Fast Techniques for Sensor Array Calibration

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Abstract

Antennas arrays have long been used on airborne reconnaissance platforms for direction finding (DF) applications. The antenna arrays used for communications signals (20–500 MHz) are usually calibrated during special calibration flights. In a calibration flight, the airborne platform flies a known flight path where the angle-of-arrival (AOA) to the calibration source varies over 360 deg. Calibration flights are extremely expensive, time consuming and often experience co-channel interference. As calibration requirements become more demanding, the duration of the calibration flights can grow dramatically. For example, if azimuth and elevation calibration is required, the calibration flight time may increase by five to ten times. In this paper, we propose a new calibration technique based on the constant-modulus, blind-adaptive beamforming methods where one can calibrate an antenna array using multiple co-channel signal sources, thereby significantly reducing the calibration time. The effectiveness of this technique with real data collected from an airborne platform is shown.

1 Introduction

Airborne reconnaissance platforms provide direction finding (DF) information on intercepted communications signals. The typical frequency range for communications signals is from 20 to 500 MHz. DF measurements are computed using data received from an array of sensors together with calibration data. The calibration data are estimates of the relative response of each sensor in the array and are parameterized by frequency and angle-of-arrival (AOA). The DF accuracy greatly depends on the accuracy of the calibration data.

The DF antenna arrays consist of from four to ten monopole antennas. Two or three antenna arrays are typically used to cover the desired frequency range. The antenna arrays are usually mounted on the belly of the aircraft. The fuselage of the aircraft does not provide a uniform ground plane around the antenna arrays. In fact, the near-field scattering from the fuselage is quite complicated. To adequately characterize the response due to the complicated near-field scattering, the antenna array must be calibrated over a dense set of AOAs and frequencies.

The DF accuracy requirements for airborne reconnaissance platforms are quite demanding. The antenna arrays are precisely calibrated in order to meet the demanding DF accuracy requirements. Typically, the antenna arrays are calibrated over 360 degrees of azimuth at several hundred measurement frequencies in order to adequately characterize the antenna array response over the full frequency range. Azimuth measurements every one to two degrees are required to characterize the response.

Antenna arrays may be calibrated in one of three ways: analytically using mathematical equations, numerically using computer simulations or experimentally by directly measuring data. Analytical and numerical methods for computing antenna-array calibration data do not provide the required accuracy. For this reason, the antenna arrays are calibrated during special calibration flights. During a calibration flight, the aircraft follows a carefully planned course around the cooperative, calibration source. Two passes on the course are required for 360 degree calibration. The azimuth to the calibration source varies from 0 to 180 degrees during one pass. On the reverse path, measurements for azimuths from 180 to 359 degrees are taken. From the data collected, the antenna-array response is estimated. Estimated antenna-array responses along

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with aircraft location, attitude and heading data are combined to generate a calibration data set indexed by frequency and AOA. A calibration data set may include over 90,000 estimated antenna-array responses.

In-flight calibration of a reconnaissance aircraft is a time consuming and expensive process even for azimuth-only calibration. If the calibration data sets are expanded to include multiple elevation angles for each azimuth, the time and expense for calibration can grow significantly. In this paper, we propose a new calibration technique where the antenna array is calibrated over multiple AOAs at once. This methods utilizes multiple cooperative, co-channel calibration sources. The calibration sources are geographically separated to allow calibrating from multiple AOAs at once. For d calibration sources, the calibration time is reduced by a factor of d. The number of calibration sources must be less than or equal to the number of elements in the antenna array. Using this fast calibration technique, one can calibrate an antenna array over multiple elevation angles without increasing the length of calibration flights.

In the following sections, the data model for the calibration problem and calibration method are described. In the forth section, the performance of the new calibration method using real data is described. The conclusions are described in the final section.

2 Data Model

The data collection environment contains the d cooperative, calibration signals and possibly some unintentional interfering signals. An m-element antenna array receives energy from the co-channel signal sources. The cooperative signals have known formats, such as constant-modulus, frequency-modulation (FM) signals. For the discussion of the data model, the dependence of the antenna-array response on frequency is dropped to simplify the expressions. The frequency of operation is known for the calibration problem.

