Error Probabilities for FFH/BFSK with Ratio-Statistic Combining and Soft Decoding in a Fading Channel with Partial-Band Jamming

Anthony Y. P. Chua, Tri T. Ha, and R. Clark Robertson
Department of Electrical and Computer Engineering
Naval Postgraduate School
Monterey, California 93943-5121

Abstract

An error probability analysis is performed for a communications link with convolutional coding and soft-decision Viterbi decoding implemented on a fast frequency-hopped binary frequency-shift (FFH/BFSK) spread spectrum system. The generated signal is transmitted through a frequency non-selective, slowly fading channel with partial-band jamming and additive white Gaussian noise. To reduce the effects of partial-band jamming, ratio-statistic combining is introduced at the receiver before the soft decision Viterbi decoding. With the use of convolutional coding and soft decision Viterbi decoding, the performance of the receiver is improved dramatically when the E_b/N_j is greater than about 12 dB, and multiple hops per bit are not required in order to force the jammer to move from a partial-band to a full-band jamming strategy.

1 Introduction

The error probability analysis of a communications link employing convolutional coding with soft decision Viterbi decoding implemented on a fast frequency-hopped, binary frequency-shift keying (FFH/BFSK) spread spectrum receiver with ratio-statistic combining and a channel with Ricean fading and partial-band jamming is presented in this paper. The implementation of convolutional coding with soft decision Viterbi decoding extends the research performed in [1].

At the transmitter, convolutional coding is first implemented on the data. After that, the transmitter transmits L hops for each encoded bit. The channel is modeled as a Ricean fading channel. Both partial-band jamming and thermal noise are also assumed to affect the channel.

The FFH/BFSK receiver with ratio-statistic combining and soft decision Viterbi decoding is shown in Fig. 1. At the receiver, the signals are dehopped and demodulated by a pair of noncoherent matched filters. Before going to the soft-decision Viterbi decoder, the signals are passed through a ratio-statistic combiner [2]. This is used to nonlinearly combine the outputs of the detectors of the two branches of the BFSK demodulator which are then combined to obtain the decision statistics. The largest output of the two envelope detectors is first inverted and then multiplied with the difference of the outputs of the two branches of the receiver. This normalizes the output of the receiver when a hop experience a large amount of jamming noise. Thus, the effects of partial-band jamming on the receiver are minimized.

The channel for each hop is modeled as a frequency non-selective, slowly fading Ricean process and each hop is assumed to fade independently. With these assumptions, it is implied that the channel's coherence bandwidth is larger than the signal bandwidth but smaller than the smallest frequency spacing. Furthermore, the channel coherence time is larger than the hop duration. The dehopped signal is modeled as a Ricean random variable and consists of a non-faded (direct) component and a Rayleigh-faded (diffuse) component.

There are two types of interference considered in this paper. These are partial-band jamming and wideband interference. The partial-band jamming can be caused either by a partial-band jammer or some unintended narrowband interference. This is modeled as an additive Gaussian noise. If γ is the fraction of the spread bandwidth that is jammed, then the partial-band jamming will be present in both branches of the BFSK demodulator with probability γ. If $N_I/2$ is the average power spectral density of the interference over the entire spread bandwidth, then if the partial-band interference is present, its power spectral density is $N_I/(2\gamma)$. The wideband interference is assumed to be caused by thermal noise or other wideband interferences. The power spectral density of this wideband noise is defined as $N_0/2$.

The power spectral density of the total noise, $N_T/2$, is

$$\frac{N_T}{2} = \begin{cases} \dfrac{N_I}{2\gamma} + \dfrac{N_0}{2} & \text{when jamming is present} \\[2ex] \dfrac{N_0}{2} & \text{when jamming is absent} \end{cases} \tag{1}$$

If the ratio-statistic BFSK demodulator has a noise equivalent bandwidth of B Hz, then the received noise power for a given hop k is

$$\sigma_k^2 = \begin{cases} \left(\frac{N_I}{\gamma} + N_0\right) B & \text{with probability } \gamma \\ N_0 B & \text{with probability } (1-\gamma) \end{cases} \quad (2)$$

If the bit rate is R_b, then with L hops/bit, the hop rate $R_h = LR_b$. Therefore, the noise equivalent bandwidth, B, must satisfy $B \geq R_h$. Furthermore, the overall spread bandwidth is assumed to be very large compared to the hop rate. Since we assume that the bit energy is constant, the coded bit energy, E_c, is related to the uncoded bit energy, E_b, and the code rate, r, by $E_c = rE_b$.

