

BEAMFORMING AND SIGNAL EXTRACTION IN MULTIPATH WITH MOBILE ANTENNA ARRAYS

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ABSTRACT

By exploiting the motion that occurs in mobile communications and using antenna arrays, we propose signal acquisition and beamforming methods that are applicable even in multipath rich environments. It is shown that the time-averaged correlation matrix of the resulting nonstationary signals is resistant to coherent interferences and is particularly useful for subspace methods and beamforming. Examples demonstrating the performance of the proposed methods are also shown.

1. INTRODUCTION

Due to the limited spectrum available and ever increasing demand for cellular and mobile communication services, multiple antenna arrays that provide better immunity to fading and interference are being proposed. It has been shown that antenna arrays at the base station and also at the mobile result in improved estimation [1, 2, 4],[12]-[15]. With spread spectrum multiple access systems (e.g., CDMA), co-channel interference from undesired users cannot be completely suppressed due to cross-correlation among the PN sequences assigned to the various users and in this case the additional diversity offered by multiple antennas is particularly attractive (see e.g., [11]). Typically, antenna array systems use a beamformer which exploits a characteristic of the desired signal (e.g., its direction) for enhancement [7].

The mobile and cellular radio channels suffer from undesired phenomena such as multipath due to buildings, interference from unwanted users and fading due to the motion of the receiver/transmitter [6]. In such situations, the (adaptive) beamforming methods available currently do not perform well because the operating assumptions are violated. The presence of coherent signals, e.g., due to multipath, completely degrades the performance of a conventional beamformer and results in cancellation of the desired signal. The

popular subspace based angle of arrival (DOA) estimation methods also do not perform well in coherent signal environments and thus discrimination based on DOAs is not appropriate either.

In addition to multipath, the relative motion of the transmitter/receiver results in fading due to Doppler effects. For a vehicle moving at 140km/hr and at 900 MHz carrier frequency, the Doppler frequency is roughly 120Hz, which implies fades occur approximately every 4 ms. This implies that processing times are short and for on-line processing fast convergence of adaptive algorithms is necessary [12, 13].

In this paper, extraction of signals in multipath with time-varying propagation delays is addressed. Instead of assuming short observation intervals over which Doppler does not affect the received signal, we exploit this time-variation to solve coherency problems. Such techniques have been proposed in the past for restoring the rank of the signal correlation matrix (see [5] and references therein). However, in this paper we provide an alternate interpretation to this procedure and show how it can be applied to separate overlapping coherent signals in mobile communications.

2. PROBLEM STATEMENT

The complex baseband discrete-time vector signal at the receiving antenna array under the assumption that the transmitted signals are narrowband can be expressed as:

$$\mathbf{x}(t) = s_0(t)\mathbf{a}_0(t) + \sum_{l=1}^L s_l(t)\mathbf{a}_l(t) + \mathbf{v}(t) \quad (1)$$

where, $\mathbf{a}_l(t)$ is the time-varying array response vector of the l th signal, $s_l(t)$ and $\mathbf{v}(t)$ is the noise vector with components $v_p(t)$. We do not require the condition that the signals $s_l(t)$, $l = 0, \dots, L$ are uncorrelated with each other. The problem then is to extract $s_0(t)$ given a stretch of $M \times 1$ vector data $\mathbf{x}(t)$, $t = 0, 1, \dots, T-1$.

The propagation vector $\mathbf{a}_l(t)$ in (1) depends on the location of the transmitter/receiver and the relative velocity between two. We assume that the k th element of $\mathbf{a}_l(t)$ is of the form

$$a_{kl}(t) = g_k e^{j\psi_{kl}(t)}, \psi_{kl}(t) = \beta_{kl} + \alpha_l t. \quad (2)$$

For illustration purposes, consider an antenna array at the mobile. The signals intercepted by this array consist of signals from the desired base station, reflections (multipath components) and signals from other users. Assuming that the vehicle on which the antenna array is mounted moves with a velocity \mathbf{v} , the l th signal arrives from direction $\boldsymbol{\kappa}_l$, and the vector \mathbf{u}_k defines the coordinates of the k th sensor, it can be shown that [6], [7], [5]:

$$\beta_{kl} = \frac{2\pi}{\lambda_c} \boldsymbol{\kappa}_l' \mathbf{u}_k, \quad \alpha_l = \frac{2\pi}{\lambda_c} \mathbf{v}' \boldsymbol{\kappa}_l. \quad (3)$$

where λ_c is the carrier wavelength.

