CHANNEL ORDER DETERMINATION BASED ON SAMPLE CYCLIC CORRELATIONS

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Abstract

Selecting the number of channel taps is an important task for blind equalization of fractionally-sampled FIR communications channels. We present two algorithms for selecting the channel order based on second-order cyclostationary statistics of the received data. The first algorithm uses an asymptotically constant false-alarm rate statistical test. In contrast, existing algorithms rely on rather ad hoc thresholding. The second algorithm extends our existing asymptotically best consistent channel estimator to include order determination.

Keywords: Order Determination, Fractionally-Spaced Blind Equalization, Cyclostationarity

1 Introduction

In channel equalization for digital communications, the impulse response of the channel is either directly or indirectly identified. One approach is to transmit a known training sequence and use standard input-output estimation methods to estimate the channel. An alternative approach, blind channel identification, estimates the channel based only on the observed output when the input symbol stream is i.i.d. Blind equalizers offer the advantage of channel estimation without the penalty of interrupting the input stream with the training sequence (see e.g., [2]).

Recently, second-order cyclic statistics have been exploited for blind channel equalization [6], [8], [12]. These cyclic approaches can identify even nonminimum phase systems (subject to certain constraints on the phase, e.g., [7]) with second-order statistics. In contrast, blind equalizers have relied so far on higher-(than second-) order statistics (HOS) to identify the channel. Second -order cyclic approaches, however, yield lower variance estimates than HOS methods for a given data length.

These properties have motivated considerable interest recently in the cyclic approaches. In general, the current methods [6], [8], [11], and [12] all assume the order or equivalently the length of the impulse response is known. In practice however, the order may not be known and must be estimated prior to the blind equalization algorithm. The problem of determining the order of the channel was addressed in [9] and [14] where algorithms are given for determining the channel order from the correlations and cyclic correlations. However, no statistical tests were presented and both methods require an arbitrarily determined threshold.

In this paper, we propose two methods for determining the order of an FIR channel through the use of second-order cyclic statistics. The first method is similar to [9]; however, we derive an asymptotically constant false alarm rate statistical test. The second method builds on the asymptotically optimal nonlinear channel estimator [6] by including order determination.

2 Problem Description

Consider the discrete-time system shown by Figure 1 in which $h(n)$ is the 'composite' impulse response

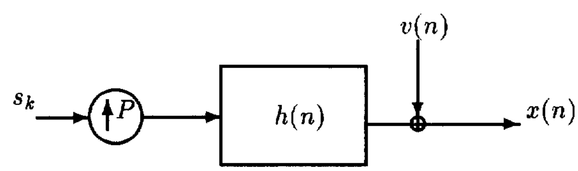

Figure 1: Oversampled Communications System

that includes the pulse-shaping filter, channel, and the receiver filters. The additive noise, $v(n)$, is stationary with covariance $c_{11v}(m)$ and the information symbols, s_k, are i.i.d. and drawn from a known, finite alphabet. This representation is the discrete-time equivalent of the 'oversampled' communication system (a.k.a. fractionally-space equalizer) considered in [7], [8], and [12]. The 'oversampling' factor is a positive integer, $P > 1$. (Note: $P = 1$ implies the continuous-time system is sampled once per symbol interval.) For T observations, the discrete-time output $x(n)$ is given

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by:

$$x(n) = \sum_k s_k\ h(n-kP)\ +\ v(n),\ n \in [0, T-1]. \quad (1)$$

The time varying output correlation $c_{11x}(n;m) := E[x(n)x^*(n+m)]$ where * denotes complex conjugate, is periodic in n with period P and is given by:

$$\begin{aligned} c_{11x}(n;m) &= \sigma_s^2 \sum_k h(n-kP)\, h^*(n+m-kP) \\ &+ c_{11v}(m)\,. \end{aligned} \quad (2)$$

