\mathbf{z}_a(n)^T\,,$$

$$\phi_N = \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{z}_a(n)\mathbf{x}(n)^T\,,$$

or in a recursive form,

$$\Phi_N = \lambda\Phi_{N-1} + \mathbf{z}_a(N)\mathbf{z}_a(N)^T\,,$$

$$\phi_N = \lambda\phi_{N-1} + \mathbf{z}_a(N)\mathbf{x}(N)^T\,,$$

where the forgetting factor $0 < \lambda \leq 1$ is a value, usually close to 1 that tunes the memory of the estimation procedure; subscript $N$ denotes the time at which $\Phi_N$ and $\phi_N$ were estimated. Using the *matrix inversion lemma* and following the derivation of Recursive Least Squares (RLS) algorithm (see, e.g., [3, pp. 306-307]) we obtain the RLS type of algorithm of Table I, where $P_N \doteq \Phi_N^{-1}$, $K(N)$ is the so-called gain matrix and $F_N$ is the optimal filter at time point $N$. The above RLS algorithm yields, for $\lambda = 1$ exactly the same solution with the batch algorithm while for $\lambda < 1$ it behaves as a pseudo-adaptive filter parameter estimation scheme.

**Table I**

| Proposed Recursive Algorithm | |
|---|---|
| I | $K(N) = \frac{P_{N-1}\mathbf{Z}_a(N)}{\lambda+\mathbf{Z}_a(N)^T P_{N-1}\mathbf{Z}_a(N)}$ |
| II | $F_N = F_{N-1} + K(N)[\mathbf{x}(N)^T - \mathbf{z}_a(N)^T F_{N-1}]$ |
| III | $P_N = \frac{1}{\lambda}\left[P_{N-1} - \frac{P_{N-1}\mathbf{Z}_a(N)\mathbf{Z}_a(N)^T P_{N-1}}{\lambda+\mathbf{Z}_a(N)^T P_{N-1}\mathbf{Z}_a(N)}\right]$ |
| Initialization: Batch estimation of $P_0 = \Phi_0^{-1}$ | |

# 4 Design in two dimensions

In the area of still and moving image compression, subband coding plays an important role. In fact, subband processing competes transform coding, mainly DCT based compression schemes, in the race of establishing a standard for lossy image coding.

Pioneering work on image compression algorithms that employ separate coding of spatial frequency bands can be found in [6] and [7]. Two-dimensional perfect reconstruction filter banks are also considered in [4]. The dominant approach is to implement 2-D decimators/interpolators using 1-D perfect reconstruction filter banks; this approach is equivalent to using 2-D filter banks with separable impulse responses. In this setup the original 2-D signal $x(m,\ n)$ is decimated into four subband components $y_{ij}(m,\ n)$ $i,\ j\ =\ 0,\ 1$ according to the following equation:

$$y_{ij}(m,n)=\sum_{k,l} h_i^c(l)h_j^r(k)x(2m-l,\,2n-k)\,. \quad (7)$$

Superscripts $c$ and $r$ denote column- and row-wise operations respectively. Here we have adopted decimation by $P=2$ which is what is commonly used in practice.

The reconstruction counterpart of eq. (7) is given by,

$$\hat{x}(m,n)=\sum_{i,j=0}^{1}\sum_{k,l} f_i^c(m-2l)f_j^r(n-2k)y_{ij}(l,k), \quad (8)$$

which yields a reconstructed version $\hat{x}(m,n)$ of the original image $x(m,n)$. Perfect reconstruction, i.e., $\hat{x}(m,n)\equiv x(m,n)$ is guaranteed provided that the 1-D filter banks applied successively on the columns are themeselves of perfect reconstruction type.

In the sequel we assume that the analysis filter banks are of perfect reconstruction type and seek optimal synthesis filter banks in the sense of minimizing the effect of additive subband noise. This means that eq. (7) is still holding true, while in eq. (8) the subband components $y_{ij}(l,k)$ should be replaced by,

$$z_{ij}(l,k)=y_{ij}(l,k)+v_{ij}(l,k)\,,$$

where $v_{ij}(l,k)$ are 2-D, zero-mean, additive noises.

