CFAR Acquisition of DS-SS Codes for CDMA Ranging

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Abstract

We present an adaptive thresholding acquisition procedure for use by a CDMA ranging system. The system provides constant false alarm rate (CFAR) in acquiring direct sequence spread spectrum signals in a multi-user environment. A bank of matched filters is used to match the received signal against the various user codes. A user is declared present when the energy in the output any filter exceeds a threshold proportional to the energy in the received signal itself. This procedure provides CFAR acquisition in a white gaussian noise level of unknown level. Most importantly, the method also provides a bounded probability of false detection as well. We present performance results which illustrate this decrease in false detections due to the CFAR thresholding with matched filtering. The proposed system is a very simple method for multiuser acquisition of CDMA signals.

1 Introduction

Direct Sequence Spread Spectrum(DS-SS) signals in ranging obtain high power, high resolution performance using a low power long duration coded pulse. Upon reception, the coded pulse is compressed using a filter that is typically matched to the coded subpulses of the ranging pulse. The book by Levanon[1] provides an excellent overview of this material and supplies a guide to the literature.

The ranging, or acquisition, information is normally obtained by determining the time of arrival of the pulse; comparing the output of the compression filter to a fixed threshold. The lower the threshold, the higher the probability of detection,(P_d), but this also increases the probability of a false alarm (P_{fa}). The filter output consists of undesired range sidelobes in addition to the peak response of the compressed pulse. Even when the codes are chosen carefully, the range sidelobes increase the probability of of falsely registering an detection, (P_{fd}).

For Code Division Multiple Access (CDMA), different users are each assigned unique codes. Ideally these are orthogonal for all time shifts, but practically, there is always a non-zero response when one users code passes through the filter of another user. This cross-correlation function, acts as interference to the detection of the desired user. As with the case with range sidelobes, cross-correlations increase the probability of a false detection, P_{fd}. If the interfering user has high signal-to-noise ratio, (SNR), P_{fd} might be very large to allow for a reasonable P_d. This is called the near-far problem. How do you set your threshold to provide a high P_d for a low SNR desired user while being resistant, low P_{fd}, to high SNR interference users? There has been considerable research in designing optimal detectors that solve this problem. Unfortunately, these optimal systems become very complex with increasing number of users.

CFAR detectors use adaptive thresholding to control the probability of false alarm. This is achieved by adapting the fixed threshold using an estimate of the energy in the received signal. This allows the system designer to set the threshold parameter without knowledge of the background noise level: CFAR. Another prime benefit of this technique is near-far resistance. This occurs naturally when scaling the threshold by the received energy. Other than at the peak response of an auto-correlation function, this energy estimate will be larger than the output of the compression filter. This is used to increase the adaptive threshold, thus reducing the probability of false detection of range sidelobes or cross-correlation interference. Remarkably, this simple adaptation provides a bounded P_{fd}, as the interference SNR increases without bound.

The scope of the paper is as follows. First, a matrix

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model is presented for the CDMA system. Next, the CFAR detector is introduced and a rigorous derivation of the calculation of the detection probabilities, (P_d, P_{fd}, and P_{fa}), is given. Performance curves of the CFAR detector are given and compared to a fixed threshold detector. Finally, a conclusion summarizes the paper.

2 Background

The performance of a CDMA ranging system is determined by the codes and filters assigned to its users. It is the response of the filters to the different codes that determine the detection probabilities P_d and P_{fd}. For optimal P_d the filters are chosen to be matched to the codes.

In a standard noncoherent ranging detector the power in the output of the compression filter is compared to a fixed threshold. In an CFAR detector the same output is compared to an adaptive threshold. To determine the performance measures of either of these detectors we need to model the output the compression filter at each decision time. In the next section we define the matrix notation which will allow us to model the system in the general case.

2.1 Modeling

To evaluate the probabilistic performance of a detector we need to model the output of the compression filter at single sample time. It is this variate that we compare to a threshold to determine the presence of a user. A matrix model of the system provides insight and ease of evaluation.