Following the standard notation, the received data with the cooperative, calibration signals on is

$$\mathbf{x}_c(t) = \begin{pmatrix} \mathbf{a}(\theta_1) & \ldots & \mathbf{a}(\theta_d) \end{pmatrix} \begin{pmatrix} s_1(t) & \ldots & s_d(t) \end{pmatrix}^T + \mathbf{n}(t)$$

and with the calibration signals off is

$$\mathbf{x}_i(t) = \mathbf{n}(t)$$

where $\mathbf{a}(\cdot)$ is the antenna-array response parameterized by AOA, $s_k(\cdot)$ is the time waveform for calibration signal k, and $\mathbf{n}(\cdot)$ is the time waveform for the background noise.

Collecting a set of L data vectors into the matrix $\mathbf{X}$, the data model may be expressed as follows—

$$\mathbf{X}_c = \mathbf{A}(\underline{\theta})\mathbf{S} + \mathbf{N} \quad (1)$$

and

$$\mathbf{X}_i = \mathbf{N}$$

where $\mathbf{A}(\cdot)$ is the $m \times d$ matrix of array response vectors for the cooperative calibration sources, $\underline{\theta}$ is the d-vector of AOAs for the calibration sources, $\mathbf{S}$ is the $d \times L$ matrix of calibration signals with each row holding data from one source, and $\mathbf{N}$ is the matrix of noise vectors. The columns of the time series matrices, $\mathbf{X}$, $\mathbf{S}$ and $\mathbf{N}$, are indexed by time.

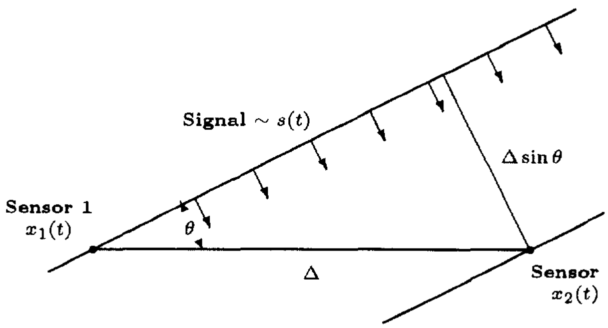

Figure 1: **Relative response between two sensors.** The relative response between two identical sensors can be accurately represented by a relative phase shift for narrowband signals. The phase shift varies smoothly in AOA.

The noise vector, $\mathbf{n}(\cdot)$, includes the interfering signals if present. The additive noise may not be white due to either a colored background noise field or the presence of interfering signals. It is important to account for nonwhite noise; otherwise, the accuracy of the calibration data will be degraded. Numerically robust methods for dealing with nonwhite noise are presented in [1].

The antenna-array response is a continuous and relatively-smooth function in AOA. Small changes in AOA induce small changes in the antenna-array response vector. Consider an antenna array of two identical sensors illuminated by plane waves from one signal. In the absence of noise, the voltage signals from the two identical sensors have a constant time shift, see Figure 1.

In the absence of noise, the voltage signals received at the two sensors are related by a constant time shift which can be modeled as a phase shift, $\omega\tau$, for narrowband signals—

$$\begin{aligned} x_1(t) &= s(t) \\ x_2(t) &= s(t-\tau) \approx \exp(-j\omega\tau)s(t) \\ \tau &= \Delta \sin(\theta)/c \end{aligned}$$

where Δ is the separation between sensors and c is the speed of propagation. Express the sensor separation in terms of wavelengths, $\Delta = \alpha\lambda$. Collecting the sensor voltages into a vector, the antenna-array response for the two sensors is—

$$\begin{aligned} \mathbf{x}(t) &= \begin{pmatrix} 1 & \exp(-j2\pi\alpha\sin(\theta)) \end{pmatrix}^T s(t) \\ \mathbf{a}(\theta) &= \begin{pmatrix} 1 & \exp(-j2\pi\alpha\sin(\theta)) \end{pmatrix} \end{aligned}$$