Decimation and interpolation for 2-D signals can be transformed into a multichannel linear filtering using polyphase analysis in a manner similar to that used in the 1-D case. Decimation can be rewritten using eq. (7) as follows,

$$\begin{aligned}\mathbf{y}(m,n) &\doteq \begin{bmatrix} y_{00}(m,n) & y_{01}(m,n)\\ y_{10}(m,n) & y_{11}(m,n)\end{bmatrix}\\ &= \sum_{k,l} H^c(l)\mathbf{x}(m-l,n-k)H^r(k)^T \quad (9)\end{aligned}$$

where, $H^c(l)$ and $H^r(k)$ are multichanel impulse responces defined similarly to $H(k)$ and

$$\mathbf{x}(m,n)\doteq\begin{bmatrix} x(2m,2n) & x(2m,2n-1)\\ x(2m-1,2n) & x(2m-1,2n-1)\end{bmatrix} \quad (10)$$

In particular for FIR decimators of order $p_c$ and $p_r$ for columns and rows respectively, eq. (9) yields,

$$\mathbf{y}(m,n)=H_a^c\mathbf{x}_a(m,n)H_a^{rT}\,,$$

where,

$$H_a^c \doteq [\ H^c(0)\ H^c(1)\ ...\ H^c(p_c)\ ]\ ,$$

$$H_a^r \doteq [\ H^r(0)\ H^r(1)\ ...\ H^r(p_r)\ ]\ ,$$

and

$$\mathbf{x}_a(m,n)\doteq\begin{bmatrix} \mathbf{x}(m,n) & \dots & \mathbf{x}(m,n-p_r)\\ \mathbf{x}(m-1,n) & \dots & \mathbf{x}(m-1,n-p_r)\\ \vdots & & \vdots\\ \mathbf{x}(m-p_c,n) & \dots & \mathbf{x}(m-p_c,n-p_r)\end{bmatrix},$$

is a $2(p_c+1)\times 2(p_r+1)$ sliding window of the input image $x(m,n)$.

Interpolation by separable 2-D filters, can be expressed, using eq. (7) and grouping together appropriate output values, as follows,

$$\hat{\mathbf{x}}(m,n)=\sum_{k,l} F^c(l)^T\mathbf{y}(m-l,n-k)F^r(k)\,, \quad (11)$$

where $F^c(n)\doteq\begin{bmatrix} f_0^c(2n) & f_0^c(2n-1)\\ f_1^c(2n) & f_1^c(2n-1)\end{bmatrix}$ and $F^r(n)\doteq\begin{bmatrix} f_0^r(2n) & f_0^r(2n-1)\\ f_1^r(2n) & f_1^r(2n-1)\end{bmatrix}$. Equation (10) can be written in a more compact form for FIR interpolators of order $(q_c,q_r)$ in a manner similar to eq. (2), namely,

$$\hat{\mathbf{x}}(m,n)=F_a^{cT}\mathbf{y}_a(m,n)F_a^r\,, \quad (12)$$

where the augmented matrices $F_a^c$, $F_a^r$ are defined from $F^c(n)$ $F^r(n)$ as in (4), and,

$$\mathbf{y}_a(m,n)\doteq\begin{bmatrix} \mathbf{y}(m,n) & \dots & \mathbf{y}(m,n-q_r)\\ \vdots & & \vdots\\ \mathbf{y}(m-q_c,n) & \cdots & \mathbf{y}(m-q_c,n-q_r)\end{bmatrix}. \quad (13)$$

Inserting eq. (9) in eq. (13) we get a direct relation between $\mathbf{y}_a(m,n)$ and the input image,

$$\mathbf{y}_a(m,n)=\mathcal{H}^c\chi(m,n)\mathcal{H}^{rT}\,. \quad (14)$$

where $\mathcal{H}^c$ and $\mathcal{H}^r$ are defined from $H_a^c$ and $H_a^r$ respectively as described by eq. (5) and

$$\chi(m,n)\doteq \begin{bmatrix} \mathbf{x}(m,n) & \dots & \mathbf{x}(m,n-p_r-q_r)\\ \vdots & & \vdots\\ \mathbf{x}(m-p_c-q_c,n) & \cdots & \mathbf{x}(m-p_c-q_c,n-p_r-q_r)\end{bmatrix}$$

is a $2(p_c+q_c+1) \times 2(p_r+q_r+1)$ window of the input image. Combining eqs. (11) and (14) we can describe the overall effect of decimation/interpolation as