Hence, $x(n)$ is cyclostationary. Upon representing this periodic correlation using the Fourier Series (FS), the k^{th} FS coefficient is:

$$C_{11x}(k;m) = \frac{1}{P} \sum_{n=0}^{P-1} c_{2x}(n;m)\, e^{-j\frac{2\pi}{P}kn}, \quad (3)$$

for $k = 0, 1, \ldots, P-1$. The quantity $C_{11x}(k;m)$ is known as the cyclic correlation at cycle $\frac{2\pi}{P}k$ (e.g., [3], [4]). From (2) and (3), $C_{11x}(k;m)$ is:

$$\begin{aligned} C_{11x}(k;m) &= \frac{\sigma_s^2}{P} \quad \sum_\ell h(\ell)\, h^*(\ell+m) e^{-j\frac{2\pi}{P}k\ell} \\ &+ c_{11v}(m)\, \delta(k). \end{aligned} \quad (4)$$

An unbiased mean-square consistent estimator for the cyclic correlation [3] is:

$$\hat{C}_{11x}(k;m) := \frac{1}{T} \sum_{n=0}^{T-1} x(n)x^*(n+m)e^{-j\frac{2\pi}{P}kn}. \quad (5)$$

In [3], $\hat{C}_{11x}(k;m)$ was also shown to be asymptotically Gaussian and the following expressions for the asymptotic covariance were provided:

$$\begin{aligned} &\lim_{T\to\infty} T \sum_{n=0}^{T-1} \text{cum}\left\{\hat{C}_{11x}(k_1;m_1), \hat{C}_{11x}(k_2;m_2)\right\} \quad (6) \\ &:= \sigma_c(k_1+k_2;k_2;m_1;m_2), \end{aligned}$$

and

$$\begin{aligned} &\lim_{T\to\infty} T \sum_{n=0}^{T-1} \text{cum}\left\{\hat{C}_{11x}(k_1;m_1), \hat{C}^*_{11x}(k_2;m_2)\right\} \quad (7) \\ &:= \bar{\sigma}_c(k_1-k_2;k_2;m_1;m_2), \end{aligned}$$

where by definition $\text{cum}\{x(n), y(n)\} := \text{E}[x(n)y(n)] - \text{E}[x(n)]\text{E}[y(n)]$. Closed form expressions for (6) and (7) in terms of the channel, $h(n)$, were given for the oversampled communication system by [6], in addition to computable sample estimates.

3 Order Estimation

Having presented the background for the channel equalization problem, we now describe two methods for estimating the order of the channel. It is assumed that $h(n)$ is FIR of unknown order L (i.e., $h(n) = 0$ for $n < 0$ and $n > L$) and that $h(n)$ satisfies the identifiability conditions for second-order cyclic statistics (e.g., [7]). The first method provides a statistical test which can be performed prior to channel estimation on either a single cyclic correlation value (scalar case) or on a vector of cyclic correlations (vector case). The second method simultaneously estimates the channel impulse response and the channel order.

3.1 Statistical test - scalar case

To determine the order of an FIR channel, consider $C_{11x}(k;m)$ for $|m| \geq L$. For this channel, we have:

$$C_{11x}(k;m) = c_{11v}(m)\delta(k) \qquad \forall\ |m| \geq L. \quad (8)$$

Unlike the time-varying correlation $c_{11x}(n;m)$, the cyclic correlation is theoretically immune to additive stationary noise provided $k \neq 0$. This noise insensitivity motivates the use of $C_{11x}(k;m)$ to select the channel order. In other words, the order of the channel can be determined by searching for m after which $C_{11x}(k;m) = 0$.

In practice, however, the estimated cyclic correlation, $\hat{C}_{11x}(k;m)$, is never exactly zero. Hence, a decision must be made as to when $\hat{C}_{11x}(k;m)$ is 'close' to zero. In [14], this decision is made based on empirical observation. In contrast, we derive an asymptotically constant false-alarm rate (CFAR) test which detects the L for which $C_{11x}(k;m) = 0$ for all $|m| > L$ based on the finite observations $\{x(n)\}_{n=0}^{T-1}$.

