Characterization of Fast Fading Vector Channels for Multi-Antenna Communication Systems

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ABSTRACT

In a wireless communication environment, multipath propagation and mobile user motion result in a dispersive time-varying communication channel. In order to analyze the performance of wireless communication systems, it is necessary to define models which reasonably approximate the time varying impulse response of the radio channel. Many widely accepted scalar channel (single antenna) statistical models have been reported. In order to analyze performance for recently proposed real time adaptive antenna array techniques, it is necessary to extend existing scalar channel statistical models to the vector channel (multiple antenna) case. This paper develops a statistical, time varying, dispersive, wireless vector channel model which is based on the physical propagation environment. The model is shown to be consistent with the known characteristics of the wireless communication channel. Antenna pattern interference rejection as a function of update rate is presented as an application example for the vector channel model.

1. INTRODUCTION

Communication between a base station and a mobile user over a wireless radio channel is difficult. Due to mobile motion, the channel response can vary rapidly in time. Radio propagation effects cause the channel signal to noise ratio to span an extremely large dynamic range. Finally, interference from other nearby radio links is usually a concern. Adaptive antenna techniques offer some of the most effective system tools for improving wireless communication link performance. First generation wireless communication systems use base station antenna selection diversity to improve radio link quality [1-3]. Second generation antenna systems use adaptive beam-forming at the base station to create radiation patterns which enhance *average* received signal power while suppressing *average* received interference power [4]. In recent years, third generation real-time adaptive base station antenna techniques have been proposed which are intended to improve *instantaneous* signal to interference power ratio [5, 6].

To support the development of real time adaptive antenna techniques, multiple antenna (vector channel) modeling tools are needed to analyze proposed approaches and predict performance. Some multiple antenna channel models have been reported [7, 8]. However, these existing model descriptions are incomplete in that they do not simultaneously account for channel time variation, fading correlation between antenna elements, distance loss and shadowing effects. The purpose of this paper is to derive a compact statistical model for the multiple user wireless vector channel which accounts for all of these effects. The new model extends existing scalar channel models to the vector channel case. The model is shown to be consistent with the known characteristics for the single antenna Rayleigh fading channel, as well as the spatial correlation behavior of the multiple antenna channel [9-13].

2. VECTOR CHANNEL MODEL

The most severe fading in a wireless channel occurs when there is no line of sight between the mobile and the base station. This Rayleigh fading case often arises in practice and is the most difficult environment in which to apply adaptive antennas. The model developed below describes a Rayleigh fading vector channel. The model considers multiple (single antenna) co-channel mobile users and one (multiple antenna) base station. For simplicity, only azimuth angles are considered in the propagation geometry but the results can be generalized to three dimensions. It is assumed that the mobile antennas radiate uniformly in azimuth angle. The mobile to base station radio up-link is emphasized. The results can easily be applied to a multiple antenna base station down-link [14], or a multiple antenna mobile.

The propagation environment under consideration is densely populated with large buildings and other structure. An illustration of the propagation channel is presented in Figure 1. The mobile radiation pattern illuminates all local structure surrounding the mobile which is within line of sight. In any given angular direction (with respect to the mobile coordinate system), a finite number of features in the local structure contribute to radiation in that direction. These features are termed *local reflectors*. The magnitude of the radiation intensity in a certain angular direction is

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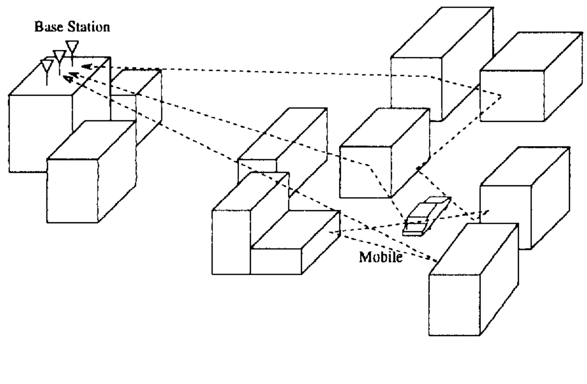

Figure 1: An illustration of the propagation channel

given by

$$\beta(\phi) = K \sum_{n=1}^{N_\phi} C_n(\phi) \qquad (1)$$

The complex radiation intensity in the direction ϕ is $\beta(\phi)$. The complex reflection coefficient of each local reflector in a direction ϕ is $C_n(\phi)$ and is assumed to exhibit a complex Gaussian distribution function $\frac{1}{\pi\sigma_C^2} \exp\left(-\frac{\|C_n(\phi)\|^2}{\sigma_C^2}\right)$. The total number of local reflectors contributing significantly to radiation in the direction ϕ is N_ϕ. The propagation constant K includes the effects of mobile antenna gain, and transmit power. In all that follows, for a given radiation angle ϕ_l, $\beta(\phi_l)$ will be expressed β_l to simplify the notation.