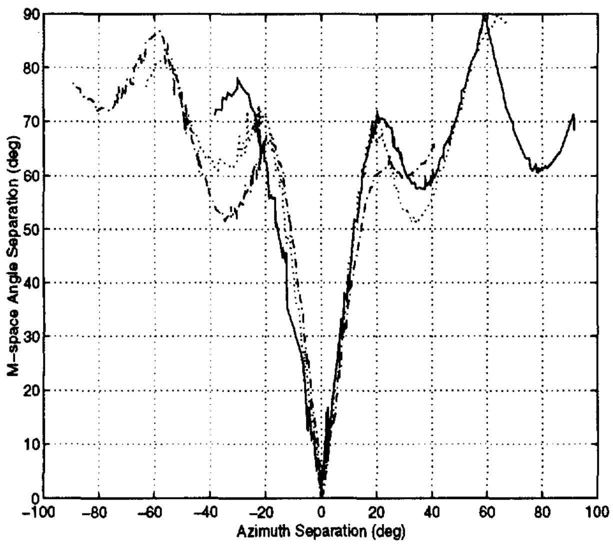

Figure 2: m-**space Angle Between Calibration Vectors.** Each of the three plots used a different calibration vector as the reference vector. All three plots indicate the high-quality of the calibration data. The antenna-array response moves quickly away from the reference vector and never comes close to the reference vector.

For this ideal case, the antenna-array response vector is a very smooth function of AOA. The response vector for antenna arrays mounted on the belly of aircraft is much more complicated. The response vector is superposition of many vectors due to near-field scattering and mutual coupling between antennas as well as the response to the illuminating plane wave. However, each of the contributions to the response vector is a smooth function in AOA so one expects the measured response vector to be a smooth function in AOA. The measured response vector must be sampled often enough in AOA in order to show this smoothness. A convenient measure of smoothness among a collection of array response vectors is the m-space angle between a reference vector and other vectors in the collection. The angle, ϕ, in degrees between two unit vectors, **a** and **b**, may be computed from the inner product of the two vectors—

$$\phi(\mathbf{a}, \mathbf{b}) = (180/\pi)\arccos(\|\mathbf{b}^H\mathbf{a}\|)$$

The magnitude of the angle should be small for AOAs in the vicinity of the reference vector and become large and stay large for AOAs outside of a small neighborhood around the reference vector. In Figure 2, the angle between each array response vector and the three reference vectors are shown. The antenna array was mounted on the belly of an aircraft and calibrated in-flight. The azimuths for the calibration data ranges from -75.8 to 53.9 degrees. The three reference vectors had azimuths of -37.6, -13.1 and 13.2 degrees. The calibration data set exhibits the desired properties for all three reference vectors.

The smoothness in m-space angle will allow us to properly associate calibration vectors with AOA. This is an important consideration when one computes multiple calibration vectors from a given data set. This will be described in the next section.

3 New Calibration Technique

It is well-known that one can exploit known temporal properties of desired signals in order to remove co-channel interference. These techniques are generally called property-restoral techniques. One can extract multiple co-channel signals using suitable property-restoral techniques. Exploitable temporal properties include the constant modulus waveform [2] for angle-modulation signals, such as FM, finite alphabet [3] for digital-modulation signals and self-coherence properties [4] for many digital-modulation types.

Property-restoral techniques are called blind-adaptive techniques because they do not use antenna-array calibration data. The property of the signal is sufficient information to extract individual signals from co-channel interference. Generally, property-restoral techniques require the number of co-channel signals with and without the property is less than or equal to the number of antennas in the array.

Our calibration technique is based on the multi-target modulus restoral (MT-MORE) algorithm which exploits the constant-modulus property. The output of the algorithm is a collection of beamforming weight vectors that can be applied to the input data to individually extract the desired signals. We used FM signals as calibration sources.

