Constrained Partially Adaptive Space-Time Processing for Clutter Suppression

Amin G. Jaffer

Hughes Aircraft Company, Fullerton, California 92634

Abstract

A major problem in airborne radar or moving platform active sonar systems is the detection of low-Doppler target returns embedded in spatially distributed and Doppler-spread clutter returns received in a radar antenna or sonar array. This paper presents two novel partially adaptive space-time processing techniques for improving low-Doppler target detection which are based on employing an adaptive preprocessing structure composed of transversal FIR filters in each subarray of a partitioned planar array. Both methods use substantially reduced adaptive degrees of freedom compared to the "optimal", fully adaptive space-time processing method with attendant significant computation advantages and improved performance in a nonstationary environment. Computer simulation results are presented that demonstrate the improved performance of the proposed techniques over conventional beamforming and filtering methods.

1: Introduction

A major problem in airborne radar or moving-platform active sonar systems is the detection of low-Doppler target returns embedded in spatially distributed and Doppler-spread clutter returns received in a radar antenna or sonar array. Conventional signal processing techniques for narrowband signals, represented by fixed beamforming followed by Doppler filtering, are generally of limited effectiveness due to the doubly-spread nature of the clutter (or reverberation). Similarly, decoupled adaptive beamforming or adaptive filtering methods are also of limited use in such environments [1]. The distributed spatial extent and attendant Doppler spread of the clutter necessitate the use of adaptive space-time processing methods which jointly process return time series data in individual array elements (corresponding to a narrowband CW pulse or coherent pulse train transmission) to create two-dimensional filters in angle and Doppler to best exploit the intrinsic small separation that exists between a low-Doppler target and clutter in these domains[1],[2]. A straightforward method is to concatenate the time series data in each array element and apply adaptive beamforming methods to this extended vector data. This fully adaptive "optimal" space-time processing method suffers from two major drawbacks: 1) Because of the product dimensionality of the resulting weight vector, the method requires the inversion of very large covariance matrices for even moderately large arrays and time series lengths, which can be computationally prohibitive and 2) It requires the estimation of very large clutter covariance matrices from a limited set of clutter data yielding a poor estimate of these matrices and attendant degradation in the adaptive space-time filter performance.

This paper presents two new partially adaptive space-time processing methods for improving the detection of low-Doppler targets in clutter which is spread in angle and Doppler frequency. Both methods are based on the following structure: The sensor array, which may be a planar or conformal array, is partitioned into M subarrays and the uniformly sampled return time series of length N corresponding to the entire CW pulse duration is passed through adaptive transversal finite impulse response (FIR) filters of length L in each subarray, where, typically, L is much less than N. This constitutes an adaptive joint spatial-temporal preprocessing structure which is then followed by Doppler filtering matched to the target Doppler. The criteria for determining the preprocessor filter coefficients are different for the two methods which are both applicable to arbitrary arrays. Both methods result in substantial reduction of degrees of freedom while incurring only a modest loss in performance relative to the theoretical, unrealizable, fully adaptive optimal processor. Although a similar preprocessor structure has been considered by Klemm[3], the techniques presented here for determining the preprocessor coefficients are quite different.

Computer simulation results are presented based on a surface reverberation model that demonstrate substantial improvement in performance of the proposed new techniques relative to conventional beamforming and filtering methods.

1058-6393/95 $4.00 © 1995 IEEE

2: Model definition

We consider here the problem of detecting a low-Doppler (low range-rate) target from returns obtained using a forward-looking active sonar mounted on a moving platform and operating in the presence of reverberation generated from the surrounding shallow water environment. These undesired environmental returns (called reverberation or clutter) originate from the surface, bottom and volume of water in the underwater environment. Because of the relative motion between the high speed platform and the spatially distributed environment, the total clutter reurn is Doppler spread rendering difficult the detection of low-Doppler targets. An illustration of the scenario is shown in Figure1. The methods presented here are also directly applicable to the low-Doppler target detection problem for side-looking airborne radars encountering surface clutter returns.