As the vehicle position varies, the phase length to each local reflector changes, resulting in a varying complex radiation intensity. For a mobile moving with velocity v, the maximum Doppler shift is $\omega_d = 2\pi\frac{v}{\lambda}$ where the RF carrier wavelength is λ. The Doppler shift results in a time varying radiation pattern with complex intensity as a function of time in a certain direction given by

$$\beta_l(t) = K \sum_{n=1}^{N_{\phi_l}} C_n(\phi_l) e^{j\omega_d \cos(\Omega_n^{(l)})t} \qquad (2)$$

where Ω_n is the angle with respect to the velocity vector of local reflector n. Equation (2) results in a radiation pattern with a complex Gaussian distribution in all directions.

Radiation from the local reflector structure reaches the base station after it is again reflected from large reflecting objects present in the environment. These objects are termed *dominant reflectors*. Scalar channel models do not explicitly account for the dominant reflector contributions to the radio path. For the vector channel, the angle of arrival of each path with respect to the base station coordinate system determines the essential nature of the channel. The angle of arrival for each path is determined by the physical location of a dominant reflector. Therefore, we must consider these secondary reflections in our vector channel model. Dominant reflectors are, in general, large reflecting surfaces such as large buildings, collections of several buildings, or natural geographic features. Several dominant reflectors often contribute strongly to the radio link. The large distance separation between dominant reflectors gives rise to a time delay difference for each dominant reflector path.

The transmitted mobile communication signal is modeled as a complex analytic baseband signal $s(t)$. For a particular dominant reflector propagation path, the received signal at the base station is given by

$$\begin{aligned} \mathbf{x}_l(t) &= \mathbf{a}(\theta_l) \cdot \beta_l(t) \cdot \sqrt{\Gamma_l \cdot \psi(\Delta_l)} \cdot s(t - \Delta_l) \\ &= \mathbf{a}(\theta_l)\alpha_l(t)s(t - \Delta_l) \qquad (3) \end{aligned}$$

where

$$\mathbf{a}(\theta_l) = [a_{l,1} \quad a_{l,2} \cdots a_{l,M}]^T$$

The angular position of dominant reflector l with respect to the mobile coordinate system is ϕ_l. The angular position of dominant reflector l with respect to the base station coordinate system is θ_l. The array response vector, $\mathbf{a}(\theta_l)$, is given by an array manifold mapping which characterizes the complex array response values for each element of the array as a function of θ [15]. The time delays Δ_l are generated from a random distribution which is usually taken as Poisson [16], or modified Poisson [17]. In most modeling applications, the difference between these two distributions is unimportant. A simplification to the Poisson distribution can be made by conditioning on a maximum possible delay. This results in a uniform time delay distribution. In practice, this uniform distribution is a worst-case assumption for most channel equalization schemes and is thus not only simple to implement, but is also a good simulation test case. The effects of propagation loss result in a reduction in average impulse energy as impulse time delay increases. This average effect is accounted for by the deterministic power-delay profile variable $\psi(\Delta_l)$. Based on analysis of empirical data [18], we have chosen $\psi(\Delta_l)$ as an exponentially decaying function of Δ_l in the log domain, *i.e.*

$$\psi(\Delta_l) = 10^{e^{-\frac{\alpha\Delta_l}{10}}} \qquad (4)$$

Experimental data for wireless channels show that paths which arrive with long delays can often have large energy relative to the first arriving path, even though there is an average trend which decays with arrival time [18]. This effect is accounted for by the partially uncorrelated log-normal variable Γ_l. This is the variable which models distance loss and shadow (log-normal) fading with the distribution function

$$\begin{aligned} \mathbf{\Gamma} &= [\Gamma_1 \quad \Gamma_2 \cdots \Gamma_L] \\ \mathbf{\Gamma} &\sim 10^{\gamma/10}, \quad \gamma = \mathbf{R}_\gamma \zeta \qquad (5) \\ \zeta_l &\sim \mathcal{N}\left(-10\eta \log d, \sigma_\gamma^2\right) \end{aligned}$$

The coloring matrix $\mathbf{R}_\gamma$ determines the correlation in log-normal fading between paths and ζ is a white jointly Gaussian random vector with a mean of $-10\eta \log d$ and standard deviation σ_γ where d is the distance from the mobile to the base station. The path loss exponent η and the log-normal standard deviation σ_γ are empirically derived constants [1].

