A Bayesian Marginalization Approach for Improved EEG Dipole Localization †

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ABSTRACT

A Bayesian framework is utilized to derive a new EEG dipole localization method. Dipole orientation/amplitude parameters are integrated out of the full observation probability density function, yielding a proper marginal. The derived cost function essentially reduces to the condensed conditional maximum likelihood for a large number of electrodes, relative to the number of sources. In addition, the reduction of nuisance parameters by marginalization leads naturally to a model selection/source number detection scheme based on the concept of Bayesian evidence. Simulation results are included to show the superior estimation characteristics of the proposed method over maximum likelihood in low SNR, small spatial/temporal sample situations, and to provide a preliminary comparison of the proposed detection technique to AIC and MDL.

1 INTRODUCTION

Current dipole localization from EEG (Electroencephalogram) data is an increasingly popular approach for extracting functional brain information from scalp electrode data. However, the estimation of dipole locations (primary parameters of interest) depends also on a number of secondary, or nuisance parameters. These nuisance parameters may be associated with each dipole source (such as dipole orientation and amplitude) or with the physical head model (electrode gains/locations, and tissue layer thicknesses and conductivities). In previous work we have presented the MAP approach for the joint estimation of location and uncertain secondary parameters associated with the head model [1]. The results of these studies indicate that stochastic variations in the radii/conductivity parameters of a 4-sphere model of the human head ([2]) result in significant dipole source location estimation error primarily in higher SNR regions (this corresponds most often to cortical sources near the electrodes).

In this paper, a Bayesian marginalization approach is used to derive a new dipole localization cost function, where secondary parameters associated with the *source* (time varying dipole orientations/amplitudes) are integrated out, yielding a marginal density function that depends only on the observed data and dipole locations. In contrast to the performance of the MAP technique for uncertain head model parameters, the marginal cost function introduced here is shown to outperform maximum likelihood/least squares methods in low SNR and other stressful circumstances; for instance, when dipoles are located deeply in the head, and/or the number of sensors/snapshots is restricted. In addition, the use of proper priors for the secondary parameters leads to a model selection criterion based on the Bayesian evidence for each considered model.

2 Background

2.1 Forward Model and Notation

Assume an array of M sensors, receiving information from P dipole sources. Denote the gain matrix that describes the transfer function from dipole source locations to the external sensors as G. Different head models for EEG are incorporated into the forward model by specification of G. The general forward model for EEG is

$$\vec{f}(t) = G(L)q(t) + \vec{n}(t), \text{ for } t = 1, ..., N, \quad (1)$$

where $\vec{f}(t)$ represents the $(M \times 1)$ array observation at the time instant t, $\vec{n}(t)$ is the vector of sensor measurement noise, $G(L)$ is the $(M \times 3P)$ gain matrix corresponding to the $(P \times 3)$ matrix L of stacked row vectors $\vec{L}_i$, where each row defines a 3-d dipole location:

$$L = \begin{bmatrix} \vec{L}_1 \\ \vdots \\ \vec{L}_P \end{bmatrix}$$
$$\vec{L}_i = [L_{xi}, L_{yi}, L_{zi}],$$

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and

$$q(t) = \begin{bmatrix} \vec{q}_1(t) \\ \vdots \\ \vec{q}_P(t) \end{bmatrix} \quad (2)$$

represents the $(3P \times 1)$ composite dipole moment vector due to P current dipoles,

$$\vec{q}_i(t) = \begin{bmatrix} q_{xi}(t) \\ q_{yi}(t) \\ q_{zi}(t) \end{bmatrix}, \text{ for } i = 1, ..., P. \quad (3)$$

The sensor noise is assumed spatially and temporally white, with covariance $E\{\vec{n}(t)\,\vec{n}^T(t)\} = \sigma^2 I$.