Let $\mathbf{W}$ be the matrix of beamforming weight vectors with each column holding a vector. Given the input data matrix $\mathbf{X}$, the estimate of the desired signals is

$$\widehat{\mathbf{S}} = \mathbf{W}^H \mathbf{X}.$$

Using the desired signal estimates or the beamforming weight vectors, one can estimate the array-response vectors for each of the signals. Using equation 1, the least-squares (LS) estimate of the array-response matrix, $\widehat{\mathbf{A}}(\underline{\theta})$, is

$$\begin{aligned} \widehat{\mathbf{A}}(\underline{\theta}) &= \mathbf{X}\widehat{\mathbf{S}}^H \left(\widehat{\mathbf{S}}\widehat{\mathbf{S}}^H\right)^{-1} \\ &= \mathbf{R}\mathbf{W}\left(\mathbf{W}^H \mathbf{R}\mathbf{W}\right)^{-1} \end{aligned}$$

where $\mathbf{R}$ is the sample covariance matrix of the input data. The estimated array-response matrix is the matrix product of the input data matrix and the pseudo-inverse of the estimated desired signals.

LS techniques provide optimal solutions if the residue noise is white. If strong, non-constant modulus interference is present, it is important to remove or whiten the interference. If the interference is not handled, the array-response vectors may include significant bias errors. We can restate the problem as follows—

$$\arg\min_{\mathbf{A}} \|\mathbf{T}(\mathbf{X} - \mathbf{A}\mathbf{S})\|$$

This is a weighted LS problem where $\mathbf{T}$ is a column weighting matrix [5].

We can use the matrix $\mathbf{T}$ to whiten the residuals and remove interference. In the calibration problem, we are using cooperative sources. The whitening matrix can be computed from data with the calibration sources off.

When pre-whitening the input data, it is important to avoid emphasizing the noise. A promising choice [6] for the whitening filter is

$$\mathbf{T} = (\mathbf{R} + \gamma \mathbf{I})^{-1/2}$$

where γ is chosen near smallest eigenvalue of the signal subspace. This whitening matrix will reduce the interference to the level of the weakest desired signal and not emphasize the noise.

The next problem is associating the estimated array-response vectors with the proper AOA. During a calibration flight, we know the positions of the stationary calibration sources and the position of the moving aircraft. As discussed in the previous section, the array-response of a well-designed antenna array is a smooth function in azimuth. In a small neighborhood around a given azimuth, the array-response vectors closely resemble each other; the m-space angle between array-response vectors from this neighborhood is small. Array-response vectors from outside this neighbor are well-separated from vectors from within the neighborhood.

We will use the smoothness property to associate AOA with array-response vectors. First, data are collected with all calibration sources off. Next, data are collected with calibration source one on. The array-response vector computed from this data set is assigned the AOA to source one. Collect data with two sources on; source one is one of the two. The array-response vector closest in m-space angle to the previous vector is assigned the AOA of source one and the other array-response vector is assigned the AOA of the other source. Continue in this manner until all calibration sources are on. Data with all calibration sources off should be collected periodically to determine if significant interference or colored noise are present.

4 Performance Evaluation

Slot Identification Number						
38	39	40	41	42	43	44
E1 E2	E1 E2 E3	E3	E2 E3	E1 E3	E1 E2 E3	E2
45	46	47	48	49	50	51
E1 E2	E1	E1 E2	E2 E3	E1 E2 E3		E2

Table 1: **Active Signals by Slot.** There were fours slots with one active signal, and ten slots with two or three active signals. Data with only signal two active was collected for two slots.

A government organization provided data collected from an antenna array mounted on an aircraft. The antenna array consisted of nine antennas. The aperture of the antenna array was about 2.2 wavelengths. The nominal range from the aircraft to the calibration sources was 30 Km, and the aircraft altitude was 5.5 Km. The aircraft flew through the calibration course three times.