For simplicity, if we assume an M element uniformly spaced linear array with separation d and a constant velocity $v_x = v$ km/hr along the x -axis and an arrival angle θ_l with the sources in the far-field, we obtain

$$\beta_{kl} = \frac{2\pi}{\lambda_c} d(k-1) \sin \theta_l, \quad \alpha_l = \frac{2\pi}{\lambda_c} v \sin \theta_l = \omega_d \sin \theta_l. \quad (4)$$

However, in this paper we do not assume that β_{kl} and α_l are restricted to be of the form (3). It only suffices that the array propagation vectors be linearly independent for different signals and that they be known *a priori*. Note that the model in (3) also covers the scenario where the base station antenna array is moving. The case when the antenna is stationary but is receiving signals from moving transmitters is similar and will not be elaborated here [9].

In the sequel, we assume the following conditions to hold with respect to the signal model in (1):

- (a1) The signals $s_l(t)$, $l = 0, \dots, L$ are stationary, zero-mean and narrowband with center frequency f_c , and $\alpha_l - \alpha_m \neq 2\pi r$, $l \neq m$, and integer r .
- (a2) The sensor noise $v_p(t)$ is zero-mean, stationary and uncorrelated with $v_q(t)$, $q \neq p$ and with $s_l(t)$.
- (a3) The number of antenna elements $M > L+1$, the total number of signals arriving at the array.
- (a4) The set of vectors $\{\mathbf{a}_l(0), \forall l\}$ is linearly independent.

3. CYCLIC CORRELATION BASED SIGNAL SEPARATION

Under (a1) and (a2), starting from (1) and using (2), the cross-correlation between the antenna outputs is,

$$r_{x_p x_q}(t) = \mathcal{E}\{x_p(t)x_q^*(t)\} = \sum_{l=0}^L g_p g_q^* \sigma_{s_l}^2 e^{j(\beta_{pl} - \beta_{ql})}$$

$$+ \sum_{m,n=0, m \neq n}^L g_p g_q^* r_{s_m s_n} e^{j(\beta_{pm} - \beta_{qn} + (\alpha_m - \alpha_n)t)} + r_{v_p v_q}, \quad (5)$$

where $\sigma_{s_m}^2 = \mathcal{E}\{|s_m(t)|^2\}$, and $r_{s_m s_n} \triangleq \mathcal{E}\{s_m(t)s_n^*(t)\}$. From (5), we observe that $r_{x_p x_q}(t)$ is time-varying if (i) the signals are correlated, and (ii) $\alpha_m \neq \alpha_n$, $m \neq n$. The latter is satisfied for example when at constant speeds v , no two signals arrive from the same angle [c.f. (4)]. The periodically time-varying correlation in (5) when expanded in Fourier series yields the cyclic correlation [3],

$$c_{x_p x_q}(\alpha) \triangleq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} r_{x_p x_q}(t) e^{-j\alpha t} \quad (6)$$

$$= \left[\sum_{l=0}^L g_p g_q^* \sigma_{s_l}^2 e^{j(\beta_{pl} - \beta_{ql})} + r_{v_p v_q} \right] \delta[\alpha] \\ + \sum_{m=0}^L \sum_{n=0, m \neq n}^L g_p g_q^* r_{s_m s_n} e^{j(\beta_{pm} - \beta_{qn})} \\ \times \delta[\alpha - \alpha_m + \alpha_n]. \quad (7)$$

Thus, if we consider only the cyclic correlation at $\alpha = 0$, we obtain from (6) and (7), the time-averaged correlation

$$\bar{c}_{x_p x_q} \triangleq c_{x_p x_q}(0) = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} r_{x_p x_q}(t) \quad (8)$$

$$= \sum_{l=0}^L g_p g_q^* \sigma_{s_l}^2 e^{j(\beta_{pl} - \beta_{ql})} + r_{v_p v_q}. \quad (9)$$

