# Blind Identification Algorithms For Co-Channel Systems Using Higher-Order Statistics *

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## Abstract

*In this work, we describe a method for the blind identification of co-channel systems. More specifically, two independent communication signals propagate through multipath channels, and their sum is picked up by the receiver. This type of a problem occurs in communication systems which employ frequency reusage. In some cases, there is no training signal available to aid the receiver for channel estimation, hence the multipath channel transfer functions have to be estimated blindly. The algorithm described in this work uses second- and fourth-order cumulants of the received signal. First, the magnitude responses of the channels are estimated, then using these, the phase responses are estimated.*

## 1 Introduction

Co-channel interference is an important problem in communication systems employing frequency reusage (such as in mobile communication systems). In this work, we consider the problem of blindly identifying the channel transfer functions in a co-channel system. We assume that two independent communication signals propagate through multipath channels, and their sum is picked up by the receiver.

The mathematical model of the co-channel system is as shown in Fig. 1(a). The unknown data sequences, denoted by $x_{1,k}$ and $x_{2,k}$, have the common sampling period $T$. The output signals of the continuous-time channels $h_1(t)$ and $h_2(t)$ are denoted as $y_1(t)$ and $y_2(t)$. The receiver observes the sum of $y_1(t)$ and $y_2(t)$ given by

$$y(t) = \sum_{k=-\infty}^{\infty} x_{1,k}h_1(t-kT) + \sum_{k=-\infty}^{\infty} x_{2,k}h_2(t-kT) \quad (1)$$

We assume that the channel impulse responses $h_1(t)$ and $h_2(t)$ are unknown. They represent the effects of baseband shaping filters (such as the baseband pulse, partial response filter, etc.) in the transmitter and the multipath channels. The receiver picks up the continuous-time signal $y(t)$ and samples it at the baud rate $1/T$ to yield the sequence $y_k = y(kT)$. The above described continuous time system can be represented by the discrete-time model shown in Fig.1(b). For this model, we have

$$y_k = h_{1,k} * x_{1,k} + h_{2,k} * x_{2,k} \quad (2)$$

where we have $h_{i,k} = h_i(kT)$, $i = 1, 2$.

Our aim is to identify the impulse responses $h_{1,k}$ and $h_{2,k}$. If training sequences were available, the receiver could estimate the channels using the well known system identification techniques such as LMS algorithm, FTF algorithm, or other least-squares techniques. In many applications, the receiver doesn't have access to training sequences. Furthermore, there may not be sufficiently good initial estimates of the channels, so the receiver cannot operate reliably in the decision-directed mode. Hence the channel impulse responses (or, equivalently, the transfer functions) must be identified blindly, solely based on the observation of $y_k$ and some statistical information about the model. It is reasonable to assume that the input sequences $x_{1,k}$ and $x_{2,k}$ are statistically independent from each other.

In this paper, we describe a blind co-channel system identification algorithm based on higher-order statistics. The algorithm consists of two steps: first the magnitude responses of the channel transfer functions are estimated; second, using these magnitude

*This work is supported by the National Science Foundation under grant MIP-9206829 and the Office of Naval Research under contract N00014-92-J-1034.



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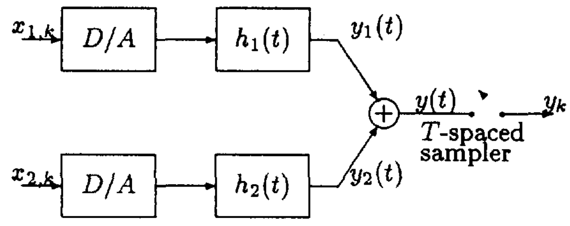
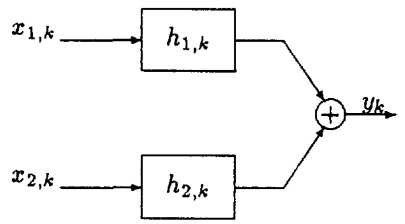



(a)




(b)

Figure 1: The co-channel system model: (a) the continuous time model, (b)the discrete time model.

estimates, the phase responses are estimated. The magnitude responses are estimated using the power-spectrum together with a slice of either the bispectrum or the trispectrum of the received signal. The phase responses are estimated from a slice of the bispectrum or trispectrum only. In most communication systems, the signals have symmetrical probability distributions. For symmetrical distributions, the bispectrum vanishes, hence trispectrum has to be used. In this approach, there is no requirement for the input signals to be discrete-time or discrete-state. However, the distribution of these signals must be non-Gaussian.

The organization of the paper is as follows. In Section 2, the higher-order statistics based approach for blind identification of co-channel systems will be described. In Section 3, experimental results will be given. Finally conclusions will be given in Section 4.