2 Analysis

The analysis of bit error probability for the receiver depicted in Fig. 1 is performed in this paper. This analysis requires the statistics of the sampled outputs x_{ik}, $i = 1, 2$ of each envelope detector for a given hop k of a bit.

In order to determine the bit error probabilities for a radio-statistic combined receiver and a Ricean fading channel, we need to first determine the probability density function (pdf) of Y, the normalized decision statistic which is formed by summing the normalized, sampled outputs for each hop over j coded transmissions.

2.1 Probability of Bit Error, P_b

It is well known that the probability of bit error for soft-decision Viterbi decoding is upper bounded by [3–5]

$$P_b \leq \frac{1}{k} \sum_{j=d_{\text{free}}}^{\infty} \omega_j P_j \quad (3)$$

where ω_j is the number of paths of weight j times the number of ones in the information sequence corresponding to one of these paths, k is the number of input bits to the encoder, P_j is the probability that the all-zero path is eliminated by a path of weight j, and d_{free} is the free distance of the code. This is the normal union bound used to evaluate the performance of coded systems. To determine the tightness of the bound, we need to compare it with a simulation of the system under consideration since an analytic solution does not exist. This bound is reasonably tight for a conventional MFSK system with no fading [3]. Therefore, we can expect the bound to be tight for the normalized receiver, especially when the number of hops per bit is small.

As the hops are independent and identically distributed, P_j is derived from

$$P_j = \prod_{k=1}^{j} \sum_{i_k=0}^{L} \binom{L}{i_k} \gamma^{i_k} (1-\gamma)^{L-i_k} p_k(i_k) \quad (4)$$

where $p_p(i_p)$ is the probability of coded bit error given that i_p of L hops in p^{th} coded transmission are jammed. This is simplified to a simple binomial expression of the form

$$P_j = \sum_{i_j=0}^{jL} \binom{jL}{\sum_{p=1}^{j} i_p} \times \gamma^{\sum_{p=1}^{j} i_p} (1-\gamma)^{jL - \sum_{p=1}^{j} i_p} P_j(\underline{i}) \quad (5)$$

where $\underline{i} = [i_1 \; i_2 \; \cdots \; i_j]$ and $P_j(\underline{i})$ is the conditional probability that the all-zero path is eliminated by a path of weight j given that $\underline{i}$ hops are jammed and equals $p_1(i_1)p_2(i_2)\cdots p_j(i_j)$.

The conditional probability $P_j(\underline{i})$ is equal to the probability that the sum of j samples from the signal-present detector branch is smaller than the sum of j samples from the signal-absent detector branch [4]. Thus,

$$P_j(\underline{i}) = \Pr(-L \leq Y \leq 0) = \int_{-L}^{0} f_Y(y|\underline{i})dy \quad (6)$$

where Y is the random variable representing the ratio-statistic combined outputs summed over j coded transmissions. Without loss of generality, it is assumed that the signal is present in branch 1.

It is observed that in order to determine the bound on bit error probabilities given by (3) for a ratio-statistic receiver and a Ricean fading channel, we need to first determine the conditional probability density function of Y.