Consider the received signal vector $\mathbf{x}$ to be a $N \times 1$ complex vector which corresponds to the last N samples of the received signal. These vector components are the last N samples contained in the tapped delay line,

$$\mathbf{x}^t = [x[n], x[n-1], \cdots, x[n-(N-1)]] . \quad (1)$$

Let the the filter vector, $\mathbf{h}$, be the $N \times 1$ complex vector containing the compression filter in correlation form,

$$\mathbf{h}^t = [h[0], h[1], \cdots, h[N-1]] . \quad (2)$$

The output of the compression filter at a single point in time is the inner product of the received signal vector and the compression filter vector: $\mathbf{y} = \mathbf{h}^{\dagger}\mathbf{x}$, where † represents the operation of conjugate transpose.

The filter vector is normalized to have unity energy, $|\mathbf{h}|^2 = 1$, and can be one of M different filters of the CDMA system, $\mathcal{H} = \{\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_M\}$.

We model the received signal as the combination of a signal component and additive gaussian white noise,

$$\mathbf{x} = A\mathbf{s} + \mathbf{w} \quad (3)$$

$\mathbf{s}$ and $\mathbf{w}$ are $(N \times 1)$ complex column vectors. These are the signal vector and gaussian white noise vectors respectively. The noise vector is complex multivariate gaussian white noise with zero mean and correlation matrix $\frac{1}{2}E\left\{\mathbf{w}\mathbf{w}^{\dagger}\right\} = \sigma^2\mathbf{I}$.

A is a real scalar amplitude that is related to the SNR of the signal. The SNR of the received code is defined with respect to the peak response of a matched filter, $SNR = A^2/2\sigma^2$. For the peak response the signal vector, $\mathbf{s}$, contains the code of the desired user:

$$\mathbf{s} = \mathbf{c} = [c[0], c[1], \cdots, c[N-2], c[N-1]] . \quad (4)$$

with $|c|^2 = 1$.

The signal vector, $\mathbf{s}$, is taken from the set of $M(2N-1)+1$ vectors representing the different possible snapshots of the codes as they pass through the tapped delay line. This is mathematically shown as $\mathbf{s} \in \mathcal{C}$, where $\mathcal{C}$ is made of different subsets corresponding to the different CDMA users, $\mathcal{C} = \left\{\left\{\mathcal{C}^1\right\}, \left\{\mathcal{C}^2\right\}, \cdots, \left\{\mathcal{C}^M\right\}, \mathbf{0}\right\}$. Since the code vector, $\mathbf{c}$, has unity the energy the signal vector $\mathbf{s}$ has variable energy between zero and one. The inner product of the signal vector, $\mathbf{s}$, and the compression filter, $\mathbf{h}$, define a location in a correlation function. We can define the absolute value of this inner product as the correlation coefficient of the filter and the signal vector,

$$\rho = \left|\mathbf{h}^{\dagger}\mathbf{s}\right| = |\rho_R + j\rho_I| . \quad (5)$$

If we compute ρ for each possible component in the subset $\mathcal{C}^i$ with filter vector $\mathbf{h}_j$ we obtain the correlation function between user i and user j. If $i = j$ the function is called an auto-correlation function, otherwise it is referred to as a cross-correlation function.

Table 2.1 summarizes the different choices of $\mathbf{s}$ for a fixed choice in the compression filter, $\mathbf{h} = \mathbf{h}_i$. The corresponding performance measure for the choice of $\mathbf{s}$ is also given.

3 CFAR Adaptive Threshold Detector

In this section we define the CFAR detector and present the computation of the probability performance measures for this detector. The constant false

Table 1: Properties and examples of s given $\mathbf{h} = \mathbf{h}_i$

Subset	Prob.	Example
$s \in \{C^i\}$	P_d	$s^T = \left[c^i[0], c^i[1], \cdots, c^i[N-2], c^i[N-1]\right]$
	P_{fd}	$s^T = \left[c^i[2], c^i[3], \cdots, c^i[N-1], 0, 0\right]$
$s \in \{C^j\}$	P_{fd}	$s^T = \left[c^j[0], c^j[1], \cdots, c^j[N-2], c^j[N-1]\right]$
		$s^T = \left[0, 0, c^j[0], \cdots, c^j[N-3]\right]$
$s \in \{0\}$	P_{fa}	$s^T = [0, 0, \cdots, 0, 0]$

alarm rate is obtained by estimating the energy in the received signal and using this to adjust the threshold. Figure 1 shows a block diagram of the CFAR detector.