## 2 Algorithm Description

In this section, we will describe the proposed algorithm. We will consider the discrete-time equivalent model of the co-channel system shown in Figure 1(b). We will assume that the input sequences $x_{1,k}$ and $x_{2,k}$ are stationary, statistically independent from each other, and have non-Gaussian distributions.

We will denote the power spectrum, bispectrum, and trispectrum of the input sequences $x_{i,k}$, $i = 1, 2$ by $S_{x_i}(\omega)$, $B_{x_i}(\omega_1, \omega_2)$, and $T_{x_i}(\omega_1, \omega_2, \omega_3)$, respectively. It is assumed that these spectra are known by the receiver. If the input sequences can be modeled as independent identically distributed, then these spectra will reduce to the variance, skewness, and kurtosis, respectively. So we have

$$S_{x_i}(\omega) = \gamma_2^{x_i} \quad (3)$$
$$B_{x_i}(\omega_1, \omega_2) = \gamma_3^{x_i} \quad (4)$$
$$T_{x_i}(\omega_1, \omega_2, \omega_3) = \gamma_4^{x_i} \quad (5)$$

Let $H_1(\omega)$ and $H_2(\omega)$ be the Fourier transforms of $h_{1,k}$ and $h_{2,k}$ respectively. That is,

$$H_1(\omega) = |H_1(\omega)|e^{j\phi_1(\omega)}$$
$$H_2(\omega) = |H_2(\omega)|e^{j\phi_2(\omega)} \quad (6)$$

where $\phi_1(\omega)$ and $\phi_2(\omega)$ represent the channel phase responses. We want to estimate $H_1(\omega)$ and $H_2(\omega)$ solely based on the received signal $y_k$, and using the statistical information about the input sequences. The algorithm described in this section consists of two steps: (1) estimation of magnitude responses, and (2) estimation of phase responses.

### 2.1 Estimation of Magnitude Responses

Let $S_y(\omega)$, $B_y(\omega_1, \omega_2)$, and $T_y(\omega_1, \omega_2, \omega_3)$ be the power spectrum, bispectrum, and trispectrum of $y_k$, respectively. Then, using the statistical independence of the input sequences, we obtain the following relationships:

$$\begin{aligned} S_y(\omega) &= S_{x_1}(\omega)|H_1(\omega)|^2 \\ &+ S_{x_2}(\omega)|H_2(\omega)|^2 \quad (7) \\ B_y(\omega, 0) &= H_1(0).B_{x_1}(\omega, 0)|H_1(\omega)|^2 \\ &+ H_2(0).B_{x_2}(\omega, 0)|H_2(\omega)|^2 \quad (8) \\ T_y(\omega, 0, 0) &= H_1^2(0).T_{x_1}(\omega, 0, 0)|H_1(\omega)|^2 \\ &+ H_2^2(0).T_{x_2}(\omega, 0, 0)|H_2(\omega)|^2 \quad (9) \end{aligned}$$

Note that, for each value of $\omega$, we have a system of linear equations in terms of the unknowns $|H_1(\omega)|^2$ and $|H_2(\omega)|^2$. Hence, using $S_y(\omega)$ together with either $B_y(\omega, 0)$ or $T_y(\omega, 0, 0)$, one can obtain non-parametric estimates for the magnitude responses $|H_1(\omega)|$ and $|H_2(\omega)|$ for each $\omega$. It is also possible to develop parametric methods to obtain smoother estimates by letting

$$|H_i(\omega)|^2 = \sum_{\ell=0}^{L} a_{i,\ell} \cos(\ell.\omega) \qquad i = 1, 2 \quad (10)$$

and solving for the coefficients $a_{i,\ell}$. More specifically, we may write a system of linear equations as

$$\begin{bmatrix} 1 & \cos(\omega_1) & \dots & \cos(L.\omega_1) \\ 1 & \cos(\omega_2) & \dots & \cos(L.\omega_2) \\ \vdots & \vdots & & \vdots \\ 1 & \cos(\omega_P) & \dots & \cos(L.\omega_P) \end{bmatrix} \cdot \begin{bmatrix} a_{i,0} \\ a_{i,1} \\ \vdots \\ a_{i,L} \end{bmatrix} = \begin{bmatrix} |H_i(\omega_1)|^2 \\ |H_i(\omega_2)|^2 \\ \vdots \\ |H_i(\omega_P)|^2 \end{bmatrix} \quad (11)$$

where $P > L$ and $\{\omega_j \; ; \; j = 1, \dots, P\}$ is an arbitrary set of values for $\omega$. The above system of overdetermined linear equations yield the coefficients $\{a_{i,\ell}\}$.

For the estimation of the power spectrum and bispectrum (or trispectrum), one of the techniques discussed in [1] may be used.