There were three, cooperative FM signal sources. The operating center frequency and bandwidth were

49.72 MHz and 14.5 KHz. The collected data were organized into fourteen-slot patterns called frames. The slot patterns were repeated every ten seconds. Data from 64 slot patterns or frames were used in the performance evaluation.

During a slot, 400 time samples, or 16 msec., of data were collected. In Table 1, the active signals are shown for each slot. Every 0.1 second, a new slot starts. Data for all fourteen slots were collected in about 1.5 seconds. The data from slots 40, 44 and 46 were used to compute the one-signal calibration data shown in Figure 2.

For the multi-signal slots, the MT-MORE algorithm was used to compute the beamforming weight vectors and array-response vectors. For each of the four single-signal slots, the dominant eigenvector estimated the array-response vector. We computed the m-space angle between the multi-signal and one-signal array-response vectors for each frame. With complex data, we can only compute the absolute acute angle between two vectors. This prevents us from analyzing the bias of the proposed technique. However, it is a meaningful quality measure for calibration vectors. If this angle was small, the performance of the multi-signal calibration technique is good.

We used the two single-signal slots for emitter E2 in order to establish a baseline for the acceptable variation in m-space angle. The two slots, 44 and 51, are approximately 0.7 seconds apart. In Figure 3, the m-space between the two calibration vectors computed for signal E2 for each of the 64 frames is shown. The overall median of the absolute angle between the two calibration vectors for signal E2 is 0.87 degrees.

In Figure 4, the absolute angle between the one-signal and multi-signal calibration vectors is shown for each frame. The multi-signal calibration vectors were computed from slot 43 in each frame. Slot 43 was chosen because it is the three-signal slot closest to the one-signal slots, slots 40, 44 and 46. The median absolute angle error for the calibration vectors for signals E1, E2 and E3 was 0.75, 0.88 and 0.86 degrees, respectively. For most slots, the median absolute angle error is comparable to the movement within a frame of the calibration vector for signal E2, see Figure 5. This is very encouraging performance.

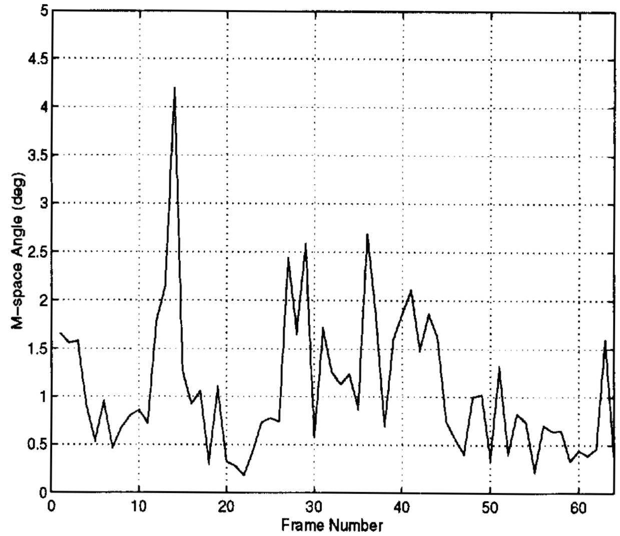

Figure 3: **Absolute m-space Angle Between One-Signal Calibration Vectors for Signal E2.** The data collects for the two calibration vectors are 0.7 seconds apart. There is a measurable angle between the two calibration vectors.