Let $\hat{\mathbf{c}}_k(m)$ denote a 1×2 vector of the real and imaginary parts of $C_{11x}(k;m)$:

$$\hat{\mathbf{c}}_k(m) := \left[\text{Re}\{\hat{C}_{11x}(k;m)\}, \text{Im}\{\hat{C}_{11x}(k;m)\}\right]. \quad (9)$$

(Note: We will assume that $C_{11x}(k;m)$ is non-zero for all $|m| < L$.) We then write:

$$\hat{\mathbf{c}}_k(m) = \mathbf{c}_k(m)\ + \boldsymbol{\epsilon}_k(m) \quad (10)$$

where $\mathbf{c}_k(m)$ and $\boldsymbol{\epsilon}_k(m)$ are the true cyclic correlation values and the estimation error values respectively. To check if $\hat{\mathbf{c}}_k(m)$ consists of a lag outside L, we formulate the following hypothesis testing problem:

$$\begin{aligned} \mathbf{H_0} &: \ \mathbf{c}_k(m) = 0, \\ \mathbf{H_1} &: \ \mathbf{c}_k(m) \neq 0, \end{aligned} \quad (11)$$

where under $\mathbf{H}_0$, $\hat{\mathbf{c}}_k(m) = \boldsymbol{\epsilon}_k(m)$, and under $\mathbf{H}_1$, $\hat{\mathbf{c}}_k(m) = \mathbf{c}_k(m) + \boldsymbol{\epsilon}_k(m)$. In general, the distribution of $\boldsymbol{\epsilon}_k(m)$ is unknown; however, from [3] we know $\boldsymbol{\epsilon}_k(m)$ is asymptotically Gaussian with asymptotic covariance, $\boldsymbol{\Sigma}_{k,m}$ [see eq. (6) and (7)]:

$$\boldsymbol{\Sigma}_{k,m} := \lim_{T\to\infty} T\text{cov}\left[\hat{\mathbf{c}}_k(m), \hat{\mathbf{c}}_k(m)\right]. \quad (12)$$

In the case of real channel coefficients and real symbols, the asymptotic covariance is given by (complex case follows similarly from [3]):

$$\boldsymbol{\Sigma}_{k,m} = \frac{1}{2}\begin{bmatrix} \mathrm{Re}\{\sigma_2+\sigma_0\} & \mathrm{Im}\{\sigma_0-\sigma_2\} \\ \mathrm{Im}\{\sigma_2+\sigma_0\} & \mathrm{Re}\{\sigma_2-\sigma_0\} \end{bmatrix} \quad (13)$$

where based on the definitions in (6) and (7)

$$\begin{array}{rcl} \sigma_2 & := & \sigma_c(2k;k;m;m) \\ \sigma_0 & := & \bar{\sigma_c}(0;k;m;m). \end{array} \quad (14)$$

Due the Gaussian nature, hypothesis testing for this case is formulated as the generalized maximum maximum likelihood detection problem [13, pp. 378-389]. Therefore, we adopt the following test statistic:

$$\Upsilon := T\hat{c}_k(m)\hat{\boldsymbol{\Sigma}}_{k,m}^{-1}\hat{c}'_k(m), \quad (15)$$

where $\hat{\boldsymbol{\Sigma}}_{k,m}$ is any consistent estimate of the asymptotic covariance of $\hat{C}_{11x}(k;m)$ (see [6]), and $'$ denotes transpose. Although in a different context, the test statistic of (15) is identical to that considered in [4]. As in [4], it follows that,

$$\begin{array}{ll} \text{under } \mathbf{H_0}: & \lim_{T\to\infty}\Upsilon \stackrel{D}{=} \chi_2^2 \\ \text{under } \mathbf{H_1}: & \lim_{T\to\infty}\sqrt{T}(\frac{\Upsilon}{T} - c_{2x}\,\boldsymbol{\Sigma}_{k,m}^{-1}\,c'_k) \\ & \stackrel{D}{=} \mathcal{N}(0, 4c_k\,\boldsymbol{\Sigma}_{k,m}^{-1}\,c'_k), \end{array} \quad (16)$$

where $\stackrel{D}{=}$ implies convergence in distribution and χ_2^2 denotes a central chi-square with 2 degrees of freedom. For a given probability of false-alarms, we can find a threshold Γ from the central chi-square tables. The test then has two results:

1. If $\Upsilon > \Gamma$, then $|m| < L$.
2. If $\Upsilon < \Gamma$, then $|m| \geq L$.

By varying the magnitude of m, one can find L by determining the smallest value of m for which case 2 ($\Upsilon < \Gamma$) holds.