The signal plus clutter return due to a CW transmit pulse (a pulsed sinusoid) is received in the M subarrays of the partitioned array and is uniformly sampled at a rate of F_s samples per second resulting in M time series, each of length N samples (corresponding to the pulse duration), received in the M subarrays. Let $\underline{X}$ denote the concatenated MN-vector of received space-time complex samples, arranged in such a way so that the first N elements correspond to time samples in subarray 1, the second N elements to time samples in subarray 2 and so on. Now $\underline{X} = \underline{S} + \underline{N}$ where the signal space-time vector correponding to a point target signal at elevation θ_s and azimuth ϕ_s and Doppler frequency f_s is given by

$$\underline{S} = a_s \underline{d}_{sf}(\theta_s, \phi_s, f_s) \tag{1}$$

and the composite space-time steering vector is given by

$$\underline{d}_{sf}(\theta_s, \phi_s, f_s) = \underline{d}_s(\theta_s, \phi_s) \otimes \underline{d}_f(f_s) \tag{2}$$

where $\otimes$ denotes the Kronecker product of matrices [4]. a_s is the signal complex amplitude, $\underline{d}_s(\theta_s, \phi_s)$ is the M dimensional steering vector of complex valued subarray beam pattern gains referenced to a common point,

$$\underline{d}_s(\theta_s, \phi_s) = [g_1(\theta_s, \phi_s), \ldots\ldots, g_M(\theta_s, \phi_s)]^T \tag{3}$$

and $\underline{d}_f(f_s)$ is the N-dimensional Doppler steering vector of the form

$$\underline{d}_f(f_s) = \left[1, e^{j2\pi f_s / F_s}, \ldots\ldots, e^{j2\pi(N-1) f_s / F_s}\right]^T \tag{4}$$

The superscript T denotes the transpose operation. Note that the subarrays can represent, as a special case, single omnidirectional identical elements (hydrophones) arranged as a uniformly spaced line array so that $\underline{d}_s(\theta_s, \phi_s)$ takes the standard form, referenced to the first sensor,

$$\underline{d}_s(\theta_s, \phi_s) = [1, e^{-j\psi_s}, \ldots\ldots\ldots, e^{-j(M-1)\psi_s}]^T$$

where $\psi_s = 2\pi d \cos\theta_s \sin\phi_s / \lambda$, d is the inter-element spacing and λ is the wavelength.

For simulation purposes, the clutter at a given range ring is modelled as the superposition of independent complex-valued returns from clutter patches in different directions with associated different Doppler frequencies (see Figure 2). The composite space-time clutter plus noise return covariance matrix for homogeneous and stationary clutter is then given by

$$R_o = P_C G_{TP}(\theta_k, \phi_k) \sum_k \underline{d}_{sf}(\theta_k, \phi_k, f_k) \underline{d}_{sf}^H(\theta_k, \phi_k, f_k) + \sigma^2 I \tag{5}$$

where P_C is the clutter power per patch, G_{TP} is the transmit beam pattern power gain, the parameters pertain to the k'th clutter patch and the summation is taken over all clutter patches in the range ring of interest. $\sigma^2 I$ represents the covariance matrix of the ambient, white noise component where I is the identity matrix. The (i,j) th block of the MN by MN block matrix R_o represents the N by N covariance matrix of the temporal data in the ith and jth subarrays, i, j = 1,...., M.

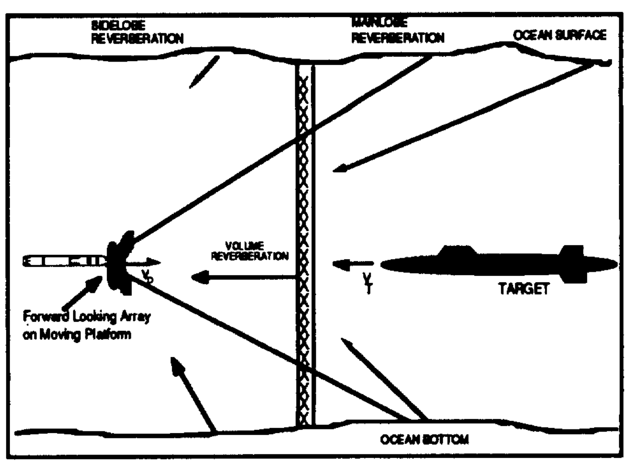

Figure 1. Underwater Reverberation Scenario Diagram

3: Adaptive space-time processing methods

We present three adaptive space-time processing methods here using ensemble values of the clutter covariance matrices. The first is the well-known "optimum " method [5] which maximizes the signal to interference plus noise ratio (SINR) in the target direction and Doppler ,