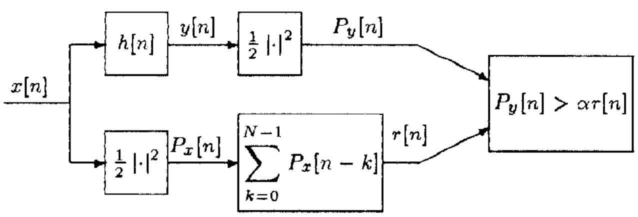

Figure 1: CFAR Adaptive Threshold Detector

Consider the case when no noise is present in the system. If we look at the ratio of $P_y[n]$ over $r[n]$ we obtain something we call CFAR normalized correlation functions. Comparing a location in this function to the fixed threshold parameter α, is equivalent to comparing $P_y[n]$ to the adaptive threshold $\alpha r[n]$. From the properties of the detector, the ratio $P_y[n]/r[n]$ will always be between zero and one, regardless of the amplitude A.

Define each value in these correlation functions as a CFAR correlation coefficient, ρ_k. Where ρ_k is defined as,

$$\rho_k = \frac{\rho^2}{1 - \frac{|k|}{N}}, \tag{6}$$

and k is the offset from the peak response.

At high SNR, no noise, the threshold should be chosen such that the peak response of the auto-correlation function will be detected while the range sidelobes and cross correlation values will not. As the energy in the noise increases a noise cloud forms around each point in the correlation function.

At low SNR, the uncertainty at each location in the correlation function becomes large. This will give locations whose ρ_k is less than α a positive probability being detection, $P_{fd} > 0$. Also this will decrease the probability of detecting the peak response, $P_d < 1$. What we would like is to set α for a given P_{fa} and then evaluate P_d and P_{fd} for varying SNRs.

3.1 Performance Evaluation

The determination of P_d, P_{fd}, or P_{fa}, consist of evaluation of the probability of one random variable exceeding another. In the case of the CFAR detector this is the probability that $P_y[n] > \alpha r[n]$. In the general case this is expressed as,

$$P = Pr\left[\frac{1}{2}\left|\mathbf{h}^\dagger \mathbf{x}\right|^2 > \alpha \frac{1}{2}\left|\mathbf{x}^\dagger \mathbf{x}\right|\right]. \tag{7}$$

Which performance measures corresponds to P depends on the choice of the signal vector s.

An initial difficulty in computing P is that the left and right hand side of the detection event are dependent. This is solved by transforming the problem into a new coordinate system. This coordinate system will allow for separation of the problem into sums of independent random variables.

The basis of this new coordinate system should include $\mathbf{h}$ as one of its members. An entire basis set is determined from the compression filter $\mathbf{h}$ and N-1 random vectors using the modified Gram-Schmidt procedure, [3]. This basis set can be represented as matrix, $\mathbf{\Phi} = \left[\phi_0, \phi_1, \cdots, \phi_{N-1}\right]$, with $\phi_0 = \mathbf{h}$. Since the columns of $\mathbf{\Phi}$ form a basis the matrix is unitary, $\mathbf{\Phi}^\dagger \mathbf{\Phi} = \mathbf{I}$.

The received signal can be represented as the linear combination of the basis set, $\mathbf{x} = \mathbf{\Phi a}$, where $\mathbf{a}$ is the $(N \times 1)$ column vector containing the coefficients of the basis vectors. The coefficients are obtained by projecting the received signal vector onto the basis set, $\mathbf{a} = \mathbf{\Phi}^\dagger \mathbf{x}$. These coefficients are independent gaussian random variables with the following distributions, $\mathbf{a} \sim \mathrm{CMVN}(A\mathbf{\Phi}^\dagger \mathbf{s}, \sigma^2 \mathbf{I})$.

Substituting for $\mathbf{x}$ in the left hand side of the detection event produces,

$$\frac{1}{2}\left|\mathbf{h}^\dagger \mathbf{x}\right|^2 = \frac{1}{2}\left|\mathbf{h}^\dagger \mathbf{\Phi a}\right|^2 = \frac{1}{2}|a_0|^2. \tag{8}$$

Doing the same for the right hand side of the detection event gives,

$$\frac{1}{2}\left|\mathbf{x}^\dagger \mathbf{x}\right| = \frac{1}{2}\left(\mathbf{\Phi}^\dagger \mathbf{a}\right)^\dagger \left(\mathbf{\Phi}^\dagger \mathbf{a}\right) = \frac{1}{2}\mathbf{a}^\dagger \mathbf{a}. \tag{9}$$

