GLRT Detection of Broadband Signals Using an Array of Electromagnetic Vector Sensors

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Abstract

The problem of detecting a point-source, broadband signal originating from a known direction is addressed using an array of vector sensors. The signal is accompanied by zero-mean noise with an unknown covariance. The broadband detector is constructed by transforming each narrowband snapshot in the frequency domain into a rank one subspace. This reduces the broadband problem into a generalization of the narrowband detection problem and yields results formerly associated only with the narrowband detection problem. Consideration is also given to a subspace extension, called a 'sparse subspace,' that exploits the symmetry inherent in the received electromagnetic field.

I. Introduction

This paper considers the general broadband detection problem using a sensor array constructed from an orthogonal set of calibrated vector electromagnetic (EM) field sensors. Each of these sensors, termed a vector sensor, has as output all six orthogonal time-varying electric and magnetic field components measured at a single location. Nehorai and Paldi [6] introduced signal processing for vector sensors in the context of a parameterized stochastic signal estimation problem. Burgess and Van Veen [3] develop a narrowband signal model from an array of vector sensors, and solve for the generalized likelihood ratio test (GLRT) in closed form. In [4], the GLRT is extended to subspace models for a vector sensor array sensing a point narrowband source with unknown amplitude.

In this paper, the results of [3] and [4] are extended to include broadband signals by transforming the data so that the broadband signals have the same form as a narrowband signal model considered in a detection problem. This is accomplished by focusing the broadband signal onto a rank one subspace in a manner analogous to the focusing approach developed by [7]. The GLRT detector assumes that the broadband signal is deterministic, at a known direction, with unknown spectral shape, and that the interference and noise are additive, colored multivariate Gaussian distributed with unknown covariance.

Secondly, the EM structure dictated by Maxwell's equations is exploited to obtain transformations, termed 'sparse subspace transformations', that reduce the data dimension prior to detection. These transformations offer the advantages of a data-compressed detector discussed in [2] and [4] while preserving the electromagnetic energy arriving from a specified direction.

This paper adheres to the following notation. Except for the vectors $\mathcal{E}$, $\mathcal{H}$, and $\mathbf{G}$, bold lower and upper case roman letters designate column vectors and matrices, respectively. Script $\mathbb{R}^{J \times K}$

and $\mathbb{C}^{J\times K}$ refer to the JxK real and complex product spaces. The plain scalar superscripts 'T' and 'H' refer to the transpose and complex conjugate operators. A J-dimensional unit vector having a "1" in the kth row and zeros elsewhere will be designated by $\mathbf{e}_{k,J}$. Let vec(**Z**) be the operator which constructs a column vector by stacking columns of the matrix **Z**. Finally, $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product of matrices **A** and **B**.

II. Problem formulation

In the proposed problem, it is assumed that a detection array will be constructed from N vector sensors that are placed in some arbitrary, but known, three-dimensional configuration. Standard assumptions will apply here¥ including unknown, stationary, zero-mean, multivariate normally distributed noise and interference, L independent snapshots, and a single possible far-field, deterministic, bandlimited signal with unknown spectral shape, but of known spatial direction **u**. In this model, other signals arriving from different directions are considered to be uncorrelated interferers. The hypothesis test is constructed from

H_0: No signal exists, H_1: A signal exists. (1)

Using the known source direction (θ_1,θ_2), we define the Euclidean direction vector

$$\mathbf{u} = \left[\cos\theta_1\cos\theta_2,\ \sin\theta_1\cos\theta_2,\ \sin\theta_2\right]^{\mathrm{T}}.$$

In addition, define $\mathbf{v}_1$ and $\mathbf{v}_2$ to construct a right orthonormal triad $(\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2)$. Let $\mathcal{E}(\mathbf{r}_j,t)$, $\mathcal{H}(\mathbf{r}_j,t) \in \mathbb{R}^3$ be the electric and magnetic field vector representations of a bandlimited EM plane wave, traveling in the $-\mathbf{u}$ direction, measured at the origin of sensor 'j'. The origin of the jth sensor, $\mathbf{r}_j$, is given relative to the first sensor $(\mathbf{r}_1 = \mathbf{0})$.