$$J(\underline{w}) = \frac{\left|\underline{w}^H \underline{d}_{sf}(\theta_s, \phi_s, f_s)\right|^2}{\underline{w}^H R_o \, \underline{w}} \qquad (6)$$

The solution is $\underline{w}_{opt} = \alpha R_o^{-1} \underline{d}_{sf}(\theta_s, \phi_s, f_s)$ where α is an arbitrary non-zero constant. The resulting expression for the SINR [6] is

$$J_{opt} = \underline{d}_{sf}(\theta_s, \phi_s, f_s)^H R_o^{-1} \underline{d}_{sf}(\theta_s, \phi_s, f_s) \qquad (7)$$

For unknown target direction and Doppler, θ_s, ϕ_s and f_s are regarded as parameters to be varied.

The optimum method produces the highest SINR but the weight vector is of dimensionality MN requiring the inversion of an MN x MN covariance matrix which can be computationally prohibitive. Also, in practice, this large covariance matrix is unknown and must be estimated from limited data resulting in inferior estimates and attendant performance degradation.

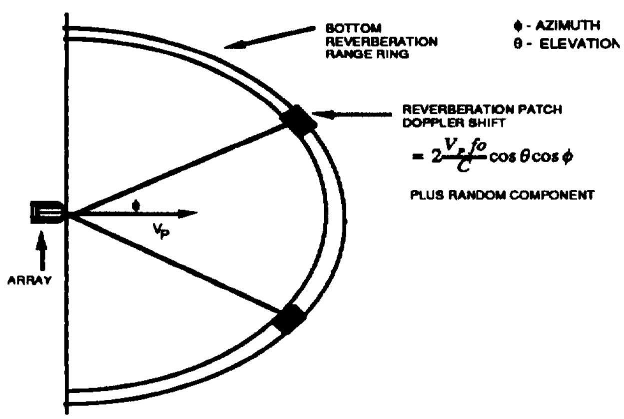

Figure 2. Surface Clutter Distribution in range ring

3.1: Joint linear prediction error / spatial constraint adaptive preprocessor

This reduced-dimensionality method developed here is based on passing the return time series data through an adaptive preprocessor which consists of adaptive FIR transversal filters with L taps in each subarray , as shown in Figure 3 where,typically, L is much less than N. Let $\underline{c}_i$ represent the L-dimensional coefficient vector for the i^{th} subarray and $\underline{c}_{ext}$ the concatenated ML-vector given by

$$\underline{c}_{ext} = \left[\underline{c}_1^T \; \underline{c}_2^T \; \ldots\ldots \; \underline{c}_M^T\right]^T$$

The criterion proposed here for estimating the $\underline{c}_{ext}$ vector is an extension of the one-dimensional minimum linear prediction error (LPE) criterion for prewhitening a clutter time series modelled as an