As we change angular direction around the mobile, different local reflectors will begin to contribute to the radiation in that direction. If we make a large enough change in angular direction, we will find that a completely different set of local reflector coefficients will determine the fading in that direction. This results in fading which is essentially uncorrelated over a relatively small change in angle with respect to the mobile. With a high probability, the dominant reflectors will be well separated in angular position with respect to the mobile. Thus, we assume that each dominant reflector path fades independently. We can now express the received time domain signal vector as the sum of all dominant reflector paths and the additive receiver noise vector $\mathbf{n}(t)$.

$$\mathbf{x}(t) = \sum_{l=1}^{L} \mathbf{x}_l(t) + \mathbf{n}(t) = \sum_{l=1}^{L} \mathbf{a}(\theta_l)\alpha_l(t)s(t-\Delta_l) + \mathbf{n}(t) \quad (6)$$

where there are L dominant reflector paths which contribute significantly to the channel. We can re-write (6) to express the received signal vector as the output of a linear time variant vector system

$$\mathbf{x}(t) = \int_{-\infty}^{\infty} \mathbf{a}(t,\tau)s(t-\tau)d\tau + \mathbf{n}(t) \quad (7)$$

where

$$\mathbf{a}(t,\tau) = \sum_{l=1}^{L} \mathbf{a}(\theta_l)\alpha_l(t)\delta(\tau-\Delta_l) \quad (8)$$

is the time varying vector channel impulse response. We can now extend (7) to include the effects of additional co-channel signals by writing the matrix equation

$$\begin{aligned} \mathbf{x}(t) &= \sum_{k=1}^{N_U} \mathbf{x}_k(t) + \mathbf{n}(t) \\ &= \int_{-\infty}^{\infty} \mathbf{A}(t,\tau)\mathbf{s}(t-\tau)d\tau + \mathbf{n}(t) \end{aligned} \quad (9)$$

where

$$\begin{aligned} \mathbf{A}(t,\tau) &= [\mathbf{a}_1(t,\tau) \quad \mathbf{a}_2(t,\tau)\cdots\mathbf{a}_{N_U}(t,\tau)] \\ \mathbf{s}(t) &= [s_1(t) \quad s_2(t)\cdots s_{N_U}(t)]^T \end{aligned}$$

and N_U is the total number of co-channel users.

The utility of the model described in (9) is that it includes the effects of multiple users in a time varying dispersive vector channel in a mathematically tractable formulation. In the next section we show that this model is consistent with the known statistical behavior of wireless channels.

3. STATISTICAL PROPERTIES OF THE MODEL

In this section we shall derive some of the statistical properties of the channel model developed in Section 2. In particular, we will derive the autocorrelation of the output of the array $\mathbf{x(t)}$ in the single user case. This derivation can also be easily extended to the multiple user scenario.

To derive the autocorrelation matrix of $\mathbf{x}(t)$ we use the expression in (6), with $\alpha_l(t)$ given by,

$$\alpha_l(t) = \beta_l(t)\cdot\sqrt{\Gamma_l\cdot\psi(\Delta_l)} \quad (10)$$

and from (2)

$$\beta_l(t) = K\sum_{n=1}^{N_{\phi_l}} C_n(\phi_l)e^{j\omega_d\cos(\Omega_n^{(l)})t} \quad (11)$$

Using (6), (10) and (11) we can find an expression for $E[\mathbf{x}(t)\mathbf{x}^H(t+\nu)]$.

$$\begin{aligned} E\left\{\mathbf{x}(t)\mathbf{x}^H(t+\nu)\right\} = &\sum_{l=1}^{L} E\left\{\mathbf{a}(\theta_l)\mathbf{a}^H(\theta_l)\right\} E\left\{\alpha_l(t)\alpha_l^*(t+\nu)\right. \\ &\left. s(t-\Delta_l)s^*(t+\nu-\Delta_l)\right\} \\ &+ E\left\{\mathbf{n}(t)\mathbf{n}^*(t+\nu)\right\} \end{aligned} \quad (12)$$

where the double sum is replaced by a single sum because the fading on each path is uncorrelated with the other paths. Let us define

$$E\left\{\mathbf{a}(\theta_l)\mathbf{a}^H(\theta_l)\right\}_{i,j} = \mathbf{Q}_{i,j} \quad (13)$$

