Image Processing Using Adaptive Aposteriori Wiener Estimation Techniques

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ABSTRACT

In this paper we describe the generalized, two-dimensional Wiener estimation algorithm in the frequency domain. An adaptive aposteriori estimation technique is then presented. The technique is applied to a specific example and results are illustrated.

1. INTRODUCTION

Two-dimensional Wiener filters and their iterative variations have been known for some time [1]. We usually make the apriori assumption that the signal and noise are not cross-correlated. This assumption is made because we have no apriori knowledge of the cross-correlation function. In actuality, from the aposteriori viewpoint there is cross-correlation and this results in an estimation error. Aposteriori filtering has been described in one dimension to estimate this cross-correlation error and has found to be quite effective [2]. We will generalize the two-dimensional Wiener filter and construct a two-dimensional adaptive aposteriori algorithm.

2. WIENER ESTIMATION FILTERS

Wiener filters are a class of least-squares filters which are optimal in the sense that they minimize the squared error between the estimated output and the desired output $[c(x,y)-d(x,y)]^2$. This leads to the general two-dimensional ideal Wiener filter with gain

$$H(u,v)=\frac{E\{D(u,v)R^*(u,v)\}}{E\{R(u,v)R*(u,v)\}} \tag{1}$$

This filter can be regarded as a spectral ratio of the expected cross-correlation between the input and desired output to the expected autocorrelation of the input. With deterministic processes, we use specific correlation spectra, instead of the expected correlation spectra.

We will use these filters to solve the classical problem of estimating an image that is corrupted with additive noise. The input R and desired output D are

$$\begin{aligned} R(u,v) &= S(u,v)+N(u,v) \\ D(u,v) &= S(u,v) \end{aligned} \tag{2}$$

where S and N are the signal and noise spectra respectively. In this example, we use the ideal prototype image as our desired output signal. Substituting (2) into (1) yields an expression for the correlated estimation filter

$$H_{ce}(u,v)=\frac{E\{|S(u,v)|^2+S(u,v)N^*(u,v)\}}{E\{|S(u,v)|^2+|N(u,v)|^2+2S(u,v)N^*(u,v)\}} \tag{3}$$

Making the apriori assumption that the signal and noise are not cross-correlated, yields that the expectation of the SN^* product is zero. Then from (3) the optimal filter equals the familiar result

$$H_{ue}(u,v)=\frac{E\{|S(u,v)|^2\}}{E\{|S(u,v)|^2+|N(u,v)|^2\}} \tag{4}$$

3. APOSTERIORI WIENER FILTERS

When using expectation spectra in (1), H is least squares optimal in an apriori sense. From the aposteriori viewpoint, when specific spectra R and D have been chosen from their respective parent ensembles, there is cross-correlation between R and D. These specific spectra can be estimated and used to synthesize a second stage filter, $H2$, that will be better than the first stage, H, in the aposteriori sense.

We will use the aposteriori scheme as shown in Figure 1. Apriori we assume that the cross-correlation between the signal and noise is zero, thus for the initial stage we will use the uncorrelated estimation filter of (4). The filters to estimate S and N are

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$$H_{S_1}(u,v) = \frac{E\{|S(u,v)|^2\}}{E\{|N(u,v)|^2 + |S(u,v)|^2\}} \qquad (5)$$

$$H_{N_1}(u,v) = \frac{E\{|N(u,v)|^2\}}{E\{|N(u,v)|^2 + |S(u,v)|^2\}} \qquad (6)$$

The first stage estimates of the signal and noise are

$$\hat{S}_1(u,v) = H_{S_1}(u,v)R(u,v) \qquad (7)$$

$$\hat{N}_1(u,v) = H_{N_1}(u,v)R(u,v) \qquad (8)$$

Now we can use $\hat{N}_1$ and our apriori assumed S to estimate the cross-correlation between the signal and noise and use this estimate in generating the next filter stage using the correlated estimation filter of (3). The *ith* stage filters are

$$H_{S_i}(u,v) = \frac{E\{|S(u,v)|^2 + S(u,v)\hat{N}_{i-1}^*(u,v)\}}{E\{|S(u,v)|^2 + |\hat{N}_{i-1}(u,v)|^2 + 2S(u,v)\hat{N}_{i-1}^*(u,v)\}} \qquad (9)$$

$$H_{N_i}(u,v) = \frac{E\{|\hat{N}_{i-1}(u,v)|^2 + S^*(u,v)\hat{N}_{i-1}(u,v)\}}{E\{|S(u,v)|^2 + |\hat{N}_{i-1}(u,v)|^2 + 2S^*(u,v)\hat{N}_{i-1}(u,v)\}} \qquad (10)$$