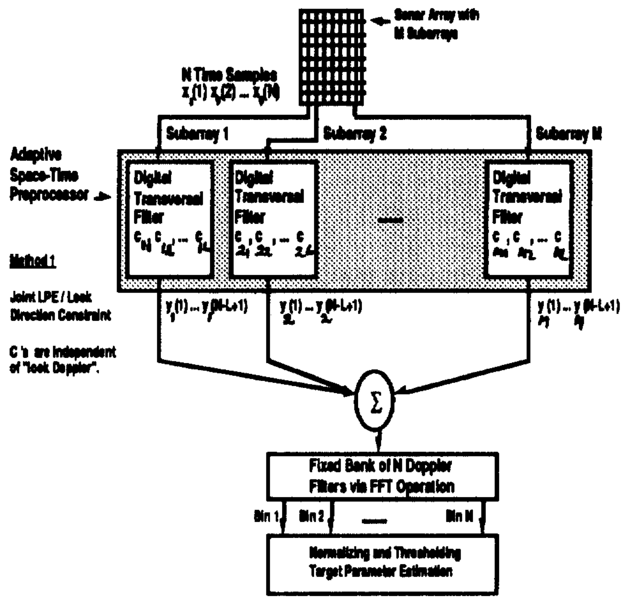

Figure 3. Block Diagram of Joint LPE/Spatial Constraint Preprocessor

autoregressive (AR) process [7],[8]. However, it is not a 2-D AR extension to the spatial-temporal domain because the sonar array is not restricted to be a uniformly spaced linear array. The time series is however assumed to be uniformly sampled in time which is usually not a restriction. The following criterion enables the space-time preprocessor to behave as a joint temporal prewhitener and spatial canceller subject to a look direction constraint. For the i^{th} time series of length N, let the "sliding" vector be

$$\underline{x}_{i,j} = [x_i(j) \;\; x_i(j+1) \;\ldots\ldots\ldots\ldots\; x_i(j+L-1)]^T$$
$$i = 1,\ldots,M, \;\; j = 1,\ldots.,N-L+1$$

We minimize the following criterion with respect to the $\underline{c}_i$'s:

$$J(\underline{c}_i,\ldots.,\underline{c}_M) = E\left\{\sum_{j=1}^{N-L+1}\left|\sum_{i=1}^{M}\underline{c}_i^H \underline{x}_{i,j}\right|^2\right\} \qquad (8)$$

subject to

$$\sum_{i=1}^{M} d_s(i)\underline{c}_i^H \underline{e}_L = 1 \qquad (9)$$

where $\underline{e}_L$ is the Lth unit coordinate vector with one in the Lth position and zeros elsewhere. It can be seen that the above criterion reduces to that of linear prediction error filtering for the special case of one subarray (M=1) and that of adaptive beamforming for the case of one tap being used (L=1). The output after the preprocessor is summed and Fourier transformed (FFT operation), possibly after shading, which may be represented as the application of a Doppler filter weight vector $\underline{w}_D(f)$ of the form (4) to the output after preprocessing and summing.

The constraint (9) can also be equivalently written in the form

$$\left[\underline{c}_1^H \ \underline{c}_2^H \ \ldots\ldots \ \underline{c}_M^H\right](\underline{d}_s \otimes e_L) = 1 \qquad (10)$$

where the dependence of $\underline{d}_s$ on elevation and bearing has been dropped for notational simplicity. Appending (10) to (8) via Lagrange multipliers yields

$$J_1 = E\left\{ \sum_{j=1}^{N-L+1} \left| \sum_{i=1}^{M} \underline{c}_i^H \underline{x}_{i,j} \right|^2 \right\} - \lambda\left\{\left[\underline{c}_1^H \ \underline{c}_2^H \ \ldots\ldots \ \underline{c}_M^H\right](\underline{d}_s \otimes \underline{e}_L) - 1\right\} \qquad (11)$$

For a fixed j, the expected value of the inner $|(\bullet)|^2$ term in (11) can be written as

$$\sum_{i1=1}^{M}\sum_{i2=1}^{M} \underline{c}_{i1}^H R_{i1,i2}^j \underline{c}_{i2}$$

where

$$R_{i1,i2}^j = E[\underline{x}_{i1,j}\, \underline{x}_{i2,j}^H]$$

Under the assumption that the clutter is homogeneous and stationary over the time duration of interest, $R_{i1,i2}^j$ is independent of j and (11) reduces to

$$J_1 = N_1 \sum_{i1=1}^{M}\sum_{i2=1}^{M} \underline{c}_{i1}^H R_{i1,i2} \underline{c}_{i2} - \lambda\left\{\underline{c}_{ext}^H (\underline{d}_s \otimes \underline{e}_L) - 1\right\} \qquad (12)$$

where N_1=N-L+1. Differentiating (12) with respect to the conjugates of the $\underline{c}_i$'s according to the rules of[9] and equating to the null vector yields

$$N_1 \sum_{i2=1}^{M} R_{i1,i2}\underline{c}_{i2} - \lambda\, d_s(i_1)\, \underline{e}_L = \underline{0} \qquad i_1 = 1,\ldots,M \qquad (13)$$