When these representations are incorporated into the overall detection event the following simplification arises,

$$\left\{\tfrac{1}{2}\left|\mathbf{h}^\dagger\mathbf{x}\right|^2 > \alpha\tfrac{1}{2}\left|\mathbf{x}^\dagger\mathbf{x}\right|\right\} \implies \left\{\tfrac{1}{2}\left|a_0\right|^2 > t\sum_{k=1}^{N-1}\tfrac{1}{2}\left|a_k\right|^2\right\} \tag{10}$$

where, $t = \alpha/(1-\alpha)$.

The probability performance measures , P, now is the comparison of two independent random variables,

$$P = Pr\left[\frac{1}{2}\left|a_0\right|^2 > t\sum_{k=1}^{N-1}\frac{1}{2}\left|a_k\right|^2\right]. \tag{11}$$

Define the random variables $v_k = \frac{1}{2}\left|a_k\right|^2$ with $k = 0, \cdots, N-1$. Since the a_k's are independent gaussian random variables the v_k's are independent noncentral chi-squared random variables with two degrees of freedom. The Moment Generating Function (MGF) of the v_k's is given by,

$$h_{v_k}(u) = E\left\{e^{-uv_k}\right\} = \frac{1}{(1+\sigma^2 u)}\exp\left\{\frac{-\frac{1}{2}\left|A\phi_k^\dagger \mathbf{s}\right|^2 u}{1+\sigma^2 u}\right\} \tag{12}$$

We can express the right and left side of the detection event in terms of the v_k's. Let $X = v_0$ and $Y = \sum_{k=1}^{N-1} v_k$. The MGF of X is just the MGF of v_0, and the MGF of Y is just the product of the MGF's of the v_k's, $k = 1, \cdots, N-1$:

$$h_X(u) = \frac{1}{(1+\sigma^2 u)}\exp\left\{\frac{-\frac{1}{2}\left|A\mathbf{h}^\dagger \mathbf{s}\right|^2 u}{1+\sigma^2 u}\right\} \tag{13}$$

$$h_Y(u) = \prod_{k=1}^{N-1}\frac{1}{(1+\sigma^2 u)}\exp\left\{\frac{-\frac{1}{2}\left|A\phi_k^\dagger \mathbf{s}\right|^2 u}{1+\sigma^2 u}\right\} \tag{14}$$

Notice that the MGF of X depends on only A^2, ρ^2, and σ^2. The MGF of Y depends on σ^2 and the energy in the signal vector in the directions other than $\mathbf{h}$. This means that the detection probabilities do not depend directly on the codes and filter of the system but only on the ρ, and thus the ρ_k, that they produce.

By subtracting tY from both sides detection event can be expressed as comparing a chi-squared random variable to a deterministic threshold,

$$P = Pr[X > tY] = Pr[X - tY > 0], \tag{15}$$

This probability function is just one minus the c.d.f of a random variable. This is something we refer to as the e.d.f. of a random variable. Since we know the MGF, P is computed by solving the contour integration problem,

$$P = \int_{C-}(-u)^{-1}h_X(u)h_Y(-tu)\frac{du}{2\pi j} \tag{16}$$

This problem is solved using the saddlepoint integration method outlined in the appendix in the paper by Iltis, Ritcey, and Milstein [4].

4 Results

All detection probability curves for the CFAR detector and the fixed threshold detector were generated assuming length 30 four phase codes with matched filtering. The threshold parameter α was set according to the P_{fa} that it generated. The secant root finding method, [5], was used to find the α which produced the desired P_{fa}.

Figure 2 shows the detection probability P for different cases. It is a plot of P for varying ρ_k at two different locations in a correlation function. ρ_k varies from 0 to 1 in intervals of 0.05 in this plot. The threshold was set to produce a P_{fa} of 10^{-4}. Whether these curves represent P_d or P_{fd} curves is up to interpretation of the reader. What value ρ_k makes sense for the system? Usually, P_d will correspond to the curve for $k = 0$ and $\rho_k = 1$, but not necessarily. Distortion in the system or use of mismatched compression filters could push down the peak response of the autocorrelation resulting in $\rho_k < 1$. The only curves that really don't fit reasonable situations are the cures for high ρ_k for the case $k = 29$, the edge of the correlation function. These were included for allowing visual comparison of the different choices of k.