2.2 Probability Density Function of the Normalized Decision Variable Y

In order to determine the pdf of the ratio-statistic combined sampled outputs of each hop, we let σ_k^2 represent the noise power in hop k of a bit and $a_k\sqrt{2}$ represent the signal amplitude. The conditional pdf of the random variable X_{1k} at the output of detector branch 1 is [6]

$$f_{X_{1k}}(x_{1k}|a_k) = \frac{x_{1k}}{\sigma_k^2} \exp\left(-\frac{x_{1k}^2 + 2a_k^2}{2\sigma_k^2}\right) \times I_0\left(\frac{a_k x_{1k}\sqrt{2}}{\sigma_k^2}\right) u(x_{1k}) \quad (7)$$

where $u(\cdot)$ is the unit step function, a_k is a Ricean random variable representing the fading of hop k of a bit, and $I_0(\cdot)$ is the modified Bessel function of the first kind and order zero.

For branch 2, fading has no effect on X_{2k} since there is no signal present. Therefore, the pdf of the random variable X_{2k} is [6]

$$f_{x_{2k}}(x_{2k}) = \frac{x_{2k}}{\sigma_k^2} \exp\left(-\frac{x_{2k}}{2\sigma_k^2}\right) u(x_{2k}) \quad (8)$$

From Fig. 1, normalization is done by choosing the largest of X_{1k} and X_{2k}, inverted and multiplied back with the two branches and differentially summed. Therefore, the normalized random variable Y_k is given by

$$Y_k = \frac{X_{1k} - X_{2k}}{\max(X_{1k}, X_{2k})} \tag{9}$$

where $1 \geq Y_k \geq -1$. Furthermore,

$$\begin{array}{lll} X_{2k} < X_{1k} & \Longrightarrow & 0 \leq Y_k \leq 1 \\ X_{2k} > X_{1k} & \Longrightarrow & -1 \leq Y_k \leq 0 \end{array} \tag{10}$$

Using (7)–(10), we obtain the conditional pdf of Y_k as [1,7]

$$\begin{aligned} f_{Y_k}(y_k|a_k) &= \frac{2(1+y_k)}{[1+(1+y_k)^2]^2} \\ &\times \exp\left[-\frac{a_k^2}{\sigma_k^2}\left(\frac{1}{1+(1+y_k)^2}\right)\right] \\ &\times \left[1+\frac{a_k^2(1+y_k)^2}{\sigma_k^2[1+(1+y_k)^2]}\right] \end{aligned} \tag{11}$$

for $-1 \leq y_k \leq 0$, and

$$\begin{aligned} f_{Y_k}(y_k|a_k) &= \frac{2(1-y_k)}{[1+(1-y_k)^2]^2} \\ &\times \exp\left[-\frac{a_k^2}{\sigma_k^2}\left(\frac{(1-y_k)^2}{1+(1-y_k)^2}\right)\right] \\ &\times \left[1+\frac{a_k^2}{\sigma_k^2[1+(1-y_k)^2]}\right] \end{aligned} \tag{12}$$

for $0 \leq y_k \leq 1$. Let the total average signal power be a_k^2, which is a summation of α^2 {average power of the direct component} and $2\sigma^2$ {average power of the diffuse component}. This total average signal power is assumed to remain constant for each hop. It should be noted that when $\alpha^2 = 0$, the channel is a Rayleigh fading channel, and if $2\sigma^2 = 0$, there is no fading. If the fading of the hop k is modeled by assuming a_k to be a Ricean random variable, the pdf of a_k is [6]

$$f_{A_k}(a_k) = \frac{a_k}{\sigma^2}\exp\left[-\frac{a_k+\alpha^2}{2\sigma^2}\right] I_0\left[\frac{a_k\alpha}{\sigma^2}\right] u(a_k) \tag{13}$$