With $[\bar{\mathbf{C}}_{xx}]_{p,q} = \bar{c}_{x_p x_q}$, we obtain the matrix relation,

$$\bar{\mathbf{C}}_{xx} = \mathbf{A} \boldsymbol{\Lambda}_s \mathbf{A}^H + \boldsymbol{\Lambda}_v, \quad (10)$$

where the diagonal matrix $\boldsymbol{\Lambda}_s = \text{diag}\{\sigma_{s_0}^2, \sigma_{s_1}^2, \dots, \sigma_{s_L}^2\}$ has rank $L+1$ and the matrix $\boldsymbol{\Lambda}_v$ has the sensor noise variances along its diagonal. With H denoting the conjugate transpose, the $M \times (L+1)$ matrix \mathbf{A} has as its l th column the propagation vector

$$\mathbf{a}_l = \mathbf{a}_l(0) = [g_{1l} e^{-j\beta_{1l}}, g_{2l} e^{-j\beta_{2l}}, \dots, g_{1M} e^{-j\beta_{1M}}]^H. \quad (11)$$

Notice that $\bar{\mathbf{C}}_{xx}$ in (10) is the same as $\mathbf{R}_{xx} = \mathcal{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\}$, the correlation matrix of $\mathbf{x}(t)$ in the absence of correlation between the various signals arriving at the receiver and with no motion present, i.e., in the absence of multipath and Doppler effects (see e.g., [7]).

In practice, under assumption (a1), the cyclic correlation $c_{x_p x_q}(\alpha; 0)$ can be consistently estimated via

sample averages as [3]:

$$\hat{c}_{x_p x_q}(\alpha; 0) = \frac{1}{T} \sum_{t=0}^{T-1} x_p(t) x_q^*(t) e^{-j\alpha t}. \quad (12)$$

The time-averaged correlation then is simply,

$$\hat{c}_{x_p x_q} = \frac{1}{T} \sum_{t=0}^{T-1} x_p(t) x_q^*(t), \quad (13)$$

which is suitable for on-line processing just as in the stationary case. Thus, adaptive algorithms can be developed to further address changing velocities, angles of arrival and other signal parameters.

If \mathbf{A} in (10) is full rank $L + 1$, then it follows that $\text{rank}[\mathbf{A}\mathbf{\Lambda}_s\mathbf{A}^H] = L + 1$. Consequently, any of the conventional subspace methods can be applied to the time-averaged matrix $\bar{\mathbf{C}}_{xx}$ to estimate the directions of arrival θ_i [9]. However, to be able to deal with cases where \mathbf{a}_i has a general form as in (2), we propose a beamformer that fits the present framework.

3.1. Beamformer for coherent signals

If the signals are uncorrelated and \mathbf{a}_0 is the desired signal propagation vector, then the optimum weight vector $\mathbf{w}_{opt,s}$ that cancels interferences is given by

$$\mathbf{w}_{opt,s} = \gamma_s \mathbf{R}_{xx}^{-1} \mathbf{a}_0, \quad (14)$$

where the value of the complex constant γ_s is dictated by the particular criterion being employed [7]. The reason a conventional beamformer fails in the coherent signal case is that $\mathbf{R}_{s,s} \triangleq \mathcal{E}\{\mathbf{s}(t)\mathbf{s}^H(t)\}$, the signal correlation matrix ceases to have rank $L + 1$. By restoring the rank by time smoothing the time-varying correlation, in contrast to the spatial smoothing proposed in [10], we have overcome this problem and at the same time taken care of the undesired effects due to motion of the receiver. Since the matrix $\bar{\mathbf{C}}_{xx}$ has exactly the same characteristics as \mathbf{R}_{xx} (in the absence of coherence), we use the former instead of the latter in (14). Therefore, the proposed beamformer that is resistant to coherent signal and motion effects is

$$\mathbf{w}_{opt,m} = \gamma_m \bar{\mathbf{C}}_{xx}^{-1} \mathbf{a}_0. \quad (15)$$

Just as in the conventional schemes (for e.g., [10]), it can be shown that with weights given by (15), the output of the proposed beamformer is

$$y(t) = \mathbf{w}_{opt,m}^H \mathbf{x}(t) = \gamma s_0(t) e^{j\alpha_0 t}, \quad (16)$$

from which the baseband signal $s_0(t)$ can be extracted after estimating α_0 via any of the frequency estimation schemes. With $\gamma_m = 1/\mathbf{a}_0^H \bar{\mathbf{C}}_{xx}^{-1} \mathbf{a}_0$, we obtain

the ‘‘MVDR’’ beamformer that presents unit gain to the desired signal. The time-averaged matrix $\bar{\mathbf{C}}_{xx}$ can also be employed instead of \mathbf{R}_{xx} in the conventional LCMV beamformer, thus giving us a choice of placing constraints on the response for desired and unwanted signals.