We can show that for the case of a uniform linear array with a uniform spatial power spectral density over $[0, 2\pi]$ that

$$\mathbf{Q}_{i,j} = J_o(2\pi d(i-j)/\lambda) \quad (14)$$

where d is the inter element distance, λ is the wavelength, and $J_o(\cdot)$ is the Bessel function of the first kind and of order zero. This result is similar to results derived in literature [1, 19] on spatial correlation between antenna elements. Using (10) and (11), we obtain the following time autocorrelation result

$$E\left\{\beta_l(t)\beta_l^*(t+\nu)\right\} = N\sigma_c^2 J_o(\omega_d\nu) \quad (15)$$

where $\sigma_c^2 = E\left\{|C_n(\phi_l)|^2\right\}$. This result is again similar to results in the literature for the time correlation of the signals in a Rayleigh fading environment [1]. We now define

$$z_l(t,\nu) = E\left\{\psi(\Delta_l)s(t-\Delta_l)s^*(t+\nu-\Delta_l)\right\} \quad (16)$$

Then putting (12) - (16) together, we have an expression for the covariance matrix of $\mathbf{x}(t)$:

$$E\left\{\mathbf{x}(t)\mathbf{x}^H(t+\nu)\right\} = L\cdot\mathbf{Q}\cdot N\cdot\sigma_c^2 J_o(\omega_d\nu)z(t,\nu)E\left\{\Gamma\right\} \quad (17)$$

This result shows that the model developed in Section 2 is amenable to statistical analysis. Moreover, we can generalize this analysis to suit various statistical assumptions on the signal $s(t)$, time delay characteristics, spatial power spectral density and so on. Thus, the vector channel model provides us with a useful tool to evaluate/predict the performance of algorithms developed for an urban propagation environment using multiple antennas.

4. SIMULATIONS

In this section, we briefly summarize the urban vector channel model in terms of a simulation procedure. We then describe some applications of this vector channel model to estimate the time, and spatial correlation functions of the channel. Finally, we provide one example of how this model can be used to analyze adaptive antenna algorithm behavior. We examine the antenna pattern update rates required to achieve a given level of performance for a 7 element linear diversity array with a high velocity mobile user and high velocity mobile interferers.

The vector channel model can be simulated in the following manner. For a given user, N_{ϕ_l} and L are generated (these are usually fixed values for all users). By changing the number of local and dominant reflector paths (N_{ϕ_l}, L) the density of the simulation environment can be varied from a sparse rural model to a dense urban model. For each path, $\beta_l(t)$ is generated from (2) with a complex Gaussian distribution on $C_n(\phi_l)$ and a uniform distribution on $\Omega_n^{(l)}$. For each dominant reflector path, the received signal component due to that path is computed from (3). The spatial power spectral density distribution function for θ_l can be taken from one of the references [9, 10], or from another source. The distribution for Γ_l is found from (5) with the coloring matrix entries chosen to match empirical data. The path delay values are generated from a uniform distribution over $[0, \Delta_{max}]$. Distributions which are more complicated but model experimental data more accurately are described in [16, 17]. The delay trend function is given in (4), where a is again determined to match empirical data. The antenna array output is then found by summing over all dominant paths and adding the noise terms as in (6).

We derived certain statistical properties of this model in Section 3. The time correlation of the model is mainly dependent on a Bessel function of the first kind. The channel simulator described in this paper approximates this description quite well. To illustrate this, we simulated a vehicle traveling at 60 mph with an RF carrier frequency of 1 GHz in the environment described this paper. This is illustrated in Figure 2. The time correlation coefficient obtained using this model is plotted along with the theoretical result from Section 3. The time-autocorrelation plot shows us that in this worst-case scenario, samples which are separated by 4-5 msecs are uncorrelated. The time correlation is a parameter studied quite extensively in literature in the context of scalar channels. The importance of this model is that it combines both the temporal and spatial models into a single compact form.

We now examine the spatial correlation function of our vector channel model. The spatial correlation is very dependent on the spatial power spectral density. In this simulation we again assume a worst-case scenario where the angular spectrum is uniform over $[0, 2\pi]$. In Figure 3 we have plotted the spatial correlation of the model obtained through simulations along with the theoretical result from Section 3. This simulation is consistent with the often stated rule-of-thumb that a separation of half-wavelength leads to uncorrelated fading.