and the *ith* stage estimates are

$$\hat{S}_i(u,v) = H_{S_i}(u,v)R(u,v) \qquad (11)$$

$$\hat{N}_i(u,v) = H_{N_i}(u,v)R(u,v) \qquad (12)$$

4. EXPERIMENTAL RESULTS

The 128 x 128 image of Lenna, Figure 2, was chosen to illustrate this method. Random phase, white noise was added to the image of Lenna to obtain the input image in Figure 3. The noise was adjusted such that the SNR of the input image is 0dB. The first stage estimate of Lenna is shown in Figure 4, while stages 2, 10, and 40, are shown respectively in Figures 5, 6, and 7. The output SNR of stage 1 is 15 dB, stage 2 is 16 dB, stage 10 is 19 dB, and stage 40 is 20 dB.

For the second example, random phase, Gaussian spectrum noise was added to the image of Lenna to obtain the input image in Figure 9. The noise was adjusted such that the SNR of the input image is 0 dB. The first stage estimate of Lenna is shown in Figure 10, while stages 2, 10, and 40, are shown respectively in Figures 11, 12, and 13. The output SNR of stage 1 is 4 dB, stage 2 is 5 dB, stage 10 is 11 dB, and stage 40 is 12 dB.

In both examples, we see that the initial stage pulls out the image partly, but once the cross-correlation terms are used, Lenna emerges very quickly. Plots of the SNR versus number of iterations are shown in Figures 14 and 15. For white noise, very little improvement is seen after 13 iterations. For Gaussian noise, little improvement is seen after 7 iterations. The white noise example shows more SNR improvement than the Gaussian noise example. Even though the overall SNR is 0dB for the input, the actual in-band SNR is quite different for the two examples. The signal image has 95% of its energy concentrated within a radius of the first four bins. White noise has its energy evenly distributed among the 128x128 bins, while the Gaussian noise has 37% of its energy concentrated within a radius of the first four bins. The in-band SNR of the input for the white noise example is much higher than the in-band SNR of the input for the Gaussian noise example

5. CONCLUSIONS

We have shown that the one-dimensional aposteriori technique may be generalized to two dimensions. This aposteriori technique has been used to estimate images from a noisy background and gives significant improvement beyond that provided by common apriori Wiener estimation filtering. Using the ideal image as the desired result, this technique gives the best least-squares estimate.

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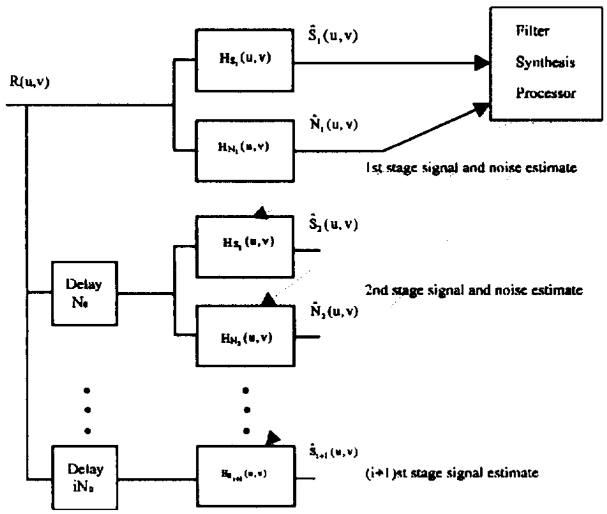