Note that we haven’t established optimality in any sense and hence the choice of γ_m is an open question. However, it can be shown along the lines of the classical theory that the beamformer in (15) minimizes the time-average of the (*time varying*) mean square error over the observation interval.

3.2. Decorrelating effects of time smoothing

The performance of the proposed beamformers is directly tied to difference in Doppler frequencies of the various signals. Let us assume for simplicity that there are only two signals arriving at the array and they are completely correlated (i.e., coherent). If $\alpha_1 - \alpha_2 = 2\pi i$, $i = 0, 1, 2, \dots$ ($e^{j\alpha t}$ is periodic in α), then we note from (7) that the signal cross correlations can affect $\bar{\mathbf{C}}_{xx}$ because $\mathbf{\Lambda}_s$ will be rank deficient. For the cross-correlations to have the least effect on $\mathbf{\Lambda}_s$, we note from (7) that $\alpha_1 - \alpha_2$ should not be close to 0, or 2π . Consequently, from (4), the optimal choice then is

$$\alpha_m - \alpha_n = \omega_d (\sin \theta_m - \sin \theta_n) = (2r - 1)\pi, r = 0, 1, \dots \quad (17)$$

Following the analysis performed in [8] for conventional beamformers, we briefly examine two possible situations.

Optimal velocity for decorrelation

Let θ_0 , the angle of arrival of the desired source be 0° . Since $\omega_d = 2\pi v/\lambda_c$, from (17), we find that the best speed for the receiver antenna array so that there is maximum ‘‘decorrelation’’ is

$$v_{opt} = \frac{(2r - 1)\lambda_c}{2 \sin \theta_i} \quad \text{m/sec}, \quad (18)$$

where λ_c is the wavelength and $\theta_i = \theta_1$ is the angle of arrival of the coherent interference. With $f_c = 900$ MHz, we have

$$v_{opt} = \frac{(2r - 1)}{6 \sin \theta_i} \quad \text{m/sec}. \quad (19)$$

We observe from (19) that reasonable decorrelation is achieved, even with small speeds and for significant angular separations. For example, if $\theta_i = 90^\circ$, then the smallest v_{opt} dictated by (19) is 0.6 km/hr with $r = 1$. If the interference is also coming from $\theta_i = 0^\circ$, then $v_{opt} = \infty$ for decorrelation to occur [see also (5)].

Closely spaced signals

If the two signals are very closely spaced, i.e., $\theta_0 = \theta$ and $\theta_1 = \theta + \epsilon$, with ϵ small, then again from (4).

$$\alpha_0 - \alpha_1 = \omega_d(\sin \theta_0 - \sin \theta_1) \quad (20)$$

$$\approx \omega_d \epsilon \cos \theta. \quad (21)$$

We conclude as in (17), that the optimal array speed with $f_c = 900$ MHz, is

$$v_{opt} = \frac{2r - 1}{6\epsilon \cos \theta} \text{ m/sec.} \quad (22)$$

Thus, small separation of coherent signals imply larger speeds are necessary to achieve good decorrelation.

In depth analysis along the lines of [8] is being investigated and will be reported in the future.

3.3. Detecting multipath components

We note from (7) that $\bar{c}_{x_p x_q} \neq 0$ only when $\alpha = \alpha_m - \alpha_n$ and $s_m(t)$ and $s_n(t)$ are correlated. We conclude that there is dependence between a fixed $s_m(t)$ and $s_n(t)$, $n = 0, \dots, L, m \neq n$ if $|\bar{c}_{x_p x_q}(\alpha_m - \alpha_n)| \neq 0$ [9]. Thus, the signals that are correlated with each other can be identified and processed coherently as in a RAKE processor.

4. SIMULATION EXAMPLES

The proposed beamforming procedure as well as the conventional that restricts itself to short observation periods that are chosen to avoid the fades were tested via computer generated simulations. The number of antennas in the uniformly spaced array was $M = 10$ and $d = \lambda_c/2$. The additive Gaussian noise was spatially white and the SNR is defined with respect to the desired signal $s_o(t)$. The array response vector \mathbf{a}_o was the steering vector which is a function of the DOA θ_o . The required correlation matrices $\hat{\mathbf{C}}_{xx}$ and $\hat{\mathbf{R}}_{xx}$ were estimated from T samples.