In MKS units, Maxwell's equations assert that

$$\mathbf{u} \times \mathcal{E}(\mathbf{r}_j,t) = -\eta\mathcal{H}(\mathbf{r}_j,t), \text{ and } \mathbf{u}^{\mathrm{T}}\mathcal{E}(\mathbf{r}_j,t) = 0 \quad (2)$$

where η is the intrinsic impedance of the medium. Thus, there exists a function $\mathbf{g}_j(t) \in \mathbb{R}^2$ (constrained in the $(\mathbf{v}_1, \mathbf{v}_2)$ plane but otherwise unknown) such that $\mathcal{E}(\mathbf{r}_j,t) = [\mathbf{v}_1 \mid \mathbf{v}_2]\mathbf{g}_j(t)$. We will call $\mathbf{g}_j(t)$ the 'signal kernel' from the jth vector sensor. Using these relationships, the (noiseless,) time-dependent, EM measurements from each vector sensor 'j' are arranged into six by one signal vectors $s_j(t)$ such that

$$s_j(t) \equiv \begin{bmatrix} \mathcal{E}(\mathbf{r}_j,t) \\ \mathcal{H}(\mathbf{r}_j,t) \end{bmatrix} = \begin{bmatrix} \mathcal{E}(\mathbf{r}_j,t) \\ (-1/\eta)\mathbf{u}\times\mathcal{E}(\mathbf{r}_j,t) \end{bmatrix} = \mathbf{V}\mathbf{g}_j(t) \quad (3)$$

where

$$\mathbf{V} \equiv \begin{bmatrix} \mathbf{v}_1, & \mathbf{v}_2 \\ -\mathbf{v}_2/\eta, & \mathbf{v}_1/\eta \end{bmatrix}.$$

Furthermore, the far-field approximation implies that $\mathcal{E}(\mathbf{r}_j,t) = \mathcal{E}(\mathbf{0},t-\tau_j)$ and $\mathcal{H}(\mathbf{r}_j,t) = \mathcal{H}(\mathbf{0},t-\tau_j)$, where $-\tau_j \equiv \mathbf{u}^{\mathrm{T}}\mathbf{r}_j/c$. Thus $\mathbf{g}_j(t) = \mathbf{g}(t-\tau_j)$ where $\mathbf{g}(t) \equiv \mathbf{g}_1(t)$.

III. Signal model

The signal component of the ith snapshot consists of measurements from all N vector sensors sampled at time t_i, and stacked into a vector labeled $\mathbf{s}_i$.

$$\mathbf{s}_i = \mathrm{vec}\left[s_1(t_i),\ s_2(t_i),\ \cdots,\ s_N(t_i)\right] = \left(\mathbf{V} \otimes \mathbf{I}_N\right)\begin{bmatrix} \mathbf{g}(t_i) \\ \mathbf{g}(t_i-\tau_2) \\ \vdots \\ \mathbf{g}(t_i-\tau_N) \end{bmatrix}. \quad (4)$$

Concatenating the L signal snapshots into a $6N$ by L matrix **S**, we have

¥ *See, for example,* [3] *without the narrowband assumption.*

$$\mathbf{S}=\left[\,\mathbf{s}_1,\ \mathbf{s}_2,\ \cdots,\ \mathbf{s}_L\,\right]$$
$$=\left(\mathbf{V}\otimes\mathbf{I}_N\right)\bullet\begin{bmatrix}\mathbf{g}(t_1), & \mathbf{g}(t_2), & \cdots, & \mathbf{g}(t_L)\\ \mathbf{g}(t_1-\tau_2), & \mathbf{g}(t_2-\tau_2), & \cdots, & \mathbf{g}(t_L-\tau_2)\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{g}(t_1-\tau_N), & \mathbf{g}(t_2-\tau_N), & \cdots, & \mathbf{g}(t_L-\tau_N)\end{bmatrix}. \quad (5)$$