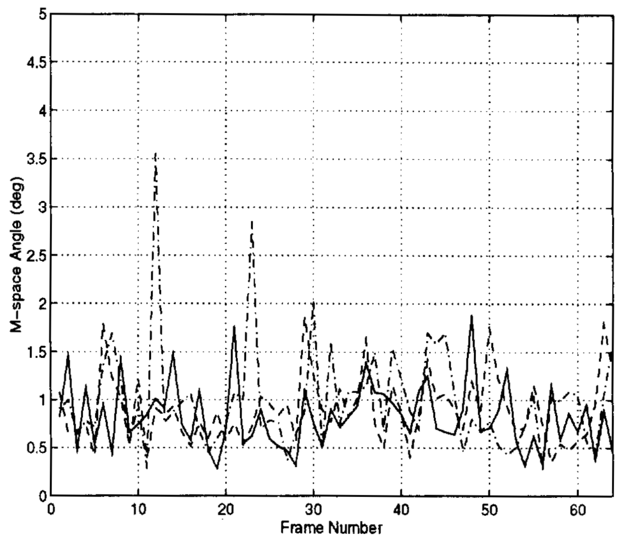

Figure 4: **Absolute m-space Angle Between One-Signal and Multi-Signal Calibration Vectors for Signals E1, E2 and E3.** The absolute angle error for the multi-signal calibration vectors is comparable to the movement between one-signal calibration vectors shown in the figure above. The error for emitters E1, E2 and E3 are plotted with solid, dash-dot and dashed lines.

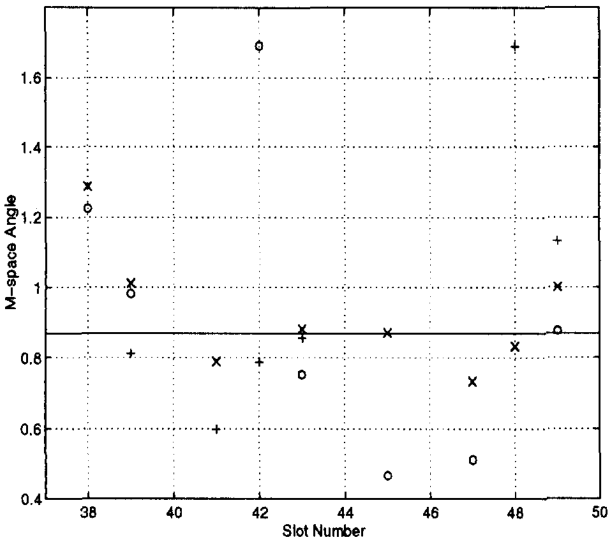

Figure 5: **Median m-space Angle Between One-Signal and Multi-Signal Calibration Vectors by Slot for Signals E1, E2 and E3.** The solid line shows the median absolute angle between the two one-signal calibration vectors for E2. The absolute angle errors for E1, E2 and E3 are plotted as 'o', 'x' and '+', respectively.

5 Conclusions

In the previous section, the proposed calibration technique achieved performance comparable to standard calibration techniques. The proposed technique computed calibration vectors for three AOAs at once; whereas, the standard calibration technique computed one calibration vector at a time. In order to compare the performance of the calibration techniques, we processed data collected from an antenna array mounted on an aircraft. The data collected was divided into frames lasting 1.5 seconds. The AOA from the aircraft to the signal sources changed little over the duration of a frame. For every frame, we computed two calibration vectors for signal E2 and one calibration vector for signals E1 and E3 using the standard calibration technique; we also computed calibration vectors using the new technique based on the MT-MORE algorithm. The data processed using the new technique had all three signals active. We compared the m-space angle between calibration vectors computed using the new and standard techniques. These m-space angles were comparable to the m-space angle between the two calibration vectors computed for signal E2 using the standard technique. This strongly indicates that the calibration vectors computed using the new technique are as accurate as the calibration vectors computed using the standard techniques.

The new calibration technique has several advantages over the standard techniques. One advantage of the new calibration technique is that much less calibration flight time is required for the same number of points or many more calibration vectors can be computed in the same flight time. Using the proposed calibration technique, one can calibrate an aircraft in azimuth and elevation in the same time required for azimuth-only calibration.

A major limitation with existing calibration sets for communications reconnaissance platforms is that the data covers only one elevation for each azimuth. The major reason for not gathering more complete calibration, *i.e.* multiple elevations for each azimuth, is the large increase in flight time. The proposed calibration technique removes the large increase in flight time.

A second advantage of the proposed technique is the ability to calibrate an antenna array in the presence of undesired, co-channel interference. This is also important because many reconnaissance aircraft are calibrated near urban areas.

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