Example 1: The MVDR beamformer with the $\hat{\mathbf{C}}_{xx}$ and $\hat{\mathbf{R}}_{xx}$ estimated from $T = 256$ samples (additive spatially white Gaussian noise, SNR= 10dB) was tested first. The desired baseband signal was $s_o(t) = \sin(0.4\pi t)$ ($\theta_o = 70^\circ$). The coherent interference was $s_i(t) = 0.1 \sin(0.4\pi t)$ with $\theta_i = -30^\circ$. An additional non-Gaussian narrowband interference $s_{ng}(t)$ ($\theta_{ng} = 15^\circ$) was also present ($10 \log_{10}[\sigma_{s_o}^2/\sigma_{s_{ng}}^2] = -3\text{dB}$). The speed of the mobile was 6.7km/hr which at a carrier frequency of 900MHz, corresponded to a maximum Doppler frequency, $\omega_d = 2\pi/3$ rad/sec. The optimum weights for the two beamformers were computed via (15) with sample correlation matrices instead of the true. The beam patterns, $g(\theta) = |\mathbf{w}_{opt}^H \mathbf{a}(\theta)|^2$, $-90^\circ < \theta < 90^\circ$ for the moving and static

array are shown in Figure 2. The performance of the beamformer using the proposed smoothed correlation matrix (Figure 2a) is superior when compared to the beamformer using the conventional statistics (Figure 2b). The MVDR beamformer fails to place nulls at the interferences in the latter case [7].

Example 2: In this example, the signal $s_o(t)$, $s_i(t)$ and $s_u(t)$ were all narrowband, non-Gaussian occupying the same frequency band. The spatially white additive Gaussian noise was added (SNR= 10dB). The third signal $s_u(t)$ was independent of the former two ($10 \log_{10}[\sigma_{s_o}^2/\sigma_{s_u}^2] = 0\text{dB}$). The interference $s_i(t)$ was correlated with $s_o(t)$ and ($10 \log_{10}[\sigma_{s_o}^2/\sigma_{s_i}^2] = 13\text{dB}$). The directions of arrival of the three signals were $\theta_o = 60^\circ$, $\theta_i = -40^\circ$ and $\theta_u = 10^\circ$. The matrix $\hat{\mathbf{C}}_{xx}$ was estimated from $T = 256$ data points and the desired signal or "look" direction was $\theta = 60^\circ$. Figure 3 shows the beam pattern of the arrays (i) when Doppler is exploited and the matrix $\hat{\mathbf{C}}_{xx}$ is used to obtain the beamformer weights (Figure 3a) and (ii) the conventional scenario where the processing is done so as to avoid the fading due to vehicle motion (Figure 3b). Clearly, the interferences (coherent and noncoherent) are better suppressed in the former than in the latter. For comparison, the MUSIC spectrum computed using the proposed time-averaged matrix and the conventional (without Doppler) is shown in Figure 1. The solid line indicates three peaks while the MUSIC spectrum computed using data intercepted from a static array shows only one peak due to coherent signals.

5. CONCLUSIONS

Methods that exploit the Doppler phenomena that shows up naturally in a mobile communications environment are proposed for the design of beamformers that are able to extract the desired signal even when it is correlated with the unwanted interferers. The time-varying statistics of the Doppler induced signals are mapped onto time-invariant correlations that can be estimated in practice via sample averages. Preliminary analysis on the optimal speeds for achieving good decorrelation for typical signal scenarios was given. A limitation of the proposed method is that it requires large number of antennas. One way to overcome this problem is by reducing the number of interferers (and hence reduce the number of antennas) by first exploiting other signal properties such as code orthogonality (or near orthogonality) to suppress the level of the interfering users. The model with transmitter motion and its effect on signal coherence needs to be studied. Asymptotic analysis of the proposed algorithms and performance studies with finite data and SNR levels