Equation (13) gives ML simultaneous equations for the determination of the ML-vector $\underline{c}_{ext}$ and may be expressed as

$$\underline{c}_{ext} = \lambda_1 [R_{i1,i2}]^{-1} (\underline{d}_s \otimes \underline{e}_L) \qquad (14)$$

where $\lambda_1 = \lambda / N_1$ and $[R_{i1,i2}]$ is a ML x ML Hermitian, positive-definite block matrix whose components are L by L matrices

$$R_{i1,i2} = E[\underline{x}_{i1,1}\, \underline{x}_{i2,1}^H]$$

The scale factor λ_1 can be evaluated so as to satisfy the constraint equation. It is noted that (14) requires the inversion of only a square matrix of order ML as opposed to MN for the optimum case, a substantial reduction when, typically, L << N. The computation given by (14) is independent of the target search Doppler (although it depends on the target search direction). For evaluating the steady-state SINR performance measure for this technique, the equivalent weight vector operating on the full length concatenated space-time data $\underline{X}$ can be derived as follows: The N_1-vector $\underline{Y}_i$ at the output of the FIR filter in the i'th subarray is given by $\underline{Y}_i = C_i^H \underline{x}_i$ and the output after applying $\underline{w}_D(f)$ is given by

$$z_i = \underline{w}_D^H(f)\underline{Y}_i = [C_i \underline{w}_D(f)]^H \underline{x}_i$$

where $\underline{x}_i$ is the N-vector in the ith subarray and C_i is the (N by N_1) matrix given by

$$C_i = \begin{bmatrix} \underline{c}_i & 0 & \ldots\ldots & 0 \\ 0 & \underline{c}_i & \ldots\ldots & 0 \\ \vdots & \vdots & \ldots\ldots & 0 \\ \vdots & \vdots & \ldots\ldots & \underline{c}_i \end{bmatrix}$$

The equivalent weight vector is then given by

$$\underline{W}_{eq} = \left[\underline{w}_{eq,1}^T \ \underline{w}_{eq,2}^T \ \ldots\ldots \ \underline{w}_{eq,M}^T\right]^T \qquad (15)$$

where

$$\underline{w}_{eq,i} = C_i \underline{w}_D(f) \qquad (16)$$

Equation (15) is used in (6) to evaluate the SINR.

3.2: Optimum space-time preprocessor

This reduced-dimensionality method is based on using the same preprocessor structure as in the previous method, however the criterion for determining the transversal FIR filter coefficient vectors is different. Specifically, we maximize the signal-to-interference plus noise ratio contingent on using FIR filters of length L in each subarray followed by fixed Doppler filtering. The criterion is equivalently one of minimizing with respect to the $\underline{c}_i$'s, the function

$$J(\underline{c}_1,\ldots\ldots,\underline{c}_M) = E\left|\underline{W}_{eq}^H X\right|^2 \qquad (17)$$

subject to $\qquad \underline{W}_{eq}^H \underline{d}_{sf} = 1 \qquad (18)$

where $\underline{W}_{eq}$ is of the form given by (15) and (16). We note that this differs from the standard adaptive beamforming optimization problem in that the optimization is carried out with respect to the $\underline{c}_i$'s which are embedded in $\underline{W}_{eq}$. Equation (17) can be written as

$$J(\underline{c}_1,...,\underline{c}_M) = \underline{w}_D^H(f)E\left[\sum_{i=1}^{M}\sum_{j=1}^{M}\underline{C}_i^H \underline{x}_i\underline{x}_j^H \underline{C}_j\right]\underline{w}_D(f)$$