Equation (5) represents a general broadband signal. Let $\mathbf{X}=\mathbf{S}+\mathbf{N}$ where $\mathbf{N}$ is a $6N$ by L matrix of noise snapshots. Assuming the noise snapshots are independent,

$$\mathrm{vec}(\mathbf{X})\sim\mathrm{N}\left(\mathrm{vec}(\mathbf{S}),\ \mathbf{R}_\mathrm{n}\otimes\mathbf{I}_L\right) \quad (6)$$

where $\mathbf{R}_\mathrm{n}$ is the covariance of each noise snapshot.

The GLRT derived in [5] assumes that the signal snapshot matrix $\mathbf{S}$ can be expressed in the form $\mathbf{ABC}$ where $\mathbf{A}$ and $\mathbf{C}$ are known matrices and $\mathbf{B}$ is an unknown matrix of signal amplitudes. If the mean can be expressed in this form, then the GLRT for (6) can be expressed as [4]

$$\Lambda(\mathbf{X})=\frac{\left|\mathbf{A}^\mathrm{T}\mathbf{P}_\mathbf{X}^{-1}\mathbf{A}\right|}{\left|\mathbf{A}^\mathrm{T}\left(\mathbf{X}\mathbf{X}^\mathrm{T}\right)^{-1}\mathbf{A}\right|}\ \overset{\mathrm{H}_1}{\underset{\mathrm{H}_0}{\gtrless}}\ \ell_0 \quad (7)$$

with $\mathbf{P}_\mathbf{X}\equiv\mathbf{X}\left\{\mathbf{I}_L-\mathbf{C}^\mathrm{T}\left(\mathbf{C}\mathbf{C}^\mathrm{T}\right)^{-1}\mathbf{C}\right\}\mathbf{X}^\mathrm{T}$ and threshold ℓ_0. The test (7) is obtained by substituting maximum likelihood estimates of $\mathbf{B}$ and $\mathbf{R}_\mathrm{n}$ into the likelihood ratios. We now transform the broadband signal model (5) into the $\mathbf{ABC}$ form with only $\mathbf{B}$ unknown.

IV. Detector model

We begin by taking the row-wise DFT of the signal matrix $\mathbf{S}\xrightarrow{\mathrm{DFT}}\mathbf{S}_D$. If $\mathbf{G}(\omega_i)$ is the DFT of $\mathbf{g}(t_i)$, and $\mathbf{d}(\omega_i)\equiv\left[\,1\ e^{j\omega_i\tau_2}\cdots e^{j\omega_i\tau_N}\right]^\mathrm{H}$ is the "steering vector" at frequency ω_i, then from (5) it follows that

$$\mathbf{S}_D=\left(\mathbf{V}\otimes\mathbf{I}_N\right)\left[\mathbf{G}(\omega_1)\otimes\mathbf{d}(\omega_1),\ \mathbf{G}(\omega_2)\otimes\mathbf{d}(\omega_2),\ \cdots,\ \mathbf{G}(\omega_L)\otimes\mathbf{d}(\omega_L)\right]. \quad (8)$$

Each column of (8) represents the narrowband response of the signal at frequency ω_i. Our strategy, similar to that in [7], is to transform each narrowband column onto the same subspace. For this purpose, define the N by N and $6N$ by $6N$ transformations

$$\mathbf{T}(\omega_i)\equiv\frac{1}{N}\mathbf{d}(\omega_i)\mathbf{d}_0^\mathrm{H},\ \text{and}\ \mathbf{T}_i\equiv\mathbf{I}_6\otimes\mathbf{T}(\omega_i). \quad (9)$$