3.2 Statistical test - vector case

For the scalar case, we assumed that $C_{11x}(k;m)$ was non-zero for all $|m| < L$. However, this may not be true. To remove this assumption, we will test a vector of cyclic correlations at various lags and cycles. Similar to the scalar case, we define a $1 \times MK$ 'test' vector as the real and imaginary parts of the candidate lags $\{m_1 < m_2 < ... < m_M\}$ and candidate cycles $k_1, ..., k_K$:

$$\hat{\mathbf{c}}_{1:M} := [\mathrm{Re}\{\hat{C}_{11x}(k_i;m_j)\}, \mathrm{Im}\{\hat{C}_{11x}(k_i;m_j)\}] \quad (17)$$

where $i = 1, \ldots, K$ and $j = 1, \ldots, M$. As in the scalar case, we then have the following hypotheses test:

$$\begin{array}{ll} \mathbf{H_0}: & \mathbf{c}_{1:M} = 0, \quad \text{all } m_1, ...m_M \geq L \\ \mathbf{H_1}: & c_{1:M} \neq 0, \quad \text{some } m_1, .., m_M \leq L \end{array} . \quad (18)$$

We therefore adopt the following test statistic:

$$\Upsilon := T\hat{\mathbf{c}}_{1:M}\hat{\boldsymbol{\Sigma}}^{-1}\hat{\mathbf{c}}'_{1:M}, \quad (19)$$

where $\hat{\boldsymbol{\Sigma}}$ is any consistent estimate of the asymptotic covariance of $\hat{\mathbf{c}}_{1:M}$. Similar to the scalar case, we have from [4]:

$$\begin{array}{ll} \text{under } \mathbf{H_0}: & \lim_{T\to\infty}\Upsilon \stackrel{D}{=} \chi_{2MK}^2 \\ \text{under } \mathbf{H_1}: & \lim_{T\to\infty}\sqrt{T}(\frac{\Upsilon}{T} - \mathbf{c}_{1:M}\boldsymbol{\Sigma}^{-1}\mathbf{c}'_{1:M}) \\ & \stackrel{D}{=} \mathcal{N}(0, 4\mathbf{c}_{1:M}\boldsymbol{\Sigma}^{-1}\mathbf{c}'_{1:M}), \end{array} \quad (20)$$

where χ_{2MK}^2 denotes a central chi-square with $2MK$ degrees of freedom. For a given probability of false-alarms, we can find a threshold Γ from the central chi-square tables. The test has two possible results:

1. If $\Upsilon > \Gamma$, then $m_1 < L$.
2. If $\Upsilon < \Gamma$, then $m_1 \geq L$.

By varying the magnitude of m_1, one can find L by determining the smallest value of m_1 for which case 2 ($\Upsilon < \Gamma$) holds.

3.3 Information theoretic criterion

For a known channel order, [6] showed the following estimator, $\hat{\mathbf{h}} := [\hat{h}(0), \hat{h}(1), \ldots, \hat{h}(L)]$, is the asymptotically best consistent estimator for $\mathbf{h} := [h(0), h(1), \ldots, h(L)]$. The estimator is of the form:

$$\hat{\mathbf{h}} := \arg\min_{\mathbf{h}} \hat{\mathbf{J}}(\mathbf{h}) \quad (21)$$

with

$$\hat{\mathbf{J}}(\mathbf{h}) := \boldsymbol{\epsilon}'_M\,\boldsymbol{\Sigma}^{-1}(\mathbf{h})\,\boldsymbol{\epsilon}_M, \quad (22)$$

where $\boldsymbol{\epsilon}_M$ is an 1 x M vector whose i^{th} entry is:

$$[\boldsymbol{\epsilon}_M]^{(i)} := \hat{C}_{11x}(k^{(i)};m^{(i)}) - C_{11x}(k^{(i)};m^{(i)}|\mathbf{h}), \quad (23)$$