After some algebraic work and using Lagrange multipliers to append the constraint, the criterion becomes one of minimizing

$$J_1 = \sum_{i=1}^{M}\sum_{j=1}^{M}\underline{w}_D^H(f)R^{ij}\underline{w}_D(f)$$

$$-\lambda\left[\sum_{i=1}^{M}\underline{w}_D^H(f)\underline{C}_i^H d_s(i)\underline{d}_f - 1\right] \qquad (19)$$

where R^{ij} Is the square matrix of orde r N_1 whose (k,l)th term is

$$R^{ij}(k,l) = \underline{c}_i^H R_{kl}^{ij}\underline{c}_j$$

The matrix R_{kl}^{ij} is defined as

$$R_{kl}^{ij} = E\left[\underline{x}_{i,k}\underline{x}_{j,l}^H\right]$$

$d_s(i)$ is the i'th component of the spatial steering vector. Further algebraic manipulation of (19) yields

$$J_1 = \sum_{i=1}^{M}\sum_{j=1}^{M}\left\{\sum_{k=1}^{N_1}\sum_{l=1}^{N_1} w_D^*(k)w_D(l)(\underline{c}_i^H R_{kl}^{ij}\underline{c}_j)\right\}$$

$$-\lambda\left[\sum_{i=1}^{M} d_s(i)\right]\left[\sum_{n=1}^{N_1} w_D^*(n)\underline{c}_i^H \underline{d}_{fn}\right] \qquad (20)$$

where * denotes complex conjugate, $w_D(l)$ is the l'th component of the fixed Doppler filter weight vector and $\underline{d}_{fn}$ is the "sliding" Doppler steering vector given by

$$\underline{d}_{fn} = \left[\underline{d}_f(n), ..., \underline{d}_f(n+L-1)\right]^T$$

Differentiating (20) with respect to the $\underline{c}_i$'s and equating to the zero vector yields ML simultaneous equations for the $\underline{c}_i$'s :

$$\sum_{j=1}^{M} R_D^{ij}\underline{c}_j = \lambda d_s(i)\left(\sum_{n=1}^{N_1} w_D^*(n)\underline{d}_{fn}\right) \quad i = 1,.....,M \qquad (21)$$

where the L by L matrix

$$R_D^{ij} = \sum_{k=1}^{N_1}\sum_{l=1}^{N_1} w_D^*(k)w_D(l)R_{kl}^{ij}$$

The solution for the optimum space-time preprocessor coefficient vector can then be expressed as

$$\underline{c}_{ext} = \lambda R_{DD}^{-1}\left(\underline{d}_s \otimes \underline{d}_{wf}\right) \qquad (22)$$

where R_{DD} is a ML by Ml block matrix whose (i,j)th block is the L by L matrix R_D^{ij} given by (X) and $\underline{d}_{wf}$ is a weighted linear combination of the sliding Doppler steering vectors,

$$\underline{d}_{wf} = \sum_{n=1}^{N_1} w_D^*(n)\underline{d}_{fn}$$

The scale factor λ can be evaluated so as to satisfy theconstraint equation. As in Section 3.1, the computation of the ML dimension space-time preprocessor coefficient vector requires only the inversion of a square matrix of order ML. Also, because of our criterion used, this method yields the highest SINR at the output subject to the constraint represented by the reduced degrees-of-freedom preprocessor structure. However, now the computation of the space-time preprocessor coefficient vector given by (22) depends on both the target search Doppler as well as search direction. The computation burden posed by this may be mitigated by using this method for only the low-Doppler target detection cases.