The pdf of the normalized random variable Y_k is found as

$$f_{Y_k}(y_k) = \int_0^\infty f_{Y_k}(y_k|a_k) f_{A_k}(a_k)\, da_k \tag{14}$$

Substituting (11), (12), and (13) into (14) and defining the signal-to-noise ratio of the direct component of hop k of a bit as $\rho_k = \alpha^2/\sigma_k^2$ and the signal-to-noise ratio of the diffuse component of hop k of a bit as $\xi_k = 2\sigma^2/\sigma_k^2$, we get

$$\begin{aligned} f_{Y_k}(y_k) &= \frac{2(1+y_k)}{[1+(1+y_k)^2][1+\xi_k+(1+y_k)^2]} \\ &\times \exp\left[-\frac{\rho_k}{1+\xi_k+(1+y_k)^2}\right] \\ &\times \left[1+\frac{(1+y_k)^2}{1+\xi_k+(1+y_k)^2}\right. \\ &\times \left.\left(\xi_k+\frac{\rho_k(1+(1+y_k)^2)}{1+\xi_k+(1+y_k)^2}\right)\right] \end{aligned} \tag{15}$$

for $-1 \leq y_k \leq 0$, and

$$\begin{aligned} f_{Y_k}(y_k) &= \frac{2(1-y_k)}{[1+(1-y_k)^2][1+(\xi_k+1)(1-y_k)^2]} \\ &\times \exp\left[-\frac{\rho_k(1-y_k)^2}{1+(\xi_k+1)(1-y_k)^2}\right] \\ &\times \left[1+\frac{1}{1+(\xi_k+1)(1-y_k)^2}\right. \\ &\times \left.\left(\xi_k+\frac{\rho_k(1+(1-y_k)^2)}{1+(\xi_k+1)(1-y_k)^2}\right)\right] \end{aligned} \tag{16}$$

for $0 \leq y_k \leq 1$.

Let $Y_{kp}^{(1)}$ and $Y_{kp}^{(2)}$ denote the random variable Y_{kp} when hop k of bit p is jammed or not jammed, respectively. We need to determine the random variable for the sum of j coded transmissions for convolutional coding with soft decision Viterbi decoding [4–5]. Assuming that for a coded transmission p of the j transmissions, i_p hops are jammed, we obtain the resultant decision random variable Y

$$Y = \sum_{p=1}^{j}\sum_{k=1}^{L} Y_{kp} = \sum_{p=1}^{j}\sum_{k=1}^{i_p} Y_{kp}^{(1)} + \sum_{p=1}^{j}\sum_{k=i_p+1}^{L} Y_{kp}^{(2)} \tag{17}$$

As the $Y_{kp}^{(n)}$ are independent and identically distributed random variables, the above equation simplifies to

$$Y = \sum_{p=1}^{j}\sum_{k=1}^{L} Y_{kp} = \sum_{k=1}^{\sum_{p=1}^{j} i_p} Y_{kp}^{(1)} + \sum_{k=1}^{jL-\sum_{p=1}^{j} i_p} Y_k^{(2)} \tag{18}$$

Since all hops are independent, we obtain the conditional pdf for the decision variable Y given that $i_1, i_2, \cdots, i_j$ hops of the coded transmissions are jammed as

$$\begin{aligned} f_Y(y|i_1, i_2, \cdots, i_j) &= \left[f_{Y_k}^{(1)}(y_k^{(1)})\right]^{\otimes \sum_{i=1}^{j} i_i} \\ &\otimes \left[f_{Y_k}^{(2)}(y_k^{(2)})\right]^{\otimes\left[jL-\sum_{i=1}^{j} i_i\right]} \end{aligned} \tag{19}$$

where $a \otimes b$ = convolution of a and b, and $a^{\otimes c}$ represents a c-fold convolution. The convolutions required in (22) cannot be performed analytically. However, as Y_k is limited in range, it is easy to perform the required convolutions numerically.

3 Numerical Results

3.1 Numerical Procedure

The conditional probability of bit errors given that i hops of the j bits have partial-band jamming is obtained by numerical convolution of (19) followed by numerical integration of (6). The results of the numerical computation are then substituted into (5) to obtain P_j. This result for P_j is then substituted into (3) where the weights (ω_j), the coding free distance (d_{free}), and the number of input bits (k) are used to determine the probability of bit error.