$$\hat{\mathbf{x}}(m,n) = F_a^{cT}\mathcal{H}^c\chi(m,n)\mathcal{H}^{rT}F_a^r \quad \text{or}$$

$$\begin{aligned}\tilde{\hat{\mathbf{x}}}(m,n) &\doteq \text{vec}\{\hat{\mathbf{x}}(\text{m},\text{n})\} \\ &= (F_a^{rT}\otimes F_a^{cT})(\mathcal{H}^r\otimes\mathcal{H}^c)\tilde{\chi}(m,n), \quad (15)\end{aligned}$$

where $\tilde{\chi}(m,n) \doteq \text{vec}\{\chi(\text{m},\text{n})\}$; in eq. (15) we used two properties of Kronecker products:

i) $\text{vec}\{\text{ABC}\} = (\text{C}^\text{T}\otimes\text{A})\text{vecB}$ and

ii) $(A\otimes B)(C\otimes D) = (AC)\otimes(BD)$.

Expression (11) is the interpolation formula when separable 2-D interpolators are used. For the general 2-D case, however, i.e., when *non-separable* interpolators are used, equation (8) should be modified to,

$$\hat{x}(m,n) = \sum_{i,j=0}^{1}\sum_{k,l} f_{ij}(m-2l,n-2k)y_{ij}(l,k) \quad (16)$$

Hence,

$$\tilde{\mathbf{x}}(m,n) \doteq \text{vec}\{\mathbf{x}(\text{m},\text{n})\} \equiv \begin{bmatrix}\mathbf{x}(2m,2n)\\ \mathbf{x}(2m-1,2n)\\ \mathbf{x}(2m,2n-1)\\ \mathbf{x}(2m-1,2n-1)\end{bmatrix}$$

$$= \sum_{k,l}F(m-l,n-k)\tilde{\mathbf{y}}(l,k) = \sum_{k,l}F(l,k)\tilde{\mathbf{y}}(m-l,n-k), \quad (17)$$

where,

$$F(m,n) = [\mathbf{f}_{00}(mn)\,\mathbf{f}_{10}(mn)\,\mathbf{f}_{01}(mn)\,\mathbf{f}_{11}(mn)]$$

with

$$\mathbf{f}_{ij}(m,n) \doteq \begin{bmatrix}f_{ij}(2m,2n)\\ f_{ij}(2m-1,2n)\\ f_{ij}(2m,2n-1)\\ f_{ij}(2m-1,2n-1)\end{bmatrix} \qquad i,j=0..1,$$

and $\tilde{\mathbf{y}}(m,n) \doteq \text{vec}\{\mathbf{y}(\text{m},\text{n})\}$.

For FIR interpolators $f_{ij}(m,n)$ of order $(q_c,q_r)$ eq. (17) can be written as,

$$\tilde{\hat{\mathbf{x}}}(m,n) = \mathcal{F}\text{vec}\{\mathbf{y}_\text{a}(\text{m},\text{n})\}, \quad (18)$$

where

$$\mathcal{F} = [F(0,0)\cdots F(q_c,0)\ldots F(0,q_r)\cdots F(q_c,q_r)]\ \mathbf{E},$$

where $\mathbf{y}_a(m,n)$ is given by eq. (13) and $\mathbf{E}$ is a unitary permutation matrix. Hence, using eq. (17)

$$\begin{aligned}\tilde{\hat{\mathbf{x}}}(m,n) &= \mathcal{F}\text{vec}\{\mathcal{H}^c\chi(\text{m},\text{n})\mathcal{H}^{\text{rT}}\} \\ &= \mathcal{F}(\mathcal{H}^r\otimes\mathcal{H}^c)\text{vec}\{\chi(\text{m},\text{n})\} \\ &= \mathcal{F}(\mathcal{H}^r\otimes\mathcal{H}^c)\tilde{\chi}(m,n). \quad (19)\end{aligned}$$

Note that eq. (17) is a special case of eq. (15) with $\mathcal{F} = F_a^{rT}\otimes F_a^{cT}$.