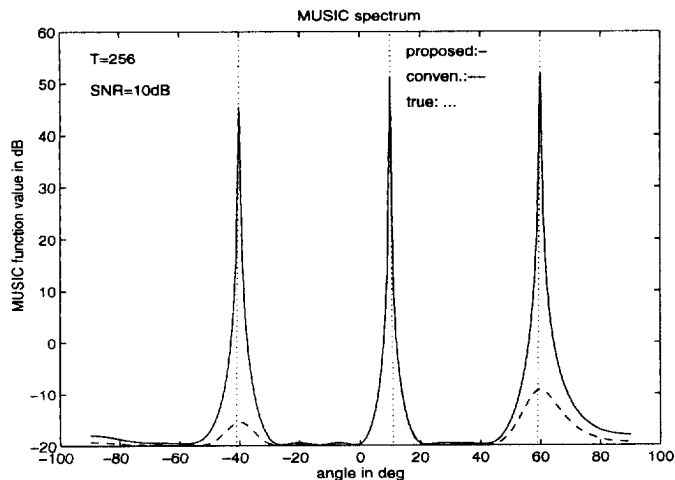


Figure 1. MUSIC spectrum with moving (solid) and static (dashed) arrays. Signal DOAs were 60° , -40° and 10° .

will be reported in the future.

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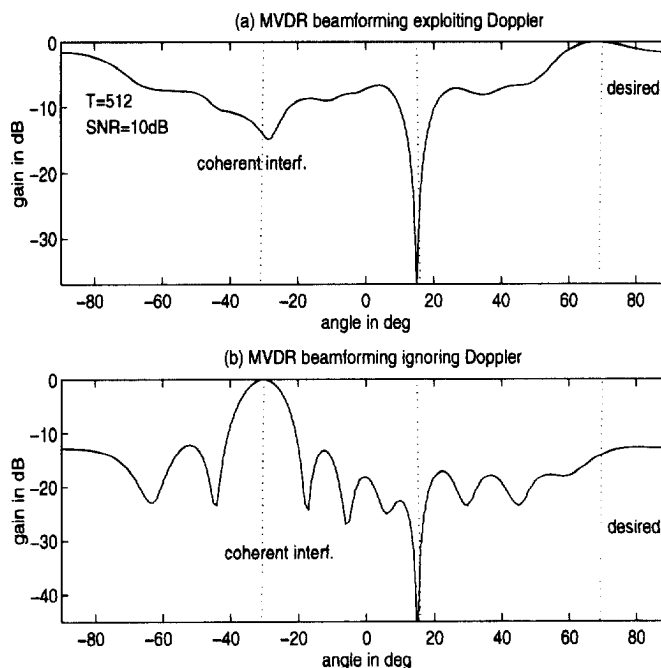


Figure 2. Beamformer responses. The desired DOA $\theta_o = 70^\circ$. In (b) there is only a single null at $\theta_{ng} = 15^\circ$, while the additional null at $\theta_i = -50^\circ$ in (a) indicates coherent interferer.

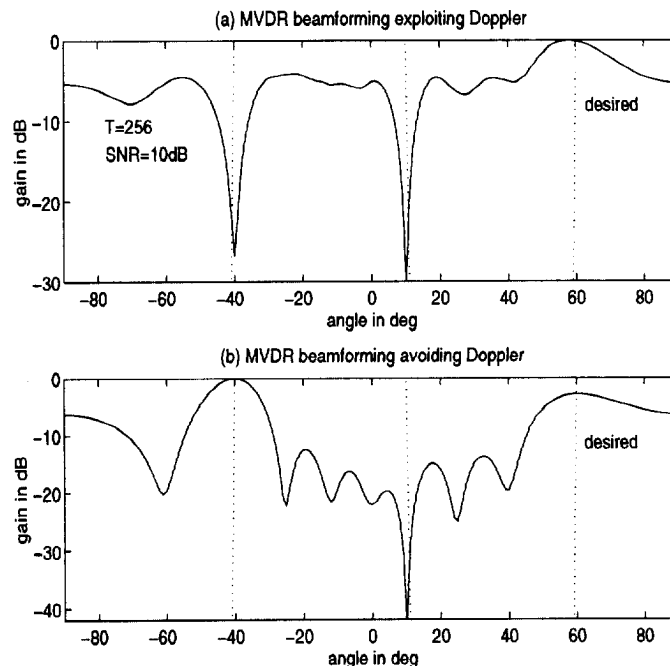


Figure 3. Beamformer responses. The desired DOA $\theta_o = 60^\circ$. In (b) there is only a single null at $\theta_u = 10^\circ$, while the additional null at $\theta_i = -40^\circ$ in (a) indicates coherent interferer.