Note that to solve the above set of equations, we need to know $|H_1(0)|$ and $|H_2(0)|$. These may be obtained by setting $\omega = 0$ in the above equations, that is

$$\begin{aligned} S_y(0) &= S_{x_1}(0).H_1^2(0) \\ &+ S_{x_2}(0).H_2^2(0) \quad (12) \\ B_y(0,0) &= B_{x_1}(0,0).H_1^3(0) \\ &+ B_{x_2}(0,0).H_2^3(0) \quad (13) \\ T_y(0,0,0) &= T_{x_1}(0,0,0).H_1^4(0) \\ &+ T_{x_2}(0,0,0).H_2^4(0) \quad (14) \end{aligned}$$

An interesting case to note is that if one of the channels is initially known to have a spectral null at some *known* frequency $\omega = \omega_0$, then $T_y(\omega, \omega_0, \omega_0)$ will be the trispecrum slice of the other channel only, hence one can directly obtain certain slices of the cumulant spectra of the other channel by setting one or more of the frequency arguments to $\omega_0$. If one of the channels has a spectral null at some *unknown* frequency, then one may first obtain magnitude estimates as described above, and estimate the frequency $\omega_0$ of the spectral null. In some cases, due to spectral shaping in the transmitter, the signal spectrum may be known to have nulls or near nulls at certain frequencies.

### 2.2 Estimation of Phase Responses

In the next step of the algorithm, we solve for the phase responses $\phi_1(\omega)$ and $\phi_2(\omega)$. If the input signals have asymmetric distributions, one may use the slice of bispectrum of the received signal given as

$$\begin{aligned} B_y(\omega,\omega) &= B_{y_1}(\omega,\omega) + B_{y_2}(\omega,\omega) = \\ & B_{x_1}(\omega,\omega).|H_1(\omega)|^2.|H_1(2\omega)|.e^{j(2\phi_1(\omega)-\phi_1(2\omega))} \\ & + B_{x_2}(\omega,\omega).|H_2(\omega)|^2.|H_2(2\omega)|.e^{j(2\phi_2(\omega)-\phi_2(2\omega))} \quad (15) \end{aligned}$$

If the input sequences $x_{1,k}$ and $x_{2,k}$ have symmetrical probability distributions, one may use the trispectrum slice given by

$$\begin{aligned} T_y(\omega,\omega,0) &= T_{y_1}(\omega,\omega,0) + T_{y_2}(\omega,\omega,0) = \\ & H_1(0).T_{x_1}(\omega,\omega,0).|H_1(\omega)|^2.|H_1(2\omega)|.e^{j(2\phi_1(\omega)-\phi_1(2\omega))} \\ & + H_2(0).T_{x_2}(\omega,\omega,0).|H_2(\omega)|^2.|H_2(2\omega)|.e^{j(2\phi_2(\omega)-\phi_2(2\omega))} \end{aligned}$$

It is also possible to use some other slices of the higher-order spectra such as

$$\begin{aligned} T_y(\omega,\omega,\omega) &= T_{y_1}(\omega,\omega,\omega) + T_{y_2}(\omega,\omega,\omega) = \\ & T_{x_1}(\omega,\omega,\omega).|H_1(\omega)|^3.|H_1(3\omega)|.e^{j(3\phi_1(\omega)-\phi_1(3\omega))} \\ & + T_{x_2}(\omega,\omega,\omega).|H_2(\omega)|^3.|H_2(3\omega)|.e^{j(3\phi_2(\omega)-\phi_2(3\omega))} \end{aligned}$$

Once the appropriate slice of the trispectrum, such as $T_y(\omega,\omega,0)$ is estimated, one can obtain estimates of

$$\begin{aligned} \angle T_{y1}(\omega,\omega,0) &= (2\phi_1(\omega) - \phi_1(2\omega)) \\ \angle T_{y2}(\omega,\omega,0) &= (2\phi_2(\omega) - \phi_2(2\omega)) \quad (16) \end{aligned}$$

by using the previously estimated magnitude responses. The geometry of this phase estimation procedure is shown in Fig.2. One can obtain a relation for the phase of channel 1 as

$$2\phi_1(\omega) - \phi_1(2\omega) = \angle T_y(\omega,\omega,0) \pm \cos^{-1} \frac{|T_{y1}(\omega,\omega,0)|^2 + |T_y(\omega,\omega,0)|^2 - |T_{y2}(\omega,\omega,0)|^2}{2|T_{y1}(\omega,\omega,0)|.|T_y(\omega,\omega,0)|} \quad (17)$$

Similarly, one can obtain a relation for $2\phi_2(\omega) - \phi_2(2\omega)$ by replacing $T_{y1}(\omega,\omega,0)$ and $T_{y2}(\omega,\omega,0)$ in the right hand side of the above equation. Note, however, that a problem with this method is the "±" ambiguity on the right hand side of the above equation. Techniques for resolving this ambiguity is under investigation. Possible approaches exploit certain continuity assumptions about the phase response and its derivative. Another approach is to use other slices of the higher order spectra simultaneously. For example, one may use $T_y(\omega,\omega,0)$ together with $T_y(\omega,\omega,\omega)$ to reduce the ambiguity problem.