Figure 1 Block diagram of adaptive aposteriori Wiener

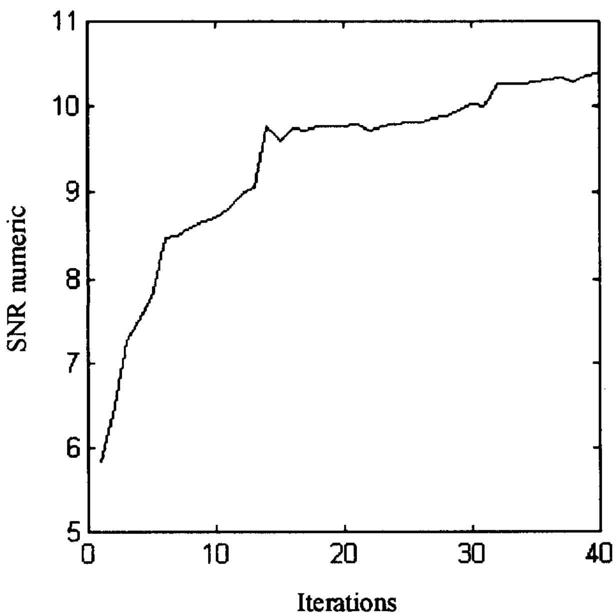

Figure 14 White noise: SNR vs. Iterations

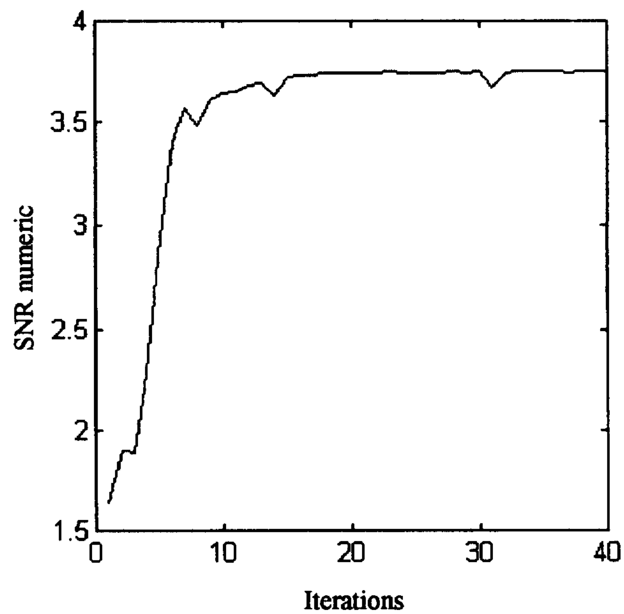

Figure 15 Gaussian noise: SNR vs. Iterations

Figure 2 Signal

Figure 3 Input

Figure 4 First Stage Estimate

Figure 5 Second Stage Estimate

Figure 6 Tenth Stage Estimate

Figure 7 Fortieth Stage Estimate

Figure 8 Signal

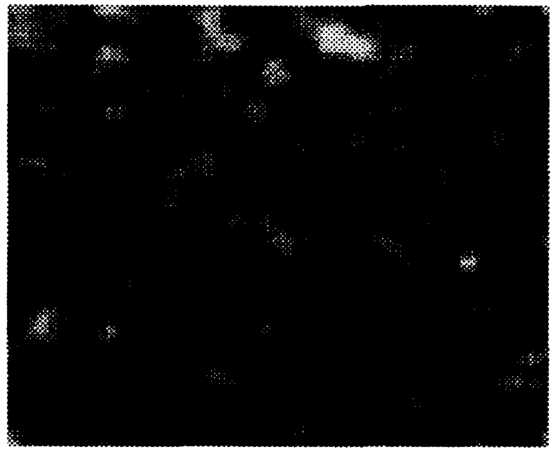

Figure 9 Input

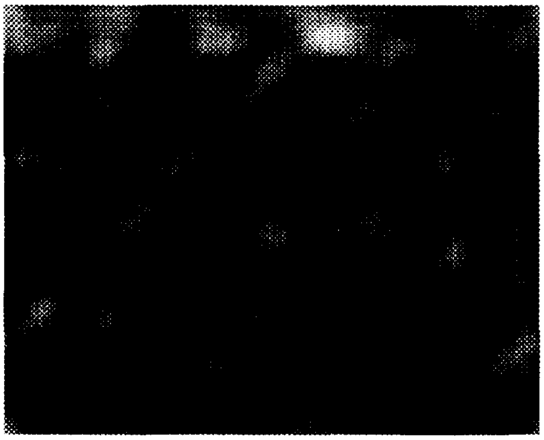

Figure 10 First Stage Estimate

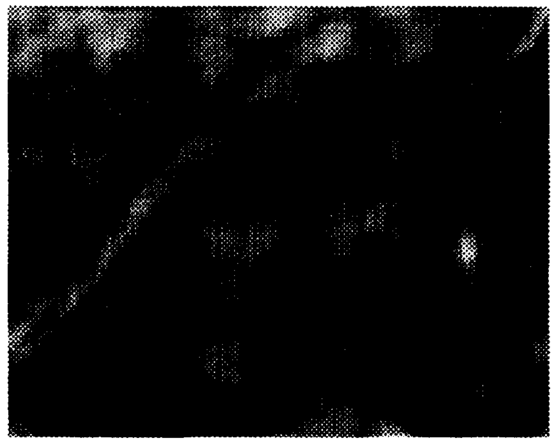

Figure 11 Second Stage Estimate

Figure 12 Tenth Stage Estimate

Figure 13 Fortieth Stage Estimate