2.2 ML Localization

Denote the collection of N data "snapshots" by the $(M \times N)$ matrix F. The dipole orientations/amplitudes are treated as deterministic parameters, thus our formulation is analogous to the *conditional* ML model [3],[4]. Under the assumption of temporally and spatially white Gaussian noise, the negative log likelihood is

$-\log p(F \mid L, q^T(1), ..., q^T(N), \sigma^2) =$

$$\frac{MN}{2}\log 2\pi + \frac{MN}{2}\log \sigma^2 + \frac{1}{2\sigma^2}\sum_{t=1}^{N} \mid \vec{f}(t) - Gq(t) \mid^2 . \quad (4)$$

It is well known that this function can be concentrated with respect to σ^2 and $q^T(1), ..., q^T(N)$ by replacing these values with their ML estimates [4], resulting in the *condensed* cost function:

$$v_{ML}(L) = \text{tr}\ \{P^{\perp}_{G(L)} F F^T\}, \quad (5)$$

where tr { . } represents the matrix trace operator, $P^{\perp}_{G(L)} = I_M - G(L)G^{\dagger}(L)$ is the projection matrix that projects onto the subspace orthogonal to the column space of $G(L)$, and $\dagger$ denotes the pseudoinverse. The dipole location estimates obtained by minimizing (5) over the $3P$ dimensional range space of L are also known as "regional dipoles" [5].

3 Bayesian Marginal Approach for Localization

Let I_p denote the prior assumptions for the model parameterized by p dipoles. In addition, let Q denote the collection of multiple dipole moment vectors, $q(1), ..., q(N)$. The marginal density function of interest for dipole localization is $p(F \mid L, I_p) =$

$$\int_{Q,\sigma} p(F \mid L, Q, \sigma, I_p) p(Q, \sigma \mid I_p) dQ\, d\sigma, \quad (6)$$

where it is assumed that $p(Q, \sigma \mid I_p) = p(Q \mid I_p)p(\sigma)$.

Because the vectors $q(1), ..., q(N)$ are free to vary over positive and negative values, the standard noninformative prior for Q is simply a constant [6],[7],[8],[9]. However, as noted in [10],[7], using an improper prior (that is, a prior that doesn't integrate to a finite value) invariably leads to erroneous model selection within the Bayesian evidence framework (to be discussed in more detail in the following section).

To remedy this situation we choose the normalizable prior

$$p(Q \mid I_p) = (2\pi\gamma^2)^{-\frac{3pN}{2}} \exp\ \{-\frac{1}{2\gamma^2}\sum_{t=1}^{N} q(t)^T q(t)\}, \quad (7)$$

under the assumption that the prior dipole orientation/amplitude variance hyperparameter, γ, satisfies $\gamma^2 >> \sigma^2$.

Performing the integration indicated in equation (6) with respect to the orientation parameters, Q, and making use of the above assumption results in

$$p(F \mid L, \sigma, \gamma, I_p) = (2\pi)^{-\frac{MN}{2}} \sigma^{N(3p-M)} \gamma^{-3pN} \quad (8)$$

$$\times \mid G^T G \mid^{-\frac{N}{2}} \exp\ \{-\tfrac{C_\sigma}{\sigma^2}\} \exp\ \{-\tfrac{C_\gamma}{\gamma^2}\},$$

where

$$C_\sigma = \frac{1}{2}\sum_t \vec{f(t)}^T P^{\perp}_G \vec{f(t)} \quad (9)$$

$$C_\gamma = \frac{1}{2}\sum_t \vec{f(t)}^T (G^{\dagger})^T G^{\dagger} \vec{f(t)} .$$

To remove the scale parameters, σ and γ, by integration we use the Jeffrey's priors [9]

$$p(\sigma) \propto \frac{1}{\sigma} \quad (10)$$

$$p(\gamma) \propto \frac{1}{\gamma}.$$

Note that these are *not* proper priors; however, the end result for model selection is not affected because the dimensionality of the integration is the same for each model class. That is, each model has a single hyperparameter and a single additive noise variance parameter, regardless of the number of assumed sources (in contrast, to integrate out the linear orientation/amplitude parameters in (6) means integrating over $Q \in \mathbf{R}^{3p \times N}$).

The following marginal results after integrating $p(F, \sigma, \gamma \mid L, I_p)$ with respect to σ and γ:

$$p(F \mid L, I_p) \propto \Gamma(\frac{N}{2}(M - 3p))\, \Gamma(\frac{3pN}{2}) \quad (11)$$

$$\times \mid G^T G \mid^{-\frac{N}{2}} (\text{tr}\ \{P^{\perp}_G F F^T\})^{\frac{N}{2}(3p-M)}$$

$$\times (\text{tr}\ \{(G^{\dagger})^T G^{\dagger} F F^T\})^{-\frac{3pN}{2}}.$$

After taking the negative-logarithm of (11) we arrive at the Bayesian cost function

$$v_B(L) = \log \mid G^T G \mid + (M-3p) \log (\mathrm{tr}\ \{P_G^{\perp} F F^T\}) + 3p \log (\mathrm{tr}\ \{(G^{\dagger})^T G^{\dagger} F F^T\}). \quad (12)$$

Note that for $M >> 3p$ the second term in (12) dominates, and thus $\hat{L}_B \approx \hat{L}_{ML}$ only for a large number of sensors relative to the number of sources.