System performance is evaluated for various values of jamming fraction γ, diversity, fading conditions, the values of bit energy-to-jamming noise density ratio (E_b/N_I), and the ratio of the direct-to-diffuse signal energy ($\alpha^2/2\sigma^2$). This ratio of the direct-to-diffuse signal energy is assumed to be the same for each hop k of the bit.

3.2 Performance for Rayleigh Fading Channels

Fig. 2 is a plot of the performance of the receiver for a Rayleigh fading channel ($\alpha^2/2\sigma^2 = 0.001$) and $\gamma = 1.0$ (full band jamming), 0.25, 0.1, and 0.01 with code of constraint length $\nu = 4$ and $L = 2$. The asymptotic probability of bit error is about 10^{-5} as compared to that of the uncoded performance of 10^{-2}. It is observed that coding improves performance when E_b/N_I is greater than 12 dB. It is also seen for Rayleigh fading that the worst case jamming strategy is full band jamming.

3.3 Performance for Ricean Fading Channels

Fig. 3 shows the performance of the receiver when $\gamma = 1.0$ (full band jamming), 0.25, 0.1, and 0.01 ($\nu = 4$, $L = 2$, and $\alpha^2/2\sigma^2 = 10$). The asymptotic bit error probability is about 10^{-11} as compared to 10^{-3} for the uncoded case. Also, coding improves the performance when E_b/N_I is greater than 10 dB. Furthermore, it can be seen that the worst case jamming strategy is quite close to full band jamming.

4 Conclusion

When convolutional coding with Viterbi soft decision decoding is employed in conjunction with FFH/BFSK utilizing ratio-statistics combining, the system performance is superior to the performance of the equivalent uncoded system when E_b/N_I is greater than 12 dB. Below 12 dB uncoded system performance is too poor for coding to be effective. In addition, at high E_b/N_I, the asymptotic probability of bit error improves dramatically as compared to the uncoded system. The jamming strategy required to cause worst case receiver performance is full band jamming. This means that the enemy jammer needs to spread its jamming power over a much wider bandwidth.

Due to non-coherent combining losses, there is some degradation in performance when the hop per bit ratio is increased and the received signal is at moderate E_b/N_I.

It is found, as expected, that when a stronger code (higher constraint length) is used that performance is improved, especially for high E_b/N_I.

References

[1] R. Clark Robertson, J. R. Riley, and T. T. Ha, "Error probabilities of fast frequency-hopped FSK with ratio-statistic combining in a fading channel with partial-band interference," *Proc. IEEE Military Communications Conf.*, vol. 3, pp. 865–869, 1992.

[2] C. M. Keller and M. B. Pursley, "Clipped diversity combining for channels with partial-band interference — Part II: Ratio-statistic combining," *IEEE Trans. Commun.*, vol. 37, pp. 145–151, Feb. 1989.

[3] J. P. Oldenwalder, "Optimal decoding of convolutional codes," Ph.D. dissertation, Univ. of California, Los Angeles (Univ. Microfilms 70-19, 875) 1970.

[4] G. C. Clark, Jr. and J. B. Cain, *Error Correction Coding for Digital Communications*, Plenum Press, 1981.

[5] J. G. Proakis, *Digital Communications*, 2nd ed., New York: McGraw-Hill, 1989.

[6] A. D. Whalen, *Detection of Signals in Noise*, New York: Academic Press, 1971.