Notice that after transforming each narrowband column of $\mathbf{S_D}$ by the corresponding $\mathbf{T}_i$, the ith column becomes

$$\mathbf{T}_i^\mathrm{H}\left(\mathbf{V}\otimes\mathbf{I}_N\right)\left(\mathbf{G}(\omega_i)\otimes\mathbf{d}(\omega_i)\right)=\mathbf{V}\mathbf{G}(\omega_i)\otimes\mathbf{d}_0=\left(\mathbf{V}\otimes\mathbf{d}_0\right)\mathbf{G}(\omega_i). \quad (10)$$

Concatenating these transformed columns yields the matrix

$$\left(\mathbf{V}\otimes\mathbf{d}_0\right)\left[\mathbf{G}(\omega_1),\ \mathbf{G}(\omega_2),\ \cdots,\ \mathbf{G}(\omega_L)\right]. \quad (11)$$

An inverse DFT applied to the matrix (11) yields the transformed $6N$ by L signal matrix in the time domain

$$\mathbf{S_T}\equiv\left(\mathbf{V}\otimes\mathbf{d}_0\right)\left[\mathbf{g}(t_1),\ \mathbf{g}(t_2),\ \cdots,\ \mathbf{g}(t_L)\right]. \quad (12)$$

Equation (12) can be expressed as $\mathbf{S_T}=\mathbf{ABC}$ with the $6N$ by 2 matrix $\mathbf{A}=\left(\mathbf{V}\otimes\mathbf{d}_0\right)$ and $\mathbf{BC}$ equal to the remainder. The matrix $\mathbf{B}$ represents unknown information about the amplitude of components of the signal, and $\mathbf{C}$ represents known temporal information about the signal.

For example, suppose some snapshots, say L–M, are known to be signal free. The detector can ascertain the existence of a signal in any of the remaining M snapshots made at times t_{i_1}, t_{i_2}, …, t_{i_M} by defining an M by L matrix $\mathbf{C}\equiv\left[\,\mathbf{c}_1,\ \mathbf{c}_2,\ \cdots,\ \mathbf{c}_L\,\right]$ with the L dimensional columns $\mathbf{c}_i$ defined by

$$\mathbf{c}_i\equiv\begin{cases}\mathbf{0} & \text{when a snapshot at time } t_i \text{ is known to be signal-free}\\ \mathbf{e}_{i,M} & \text{otherwise.}\end{cases} \quad (13)$$

The 2 by M matrix $\mathbf{B}$ in this example is defined by $\mathbf{B}\equiv\left[\,\mathbf{g}_1(t_{i_1}),\ \mathbf{g}_1(t_{i_2}),\ \cdots,\ \mathbf{g}_1(t_{i_M})\right]$.

Alternatively, suppose $\mathbf{g}(t)$ is known to be bandlimited with only M non-zero frequency components. Then $\mathbf{G}(\omega_i)=\mathbf{0}$ when $i=M+1$, $M+2, \ldots, L$. In this case,

$$\mathbf{B}=\left[\,\mathbf{G}(\omega_1),\,\mathbf{G}(\omega_2),\,\cdots,\,\mathbf{G}(\omega_M)\right] \text{ and}$$

$$\mathbf{C}=\left[\,\mathbf{v}(\omega_1),\,\mathbf{v}(\omega_2),\,\cdots,\,\mathbf{v}(\omega_L)\right]^{\mathrm{H}},$$

where $\mathbf{v}(\omega_i)\equiv\left[\,1\;\mathrm{e}^{\mathrm{j}\omega_i}\cdots\mathrm{e}^{\mathrm{j}(M-1)\omega_i}\right]^{\mathrm{H}}$. Additional choices for **B** and **C** are possible and depend upon prior knowledge about the signal characteristics.