4: Computer simulation results

The following simulation results pertain to a scenario of an underwater vehicle with a forward mounted sonar operating in a reverberant environment. A line array with eight hydrophones (M=8) was assumed with a transmit CW pulse of 16 msec duration yielding N=16 coherent signal samples (for a point target) at a sampling rate of 1 kHz. The transmit frequency was 30 kHz and the vehicle velocity was 9.4 knots. The clutter model was a single surface model as described in Section 2. The adaptive preprocessor used FIR filters with 3 taps per spatial element. Figure 4 shows the "steady-state" signal-to-interference plus noise ratio, normalized by the ideal 10 log MN dB gain, for various methods as a function of target radial velocity. This was obtained from a SINR plot vs. Doppler frequency by translating by own platform Doppler and converting frequency to target radial velocity units. The conventional method refers to fixed beamforming with 30 dB Taylor shading followed by Doppler filtering with 72 dB Blackman-Harris weighting. As can be seen from the Figure, the two adaptive space-time processing methods with reduced degrees of freedom (ML=24) developed here performed substantially better than the conventional method and only slightly worse than the optimum fully adaptive method with MN = 128 degrees of freedom. The SINR improvement of the LPE/spatial constraint space-time method over the conventional is, for example, about 35 dB for a 2 knot down-Doppler target. Numerous other simulation results and limited results using real test data, not reported here, have been obtained by the author and his colleagues at Hughes Aircraft Company which tend to support the above conclusion.

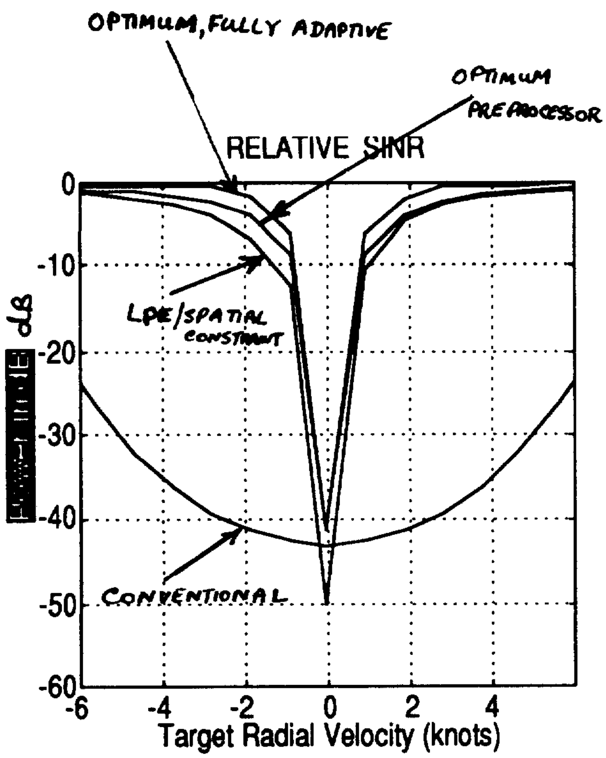

Figure 4.

References

1. R. Klemm, "Adaptive Airborne MTI: An Auxiliary Channel Approach," IEE Proceedings, Vol. 130, Pts F and H, No.1, pps 125-143, February 1983.

2. A. G. Jaffer, M.H. Baker, W.P. Ballance, J.R. Staub, "Adaptive Space-Time Processing Techniques for Airborne Radars," Final Report RL-TR-91-162 for Rome Laboratory, Griffiss Air Force Base, NY13441, July 1991.

3. R. Klemm, "Interrelations Between Matched Field Processing and Airborne MTI Radar," IEEE Journal of Oceanic Engineering, Vol. 18, No.3, pp 168-180, July 1993.

4. A. Graham, Kronecker Products and Matrix Calculus, Ellis Horwood Limited, west Sussex, England, 1981.

5. L.D. Brennan and I.S. Reed, " Theory of Adaptive Radar." IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-9, pp. 237-252, 1973.

6. R.T. Compton, Adaptive Antennas Prentice Hall, New Jersey, 1988.

7. S. Haykin, Adaptive Filter Theory, 2nd Edition, Prentice Hall, New Jersey, 1991.

8. S. Kay and J. Salisbury, " Improved Active Sonar Detection Using Autoregressive Prewhiteners," Journal of Acoust. Soc. Am. Vol.87, N0.4, pp1603-1611, April 1990.

9. D.H. Brandwood , "A Complex Gradient Operator and its Apllications in Adaptive Array Theory," IEE Proceedings Vol. 130, Pts F and H, No.1, pp 11-16, February 1983.