When additive subband noise has been added to $y_{ij}(m,n)$, $\mathbf{y}_a(m,n)$ in eqs. (11) and (18) should be replaced by $\mathbf{z}_a(m,n) = \mathbf{y}_a(m,n)+\mathbf{v}_a(m,n)$ where $\mathbf{v}_a(m,n)$ is defined similarly to $\mathbf{y}_a(m,n)$. Then, reconstruction eq. (15) transforms to

$$\tilde{\hat{\mathbf{x}}}(m,n) = \mathcal{F}\left[(\mathcal{H}^r\otimes\mathcal{H}^c)\tilde{\chi}(m,n)+\tilde{\mathbf{v}}_a(m,n)\right], \quad (20)$$

with $\tilde{\mathbf{v}}_a(m,n) \doteq \text{vec}\{\mathbf{v}_\text{a}(\text{m},\text{n})\}$. Equation (20) is of the same form as eq. (6), hence following the same steps we conclude that for fixed decimators $\mathcal{H}^r$, $\mathcal{H}^c$ the optimal in the MSE sense interpolator is given by,

$$\mathcal{F}_{opt} = \left(\mathcal{H}R\mathcal{H}^T+Q\right)^{-1}\mathcal{H}\mathbf{r},$$

where $\mathcal{H} \doteq (\mathcal{H}^r\otimes\mathcal{H}^c)$, $R \doteq \bar{E}\{\tilde{\chi}(\text{m},\text{n})\tilde{\chi}(\text{m},\text{n})^\text{T}\}$, $Q \doteq \bar{E}\{\tilde{\mathbf{v}}_\text{a}(\text{m},\text{n})\tilde{\mathbf{v}}_\text{a}(\text{m},\text{n})^\text{T}\}$ and $\mathbf{r} \doteq \bar{E}\{\tilde{\chi}(\text{m},\text{n})\tilde{\mathbf{x}}(\text{m},\text{n})^\text{T}\}$. In general the RHS of the above equation yields some $\mathcal{F}_{opt}$ which is not a "perfect" Kronecker product. Thus, in general optimal 2-D interpolators have not separable transfer functions, i.e., $\mathcal{F}_{opt}$ cannot be split into some $F_{a,opt}^{rT}$, $F_{a,opt}^{cT}$ such that $\mathcal{F}_{opt} = F_{a,opt}^{rT}\otimes F_{a,opt}^{cT}$. In practice this can be checked out, by estimating $\mathcal{F}_{opt}$ and by using Lemma 4.1 of [2]. suboptimal separable filters can be obtained, by computing $F_{a,opt}^{rT}$, $F_{a,opt}^{cT}$ via the singular vectors, of aforementioned lemma, that correspond to the largest singular value.

## 5 An application

The structure and the associated flow of data/information in a subband coding/transmission system that exploits the above tools is depicted in Figure 1. The original input signal is analyzed in, say $P = 2$, subband sequences which are next quantized. The quantizer proceeds to the determination of the number of quantization levels based on the channel load information. Both the quantized and the quantization free subband signals are fed to the next stage ("coder+filter design") which implements the design task. Since this stage has access to both signals it can estimate the covariance matrices $R$ and $Q$ that are necessary for the optimal filter design. As an alternative, the recursive algorithm of Table I can be used to adaptively compute the optimal filter taps.

In the sequel, the quantized subband signals along with information regarding the structure of the optimal synthesis filter bank are coded and transmitted through the channel. Assuming a lossless channel, the decoder's output will consist of exact duplicates of the quantized subband signals and optimal filter information. Synthesis bank uses the available optimal filter information in order to adjust the impulse responses of the reconstruction filters. This information could be either an explicit description of the optimal impulse responses or a pointer to a look-up table that is accessible from the synthesis bank and contains a variety of possible impulse responses for the synthesis filters. The latter choice consists a suboptimal solution but it doesn't require transmission of the entire filter's impulse response.

## 6 Simulations

Several simulations performed on one and two-dimensional signals both stochastic and deterministic have been performed illustrating the improvement in the quality of reconstruction by the use of the proposed optimal filters. Due to lack of space we include here only two results (Figures 2 and 3) that show the atained reconstruction MSE for signals whose subband components were disturbed by various levels of quantization noise. For extensive experimental tests the reader is refered to [2].

## 7 Conclusions

In this work we analysed the effect of subband noise to the quality of the reconstructed signals. We introduced synthesis filterbanks that taking into account the statistics of the involved signals perform optimal reconstruction in the presense of subband noise. The designed filters outperform the conventional "perfect reconstruction" ones in subband processing applications such as speech, audio and image coding and transmission through low bit-rate channels.