In the special case $\mathbf{d}_0=\left[\,1\;1\;\cdots\;1\,\right]^{\mathrm{H}}$, it can be shown that applying the focusing transformations $\mathbf{T}_i$ in the frequency domain is equivalent to inserting time delays into each sensor channel. These delays "steer" the array in the signal direction; i.e., so the signal arrives simultaneously at each sensor.

Transforming frequency domain snapshots affects the array's apparent noise field in the same way that it alters the signal's form. However, noise is assumed zero-mean and Gaussian with an unknown full-rank covariance matrix. These transformations will simply result in a different unknown, zero-mean and Gaussian, full-rank covariance matrix. Consequently, the data's resulting distribution becomes

$$\operatorname{vec}(\mathbf{X}) \sim \mathrm{N}\left(\operatorname{vec}(\mathbf{ABC}),\,\mathbf{R}_{\mathrm{n}}\otimes\mathbf{I}_L\right) \qquad (14)$$

with **A** and **C** known, and both **B** and $\mathbf{R}_{\mathrm{n}}$ unknown and unstructured. Hence, the hypothesis test (7) is applicable. Although the probability of false alarm (PFA) is known exactly for this test [5; p.223], the probability of detection is currently unknown.

V. Utilization of sparse subspaces

The detector (7) only partially takes advantage of the *a priori* information contained in the signal direction **u**. When the number of snapshots L is limited, it is often advantageous to perform signal detection in a data subspace (*see* [1] *and* [4]). Subspace detection yields three primary advantages over full-space detection. First, the size of the nuisance parameter matrix $\mathbf{R}_{\mathrm{n}}$ decreases yielding a more statistically stable maximum likelihood estimate. This stability tends to improve detector performance. Second, the computational requirements of the detector decrease because the dimension of required matrix inverses decreases. Third, the algorithm's robustness to deviations in the signal model improves [2].

Special linear transformations that individually compress the six data samples from each vector sensor are now developed by exploiting structure in Maxwell's equations. Begin by considering that (broadband) plane-wave signals (including point-source interfering signals) traveling toward the origin have a synchronized electric field intensity which leads the magnetic field intensity by 90 degrees (*see* (2)); i.e. a plane-wave signal is completely specified by its electric field. If the signal (or otherwise desired) direction is given by $\mathbf{u}=[u_{\mathrm{x}}, u_{\mathrm{y}}, u_{\mathrm{z}}]^{\mathrm{T}}$, then

$$\mathcal{E}(\mathbf{r}_j,t)=\eta\,\mathcal{U}\mathcal{H}(\mathbf{r}_j,t), \text{ where} \qquad (15)$$

$$\mathcal{U}\equiv\begin{bmatrix} 0, & -u_{\mathrm{z}}, & u_{\mathrm{y}} \\ u_{\mathrm{z}}, & 0, & -u_{\mathrm{x}} \\ -u_{\mathrm{y}}, & u_{\mathrm{x}}, & 0 \end{bmatrix}.$$

Thus measurements corresponding to $\mathcal{E}(\mathbf{r}_j,t)$ may be replaced with the three by one quantity

$$\tfrac{1}{2}\left[\mathcal{E}(\mathbf{r}_j,t)+\eta\,\mathcal{U}\mathcal{H}(\mathbf{r}_j,t)\right]. \qquad (16)$$

This has the advantage of averaging out some of the noise and (uncorrelated) interference as well as reducing a vector-sensor's measurement space from six to three dimensions. The transformation mapping the six by one quantity $s_j(t+\tau_j)$ into a three by one quantity of the form (16) is given by

$$\mathbf{T}_{\mathbf{E}}\equiv\tfrac{1}{2}\begin{bmatrix}\mathbf{I}_3\\ \eta\,\mathcal{U}^{\mathrm{T}}\end{bmatrix}=\tfrac{1}{2}\begin{bmatrix}\mathbf{I}_3\\ -\eta\,\mathcal{U}\end{bmatrix}. \qquad (17)$$