We assume that we have unwrapped estimates of $2\phi_i(\omega) - \phi_i(2\omega)$, $i = 1, 2$. We may then write the following system of equations

$$\begin{bmatrix} 2\phi_i(0) - \phi_i(0) \\ 2\phi_i(1) - \phi_i(2) \\ 2\phi_i(2) - \phi_i(4) \\ \vdots \\ 2\phi_i(N) - \phi_i(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & .. \\ 0 & 2 & -1 & .. \\ 0 & 0 & 2 & .. \\ & \ddots & & \end{bmatrix} \begin{bmatrix} \phi_i(0) \\ \phi_i(1) \\ \phi_i(2) \\ \vdots \\ \phi_i(N) \end{bmatrix} \quad (18)$$

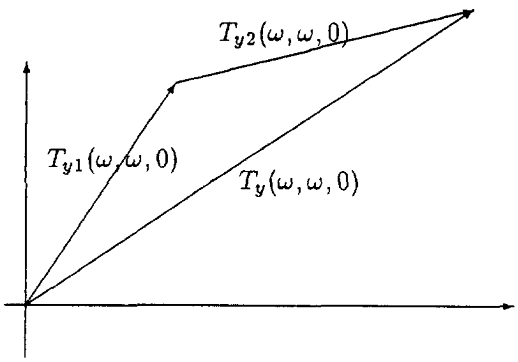



Figure 2: Geometry of phase estimation

for $i = 1, 2$. Finally, one can obtain the desired phase responses $\phi_1(\omega)$ and $\phi_2(\omega)$ by solving the above system of equations.

## 3 Experimental Results

In this section, we present an experimental result to illustrate the method described in this paper. The input signals for the channels are synthetically generated binary sequences. The sum of two channel outputs are assumed to be observed. The specified and estimated magnitude and phase responses are shown in Fig.3. In this experiment, $10^6$ data samples were used to obtain the desired spectral estimates.

## 4 Conclusions

We described a method for the blind identification of co-channel systems using higher-order spectra techniques. The method requires the input signal distributions to be non-Gaussian. We are currently investigating possible improvement techniques to obtain better estimates with less data.

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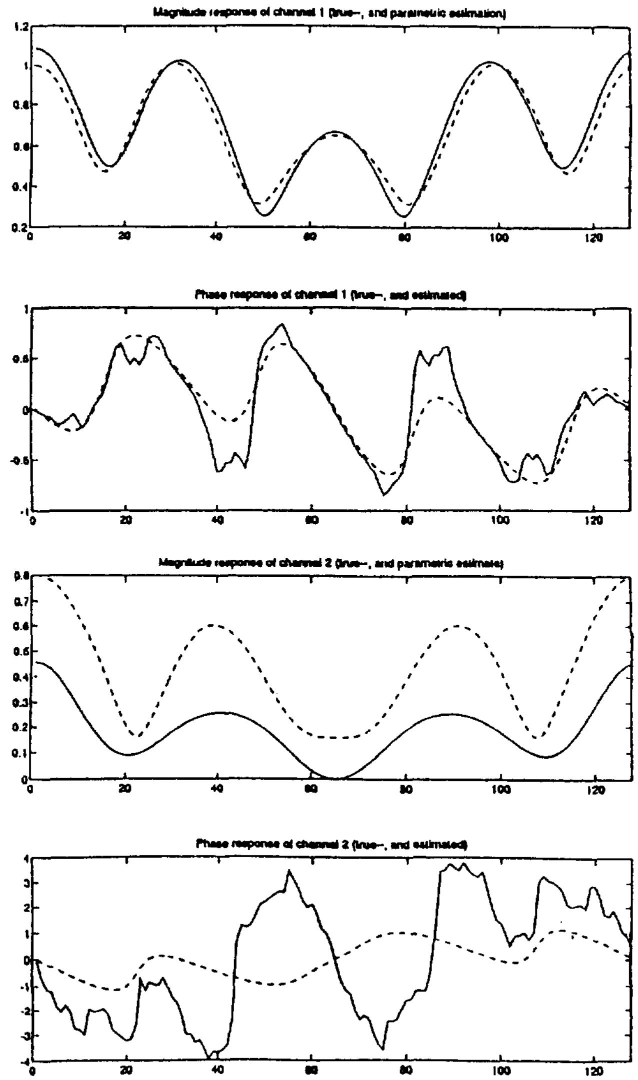

Figure 3: True and estimated magnitude and phase responses. Solid lines indicate estimated values, dashed lines indicate true values.