4 Bayesian Model Selection

The posterior probability for model I_p is given by

$$p(I_p \mid F) \propto p(F \mid I_p) p(I_p), \quad (13)$$

where $p(F \mid I_p)$ is the Bayesian "evidence" for I_p [10],[11]. Assuming no prior preference of one proposed model over another, the "optimal" model is chosen as the one that maximizes

$$p(F \mid I_p) = \int_L p(F \mid L, I_p) p(L \mid I_p)\ dL. \quad (14)$$

Note that maximizing the Bayesian evidence is equivalent to minimizing the *Stochastic Complexity* of the data F, relative to a model class [12].

The integral in (14) cannot be evaluated in an analytic closed-form because standard EEG models express an observation as nonlinear functions of L. Assuming a diffuse multivariate Gaussian normal prior for L, and using the second order Taylor series approximation for $-\log p(F \mid L, I_p)$, the following approximation can be derived [13]:

$$p(F \mid I_p) \propto p(F \mid \hat{L}_B, I_p) \mid \hat{\Sigma}_B \mid^{-\frac{1}{2}} \times \Gamma(\frac{3p}{2})(\frac{1}{2} \mid\mid \hat{L}_B \mid\mid_F^2)^{-\frac{3p}{2}}, \quad (15)$$

where $\mid \hat{\Sigma}_B \mid$ denotes the determinant of the Hessian matrix of $-\log p(F \mid L, I_p)$, evaluated at $\hat{L}_B$. The notation $\mid\mid . \mid\mid_F^2$ represents the square of the Frobenius norm.

5 Examples

To illustrate the effectiveness of the proposed Bayesian techniques, a two dipole source scenario was simulated with the 4-sphere model of the human head [2]. The simulated electrode configuration is the standard 10-20 pattern, providing $M = 21$ EEG channels. Each dipole was allowed to "rotate" freely over $N = 50$ time samples. The three scalar time series (each corresponding to a Cartesian coordinate) for source 1 are given in figure 1, and those for source 2 in figure 2. All simulation results are based on 100 Monte Carlo trials.

5.1 Localization Example

For both localization examples the additive noise standard deviation was held constant at $0.4\mu V$. In figure 3, overall RMS error for the ML and Bayesian estimators is plotted as a function of Lz for dipoles at $L_1 = [\ 3\ 0\ Lz]$ cm, and $L_2 = [\ \text{-}3\ 0\ Lz]$ cm, where the location coordinates are given with respect to a sphere with the origin at the center, and outer radius $= 8.8cm$. The Bayesian estimator exhibits lower RMS error at all location depths of the dipole pair.

Figure 4 is a plot of RMS error for a fixed-location dipole pair at $L_1 = [\ 3\ 0\ 4.5\ ]$ cm, and $L_2 = [\ \text{-}3\ 0\ 4.5\ ]$ cm, as N was allowed to vary from $N = 20$ to $N = 100$. In this case the Bayesian estimator exhibits superior performance for $N < 80$. Note that the Bayesian RMS error is relatively constant across different values of N.

5.2 Model Order Example

In this subsection we present some preliminary results for the Bayesian evidence approximation given by equations (15) and (11). For this simulation, N, $\vec{L}_1$, and $\vec{L}_2$ were held fixed at $N = 50$, $L_1 = [\ 3\ 0\ 4\ ]$ cm, and $L_2 = [\ \text{-}3\ 0\ 4\ ]$ cm, respectively. Thus, the correct number of dipoles was $P = 2$. The empirical probabilities of detection for the AIC, MDL ([14],[15]), and Bayesian methods were computed as a function of the noise standard deviation, and are displayed in figure 5.

It must be noted that these results are for the binary hypothesis $P = 1$, or $P = 2$. The evidence for $P > 2$ could not be tested for a large number of Monte Carlo trials because of the difficulty in getting stable (over-parameterized) estimates for L. However, preliminary results for a linear antenna array/narrowband source scenario suggest that the Bayesian evidence technique is also resistant to overestimation [16]. Model selection is a more complicated and subtle issue in dipole localization because there is no rigorous concept of a "true" number of generating sources. It is for this very reason that we are investigating the Bayesian framework for model selection.