and $C_{11x}(k;m|\mathbf{h})$ denotes the cyclic correlation corresponding to the model $\mathbf{h}$ [c.f. 4]. The matrix $\boldsymbol{\Sigma}(\mathbf{h})$ is the asymptotic covariance of $\boldsymbol{\epsilon}_M$ and can be computed in closed form using expressions in [6]. Motivated by methods such as Akaike's Information Theoretic criteria (AIC) [1], Rissanen's Minimum Description Length [10], and related results for ARMA order determination [5], we propose the following simultaneous channel estimation and order determination method:

$$\overline{\boldsymbol{\Theta}} = [\hat{\mathbf{h}}, \hat{L}] = \arg\min_{\mathbf{h},L} \hat{V}(\mathbf{h}, L) \quad (24)$$

where

$$\hat{V}(\mathbf{h}, L) = \hat{\mathbf{J}}(\mathbf{h}) + L\frac{\log T}{T} \quad (25)$$

The main difference between (21) and (24) is the penalty term $L(\log T)/T$. If (21) were generalized to include order estimation, it would tend to favor higher (than the true) order models. The penalty term increases with increasing order and hence discourages overparameterization.

4 Simulations

In this section, we present simulation results showing the performance of the two order selection algorithms. For all simulations, the true order of channel was $L = 2$, the impulse response was $\mathbf{h} = [1,\ 0.2,\ 0.5]$, $P = 2$, and the SNR $= 30dB$. Figures 2 and 3 show the mean and standard deviation computed for 100 Monte Carlo runs of Υ for the statistical test method with $M = 2$ and $K = 2$. Figure 2 shows the results for $T = 500$ while Figure 3 shows the results for $T = 1,000$. Also shown on Figures 2 and 3 are the thresholds, Γ, for a probability of false-alarm of 0.1 and 0.05. Figures 4 and 5 show the mean and standard deviation over 100 Monte Carlo runs of $\hat{V}(\mathbf{h}, L)$ as a function of channel length $L+1$. Figure 4 and Figure 5 used $T = 500$ and $T = 1,000$, respectively. Figure 6 indicates the number of correct decisions for both the information theoretic and statistical test procedures over 100 Monte Carlo runs as the SNR varies from 5dB to 30 dB. Tables 1 and 2 indicate the mean and standard deviation obtained for each impulse response coefficient using the information theoretic criterion as the candidate order varied from 2 to 4.

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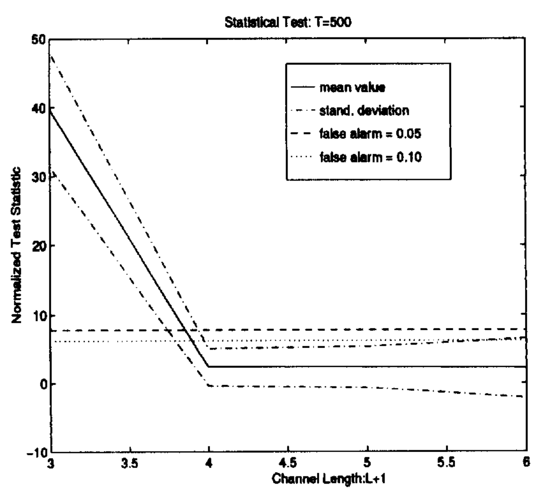

Figure 2: Υ for SNR = 30 dB, $L = 2$, T=500

		h(1)	h(2)	h(3)
L = 3	mean	0.1994	0.4987	
	std. dev.	± 0.0867	± 0.4857	
L = 4	mean	0.1998	0.5057	0.0006
	std. dev.	± 0.0583	± 0.7971	± 0.1794
L = 5	mean	0.1991	0.4795	0.0026
	std. dev.	± 0.0806	± 1.149	± 0.5143

Table 1: Estimated Impulse Response Coefficients - T = 500, SNR = 30 dB

		h(1)	h(2)	h(3)
L = 3	mean	0.2000	0.4995	
	std. dev.	± 0.0637	± 0.4319	
L = 4	mean	0.2001	0.5004	0.0003
	std. dev.	± 0.0432	± 0.8640	± 0.2283
L = 5	mean	0.1998	0.4242	0.0056
	std. dev.	± 0.0565	± 1.085	± 0.5273