[7] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

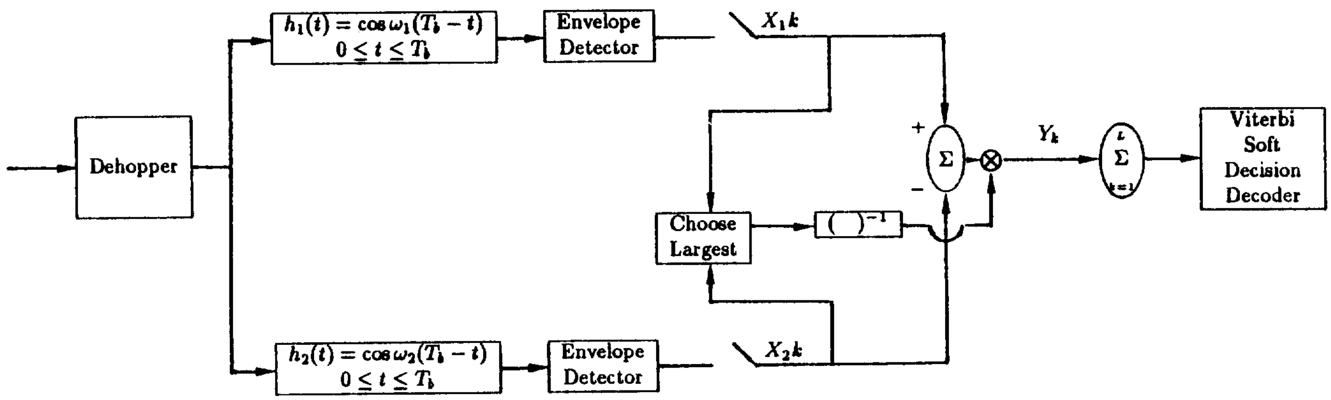

Figure 1: FFH/BFSK receiver with ratio to statistics combining.

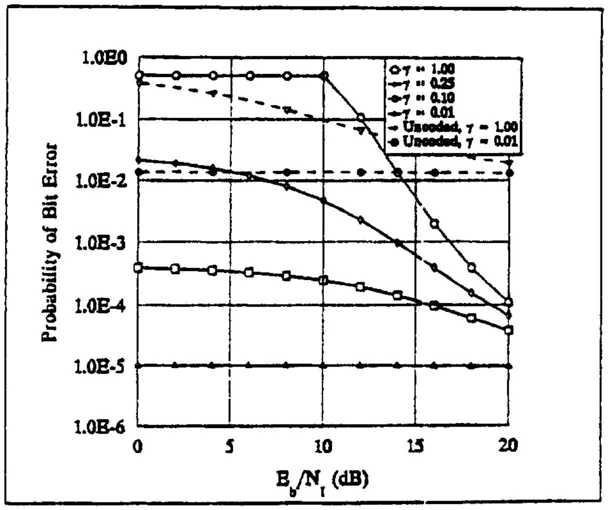

Figure 2: Performance of ratio-statistic combined receiver with soft-decision Viterbi decoding for $\gamma = 1.0,\ 0.25,\ 0.1,$ and 0.01 with Rayleigh fading ($\alpha^2/2\sigma^2 = 0.001$) and $E_b/N_0 = 16.0$ dB, $L = 2$, and $\nu = 4$.

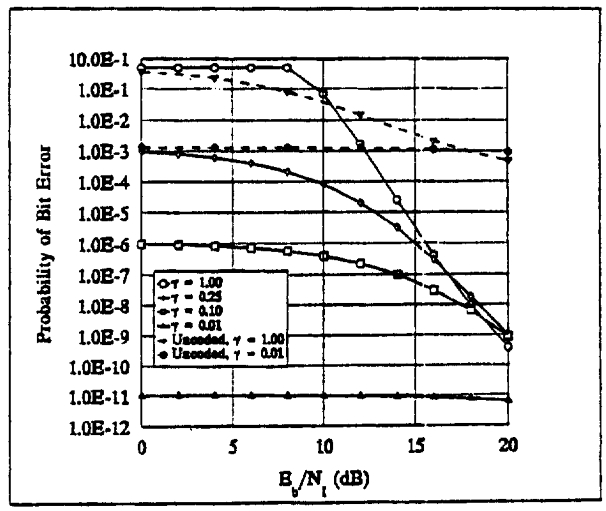

Figure 3: Performance of ratio-statistic combined receiver with soft-decision Viterbi decoding for $\gamma = 1.0, 0.25, 0.1,$ and 0.01 with Rician fading ($\alpha^2/2\sigma^2 = 10.0$) and $E_b/N_0 = 16.0$ dB, $L = 2$, and $\nu = 4$.