6 Summary

We have demonstrated the effectiveness of a Bayesian marginalization approach for combined EEG dipole source localization and model selection. Source estimation results via computer simulation show that the proposed localization scheme outperforms ML in low SNR, small spatial/temporal sample circumstances. Simulation studies also suggest that the Bayesian evidence technique for source number estimation will prove valuable under the same conditions.

References

[1] B. Radich and K. M. Buckley, "EEG dipole localization bounds and MAP algorithms for head models with parameter uncertainties," *to appear in IEEE Trans. Biom. Eng.*

[2] B. N. Cuffin, and D. Cohen, "Comparison of the magnetoencephalogram and electroencephalogram," *Electroencephalography and Clinical Neurophysiology*, pp. 132-146, 1979.

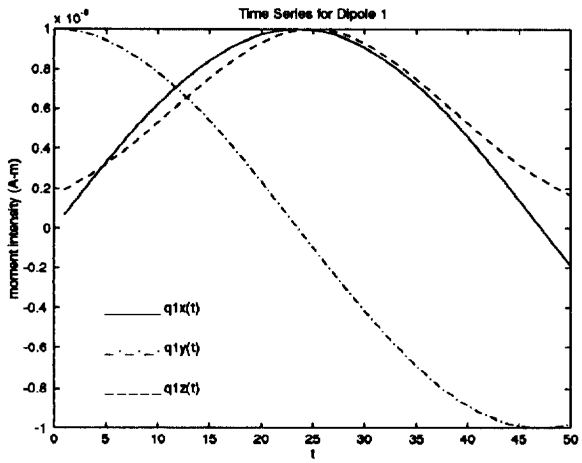

Figure 1: Rotating Dipole 1

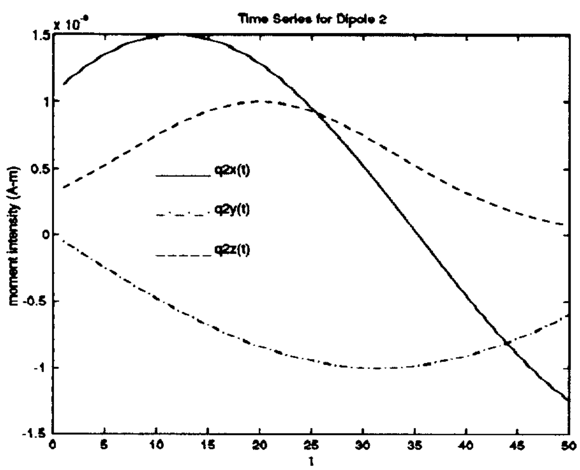

Figure 2: Rotating Dipole 2

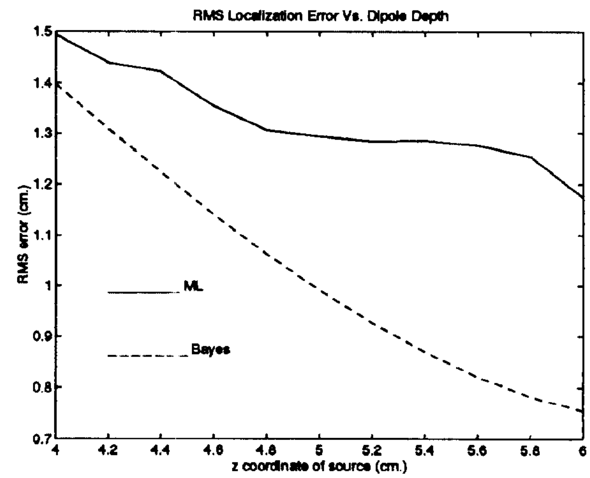

Figure 3: Combined RMS error for dipoles 1 and 2 as a function of Lz, where $\vec{L}_1$= [3 0 Lz] cm, and $\vec{L}_2$= [-3 0 Lz] cm.