Further signal-preserving compression is accomplished by mimicking the two-dimensional representation in (3). A transformation mapping $s_j(t)$ into a two by one quantity is given by

$$\mathbf{T}_\mathrm{g} \equiv \frac{\eta}{1+\eta^2}\begin{bmatrix} \eta\mathbf{v}_1, & \eta\mathbf{v}_2 \\ -\mathbf{v}_2, & \mathbf{v}_1 \end{bmatrix} = \frac{\eta^2}{1+\eta^2}\mathbf{V}. \qquad (18)$$

The transformations $\mathbf{T}_\mathrm{E}$ and $\mathbf{T}_\mathrm{g}$ operate on the data, $s_j(t)$, obtained from a single vector sensor before any additional processing. (All remaining steps easily follow.) Alternatively, equivalent results are obtained by applying the block diagonal transformations

$$\mathbf{T}_\mathrm{Es} \equiv \mathbf{T}_\mathrm{E} \otimes \mathbf{I}_N \text{ and } \mathbf{T}_\mathrm{gs} \equiv \mathbf{T}_\mathrm{g} \otimes \mathbf{I}_N \qquad (19)$$

to an entire snapshot. For example, from (11),

$$\begin{aligned} &\mathbf{T}_\mathrm{gs}^\mathrm{H}(\mathbf{V} \otimes \mathbf{d}_0)\left[\mathbf{G}(\omega_1), \mathbf{G}(\omega_2), \cdots, \mathbf{G}(\omega_L)\right] \\ &\quad = (\mathbf{I}_2 \otimes \mathbf{d}_0)\left[\mathbf{G}(\omega_1), \mathbf{G}(\omega_2), \cdots, \mathbf{G}(\omega_L)\right]. \end{aligned} \qquad (20)$$

Using (20), the hypothesis test will operate on a data model (14) where $\mathbf{A} = (\mathbf{I}_2 \otimes \mathbf{d}_0)$ is $2N$ by 2 (instead of $6N$ by 2) and $\mathbf{R}_\mathrm{n}$ is a much smaller $2N$ by $2N$ unknown matrix (instead of $6N$ by $6N$).

If $\mathbf{T}$ represents either $\mathbf{T}_\mathrm{Es}$ or $\mathbf{T}_\mathrm{gs}$ (or any other full-rank transformation), the subspace-based hypothesis test is represented by [4]

$$\Lambda(\mathbf{X}) = \frac{\left|\mathbf{A}^\mathrm{T}\mathbf{T}\mathbf{P}_\mathbf{T}^{-1}\mathbf{T}^\mathrm{T}\mathbf{A}\right|}{\left|\mathbf{A}^\mathrm{T}\mathbf{T}\left(\mathbf{T}^\mathrm{T}\mathbf{X}\mathbf{X}^\mathrm{T}\mathbf{T}\right)^{-1}\mathbf{T}^\mathrm{T}\mathbf{A}\right|} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \ell_0. \qquad (21)$$

where $\mathbf{P}_\mathbf{T} \equiv \left(\mathbf{T}^\mathrm{T}\mathbf{X}\left\{\mathbf{I}_L - \mathbf{C}^\mathrm{T}\left(\mathbf{C}\mathbf{C}^\mathrm{T}\right)^{-1}\mathbf{C}\right\}\mathbf{X}^\mathrm{T}\mathbf{T}\right)$.

VI. Summary

This paper addresses the GLRT detection problem for a deterministic, broadband signal received by an array of vector sensors. The noise is assumed distributed as colored multivariate Gaussian with unknown covariance. A focusing technique is used to transform the broadband signal model into a form that allows application of the GLRT detector developed for narrowband detection. The result is a closed form generalized likelihood ratio hypothesis test. The physical structure inherent in a plane electromagnetic wave is then exploited to develop two subspace transformations capable of compressing data prior to detection without signal loss.

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