## References


[1] R. E. Crochiere and L. R. Rabiner, "Multirate Digital Signal Processing," *Prentice-Hall, Inc.,* Englewood Cliffs, New Jersey, 1983.

[2] A. Delopoulos, and S. Kollias, "Optimal Filterbanks for Reconstruction from Noisy Subband Components," *IEEE Trans. on Signal Processing,* 1994 (submitted for publication).

[3] L. Ljung, "System Identification, Theory for the User," *Prentice-Hall, Inc.,* Englewood Cliffs, New Jersey, 1987.

[4] S. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," *IEEE Trans. on Pattern Analysis and Machine Intelligence,* Vol. 11, No. 7, pp. 674-693, July 1989.

[5] P. P. Vaidyanathan, "Multirate Digital Filters, Filter Banks, Polyphase Networks, and Applications: A Tutorial," *Proceedings of the IEEE,* Vol. 78, No. 1, January 1990.

[6] M. Veterli, "Multi-dimensional Sub-band Coding: Some Theory and Algorithms," *Signal Processing,* Vol. 6, pp. 97-112, April 1984.

[7] J. W. Woods and S. D. O' Neil, "Subband Coding of Images," *IEEE Trans. of Acoustics, Speech and Signal Processing,* Vol. 34, pp. 1278-1288, Oct. 1986.


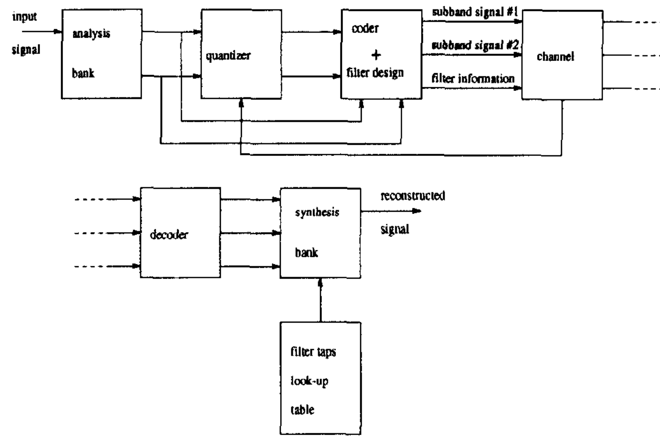


Figure 1: Subband coder-decoder implementing optimal filter design.

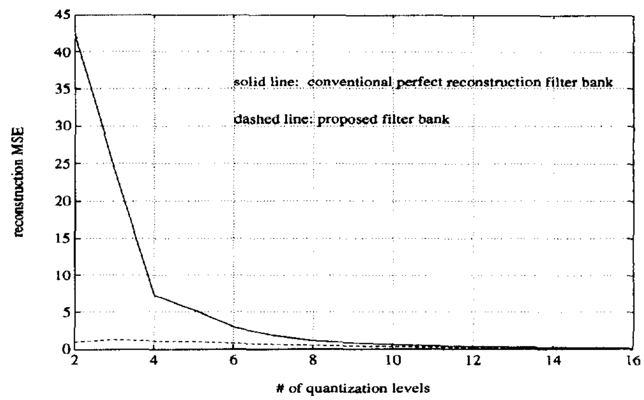


Figure 2: Reconstruction MSE vs. # of quant. levels for an 1-D non-stationary speech signal.

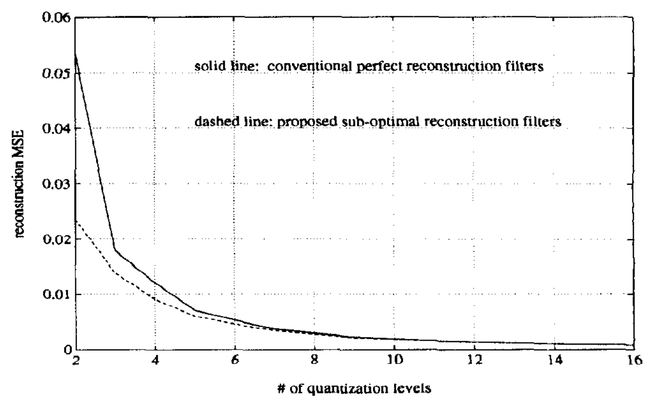


Figure 3: Reconstruction MSE vs. # of quant. levels: real life image, optimal filter design based on estimated statistics.