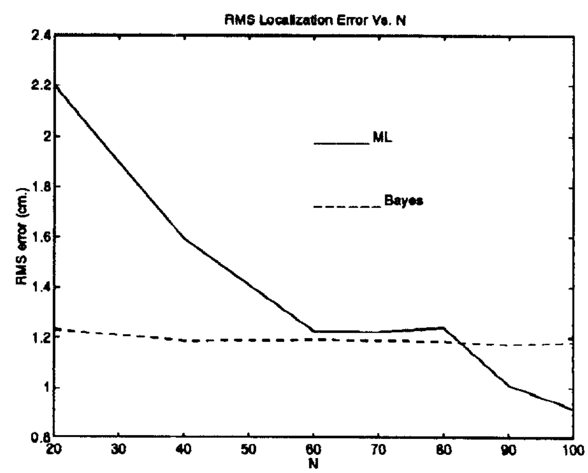

Figure 4: Combined RMS error for dipoles 1 and 2 as a function of N. Source Locations were held fixed with $\vec{L}_1$= [3 0 4.5] cm, and $\vec{L}_2$= [-3 0 4.5] cm.

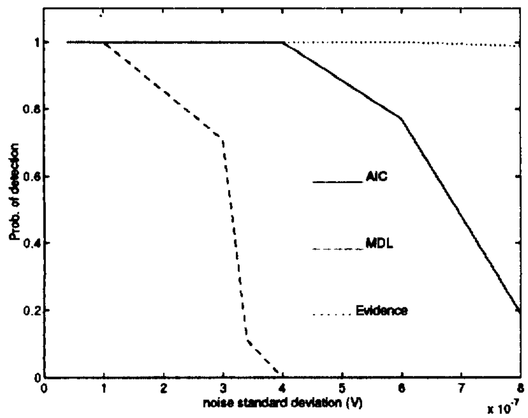

Figure 5: Probability of correct detection for $P = 2$ dipole case.

[3] Y. Bresler and A. Macovski, "Exact maximum likelihood parameter estimation of superimposed exponential signals in noise," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol. ASSP-34, pp. 1081-1089, Oct. 1986.

[4] P. Stoica and A. Nehorai, "MUSIC, maximum-likelihood, and Cramer-Rao bound," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol. ASSP-37, pp. 720-741, May 1989.

[5] M. Scherg, "Fundamentals of dipole source potential analysis," In: F. Grandori, M. Hoke, and G. L. Romani (eds): Auditory Evoked Magnetic Fields and Electric Potentials. Adv. Audiol. Basel, Karger, Vol. 6, pp. 40-69, 1990.

[6] S. Haykin, J.P. Reilly, V. Kezys, and E. Vertatschitsch, "Some aspects of array signal processing," *IEE Proceedings-F*, Vol. 139, No. 1, pp. 1-26, Feb. 1992.

[7] P.M. Djuric, S.M. Kay, "Model selection based on Bayesian predictive densities and multiple data records," *IEEE Trans. Signal Processing*, Vol. 42, No. 7, pp. 1685-1699, July 1994.

[8] A.P. Quinn, "The performance of Bayesian estimators in the superresolution of signal parameters," *Proc. IEEE Int. Conf. on Acoust., Speech and Sig. Proc.* (ICASSP92), Vol. V San Francisco, 297-300.

[9] G.E.P. Box and G.C. Tiao, *Bayesian Inference in Statistical Analysis*, Addison-Wesley, 1973.

[10] G.L. Bretthorst, *Bayesian Spectrum Analysis and Parameter Estimation*, Springer-Verlag, 1989.

[11] D.J.C. MacKay, "Bayesian Interpolation," in Maximum Entropy and Bayesian Methods, Seattle, pp. 39-66, Kluwer Academic Publishers, 1992.

[12] J. Rissanen, *Stochastic Complexity in Statistical Inquiry*, pp. 58, World Scientific, 1989.

[13] B.M. Radich and K.M Buckley, "Combined Bayesian EEG dipole localization and model selection," *in preparation, to be submitted to IEEE Trans. Biom. Eng.*

[14] H. Akaike, "A new look at the statistical model identification," *IEEE Trans. Automatic Control*, Vol. 19, No. 6, pp. 716-722, Dec. 1974.

[15] J. Rissanen, "A universal prior for integers and estimation by minimum description length," *Annals of Statistics*, Vol. 11, No. 2, pp. 416-431, 1983.

[16] B.M. Radich and K.M. Buckley, "Proper prior marginalization of the conditional ML model for combined model selection/source localization," *submitted to Proc. IEEE Int. Conf. on Acoust., Speech and Sig. Proc.* (ICASSP95), Detroit, MI.