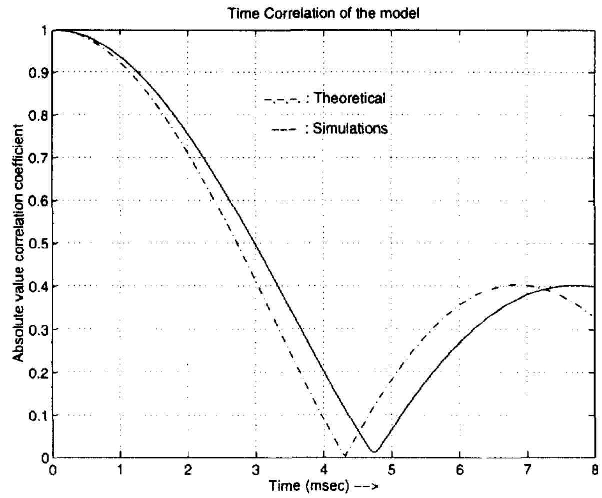

Figure 2: Time Correlation of the model.

We now briefly examine an application of the channel model. This application serves to illustrate the utility of this model in predicting performance of adaptive antenna algorithms in a urban fading environment.

The application described here is adaptive beamforming in the presence of fading and interferers. The modeled array is a 7 element linear diversity array with 10λ spacing. There is one desired mobile and six interfering mobiles all traveling at 60 mph. The spatial power spectral density for all mobiles is uniform with a $\frac{\pi}{4}$ spread and random central angles. In most adaptive beamforming techniques, a weight vector is estimated which steers the array toward the desired user. This weight vector is usually estimated either through blind algorithms or through training. The weight vector is typically estimated by using a block of data, over which the channel is assumed to be constant. For our application example, we examine the rate at which the weight vector needs to be updated. We do this by plotting the SNR degradation due to keeping the weight vector fixed over long periods of time. This is presented in Figure 4, where the relative SNR for the desired mobile is plotted against time. If the minimum acceptable degradation is about 10 dB below the initial value achieved just after the update, then update rates of the order of 1 KHz are needed.

5. CONCLUSIONS

In this paper we have developed a mathematically tractable model for fast-fading vector channels for use in urban mobile communication applications. This development is based on a physical propagation model. We have shown that several statistical properties of this model are consistent with existing wireless scalar channel models. The vector model provides a useful tool in understanding the multiple antenna wireless channel and analyzing performance of proposed adaptive antenna algorithms and architectures.

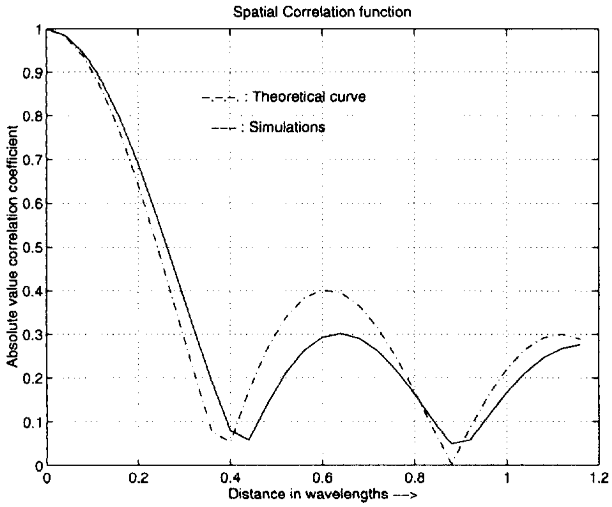

Figure 3: Spatial Correlation of the model.

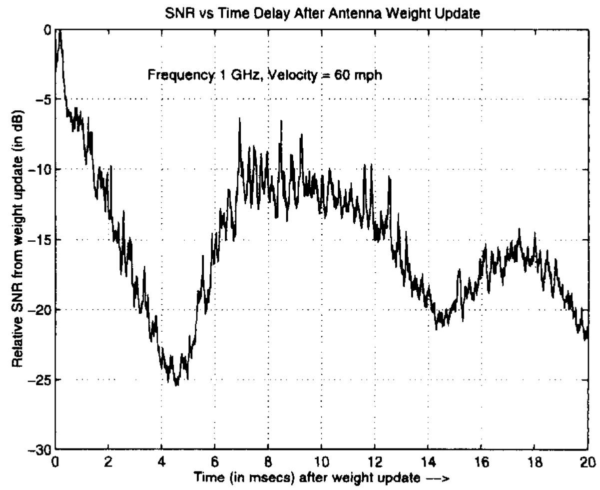

Figure 4: An application of the channel model. This illustrates the SNR vs time if the weights of an adaptive beamforming algorithm are kept frozen.

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