Table 2: Estimated Impulse Response Coefficients - T = 1,000, SNR = 30 dB

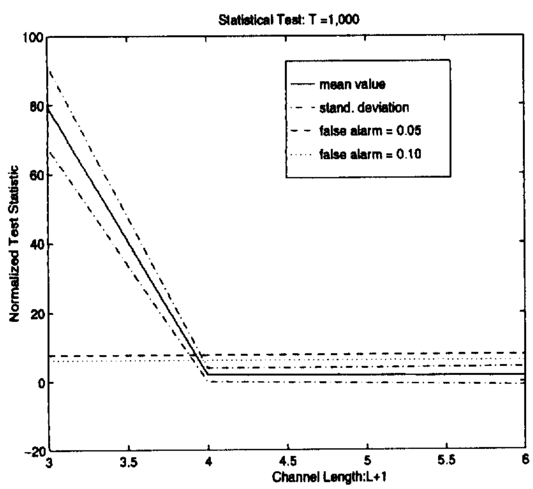

Figure 3: Υ for SNR = 30 dB, $L = 2$, $T = 1,000$

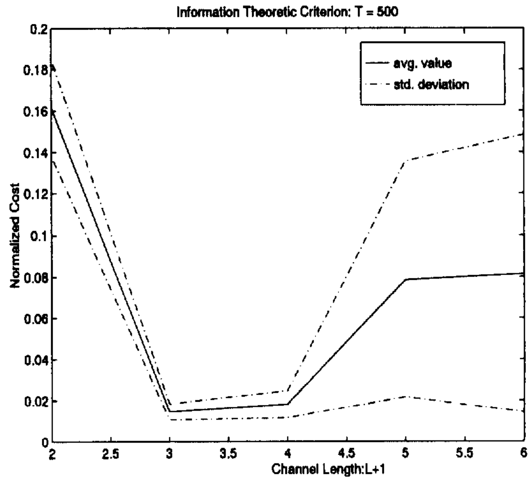

Figure 4: Normalized $\hat{V}(\mathbf{h}, L)$ for SNR = 30 dB and $L = 2$

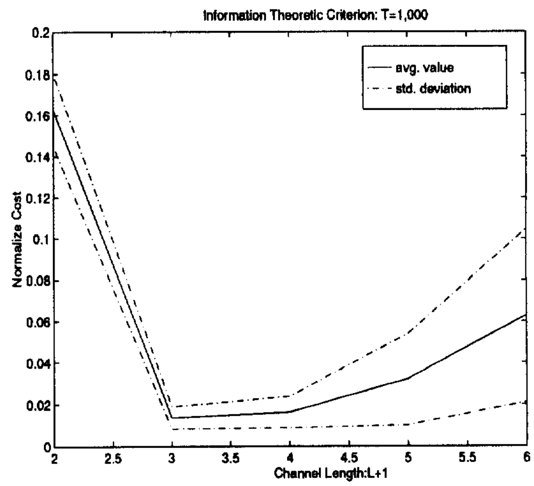

Figure 5: Normalized $\hat{V}(\mathbf{h}, L)$ for SNR = 30 dB and $L = 2$

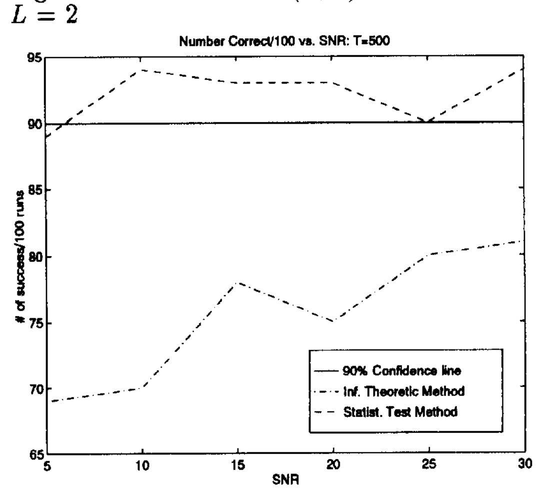

Figure 6: Number of Correct Decisions/100 runs with SNR = 30dB and Prob. of false-alarm = 0.10