The important thing to notice is that for ρ_k's that would correspond to P_{fd}, low ρ_k, the curves tend toward zero for increasing SNR. This is due to that fact that as SNR increases the variance of the noise cloud around the points in the CF normalized correlation functions approaches zero. Thus as long as the threshold α was chosen above ρ_k for that location there will never be a detection. If α was chosen to be equal to ρ_k then P would go to 0.5 as SNR increased to infinity. In a fixed threshold detector as SNR increases the correlation location will rise above the fixed threshold and the detection probability of that location will go to one. If that location is a range sidelobe or a cross-correlation location it will cause a large P_{fd}. This is the near-far problem discussed in the introduction. The CFAR detector solves this problem by normalizing the output of the compression filter by the energy in the received signal.

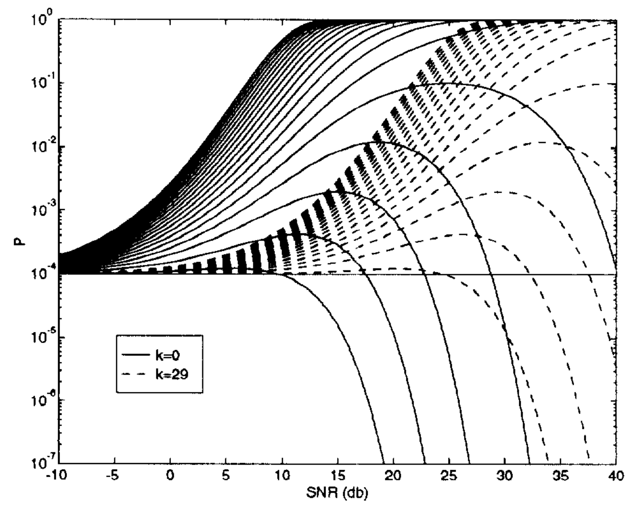

Figure 2: P versus SNR for Different ρ_k at $k = 0, 29$ for length 30 matched filter system. $\rho_k = 0 : 0.05 : 1.0$.

There is a cost in using a CFAR detector. Figure 3 shows P_d curves of both the CFAR detector and a fixed threshold detector. For both detectors, we take $\rho = 1$ and $\rho_k = 1$. From this plot it is apparent that

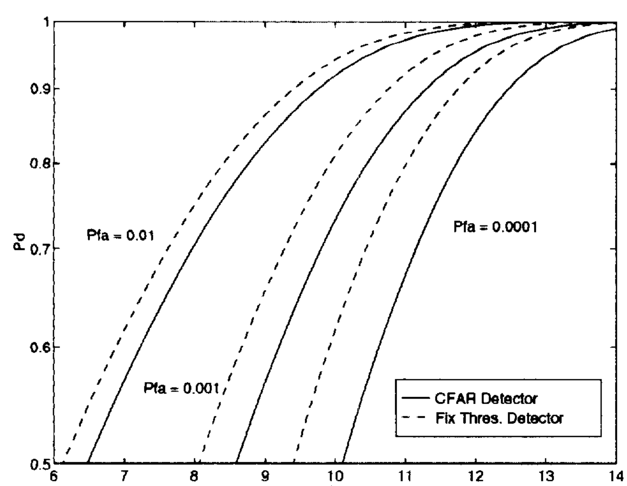

Figure 3: P_d versus SNR (dB) for CFAR detector and fixed threshold detector. Three different thresholds $P_{fa} = 10^{-2}, 10^{-3}, 10^{-4}$.

there is a slight cost in SNR to obtain the same P_d for the CFAR detector compared to the fixed threshold receiver. This is quite small considering the gain in P_{fd} performance.

5 Conclusion

We have shown how the CFAR adaptive threshold detector provides near-far resistance in a multi-user CDMA ranging system. With proper choice of the threshold parameter α the P_{fd} goes to zero with increasing SNR. The maximum value of P_{fd} depends on the codes and filters used in the CDMA system by the CFAR correlation coefficient, ρ_k.

The general method used to compute the detection probabilities allows for the study of many different situations. Distortion in the correlation functions due to frequency offset, intersymbol interference, use of mismatched filters, can be studied through analysis of their detection probabilities. These different cases correspond to a change in the value of ρ_k.

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