LINEAR CYCLIC CORRELATION APPROACHES FOR BLIND IDENTIFICATION OF FIR CHANNELS

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ABSTRACT

Blind fractionally spaced equalizers rely on output only samples taken at a rate higher than the symbol rate in order to estimate the channel taps. Simple methods for mixed phase FIR channel identification are developed by solving a linear system of equations formed by sample cyclic correlations of the received oversampled data sequence. In addition to computational simplicity over existing multivariate alternatives, the resulting linear algorithm is easily modified to yield asymptotically minimum variance weighted least squares channel estimators. A "deterministic" approach is also developed which requires only persistently exciting inputs and leads to closed form data based solutions. Comparisons and tools for performance analysis are also described.

1. INTRODUCTION

To suppress inter-symbol interference (ISI), channel estimation is a necessary first step either for constructing linear equalizers, or, for running the Viterbi algorithm for demodulation. The "combined" continuous-time filter includes the transmitter/channel/receiver filters, and is assumed to be FIR with nonzero impulse response $h_c(\tau)$ over Q symbols each of duration T_s; i.e., $h_c(\tau) \neq 0$ for $\tau \in [0, QT_s]$. With s_l denoting the i.i.d. information symbol stream, the continuous time output $x_c(t) = \sum_l s_l \, h_c(t - lT_s)$ is traditionally sampled at the symbol rate ($t = nT_s$) which renders the resulting time series $x(n)$ stationary. In the blind channel estimation scenario, where only channel output data are available, higher- (than second-) order statistics (HOS) have been employed implicitly or explicitly in order to capture not only the channel amplitude but also its correct phase characteristics (e.g., [4]).

However, there has been a considerable interest recently in oversampling (or fractionally sampling) the output, i.e. sampling at a rate higher than the symbol rate. Oversampling by a factor P yields a discrete time output

$$\begin{aligned} x(n) &:= x_c(t)|_{t=nT_s/P}, \quad n = 0, 1, \ldots, T-1, \\ &= \sum_l s_l h(n - lP), \end{aligned} \tag{1}$$

which turns out to be cyclostationary with periodically time-varying correlation

$$\begin{aligned} c_{11x}(n;m) &:= E[x(n)x^*(n+m)], \\ &= c_{11s}(0) \sum_l h(n-lP)h^*(n+m-lP), \end{aligned} \tag{2}$$

where $c_{11s}(0)$ denotes the variance of s_l and $*$ stands for conjugation.

The motivation behind fractional sampling is twofold: (i) under certain conditions [12], [11], [2] even nonminimum phase channels are identifiable using $\{c_{11x}(n;m)\}_{n=0}^{P-1}$, which, relative to HOS based methods, require less samples for reliable estimation; (ii) for the FIR channel $h(n)$, the equalizer, $g(n)$, is also FIR [10], and can be obtained exactly by solving the "zero-forcing" system of equations: $\sum_m g(m)\, h(nP - m) = \delta(n)$.

The SISO cyclostationary output $x(n)$ is equivalent to a SIMO stationary one (e.g., [8]), and most of the existing approaches recast (1) in a vector form before applying ESPRIT [12], MUSIC [7], multivariable linear prediction, conditional ML [10], and other eigen-solutions [6], to solve for the discrete-time impulse response $\{h(l)\}_{l=0}^{L}$ where $L := QP$. However, in contrast to Pagano [8] who in order to save computations, converted a SIMO stationary process to a SISO cyclostationary one, existing approaches vectorize (1).

In this paper, we develop within the SISO cyclostationary framework, linear equation based methods to solve for $h(n)$ using: least-squares or eigen-vector methods (Section 2); an asymptotically optimum (in the minimum variance sense) weighted least-squares method (Section 3) which is important for initializing our nonlinear estimator in [3]; and "deterministic" approaches (Section 4) to parallel those in [6] and [10]. Tools for performance analysis and comparisons are also delineated.

2. LINEAR CYCLIC METHODS

Since $c_{11x}(n;m)$ in (2) is periodic (in n) with period P, its kth-order Fourier Series coefficient (also known as cyclic correlation [1]) is

$$\begin{aligned} C_{11x}(k;m) &:= \frac{1}{P} \sum_{n=0}^{P-1} c_{11x}(n;m)\, e^{-j\frac{2\pi}{P}kn} \\ &= \frac{c_{11s}(0)}{P} \sum_{l=0}^{L} h(l)h^*(l+m)\, e^{-j\frac{2\pi}{P}kn}, \end{aligned} \tag{3}$$

where we have used (2) in deriving the second equality. The Z-transform of $C_{11x}(k;m)$ w.r.t. m is given by [2], [3], [4],

$$S_{11x}(k;z) = \frac{c_{11s}(0)}{L} H(z) H^*(e^{-j\frac{2\pi}{P}k}/z^*), \qquad (4)$$

and can be used to establish identifiability of the channel transfer function $H(z)$ [or $H(\omega) := H(z)|z = \exp\{j\omega\}$] based on $S_{11x}(k;z)$, or, equivalently $C_{11x}(k;m)$, or, equivalently $\{c_{11x}(n;m),\ n \in [0, P-1],\ m \in [-L, L]\}$. If ζ_i is a zero of $H(z)$, then according to [2], [11], [12], the FIR channel $H(z)$ is identifiable (ID) from $S_{11x}(k;z)$, $k = 0, 1, \ldots, P-1$ iff $\forall i$ either (ID$_a$) $\exists k$: $|\zeta_k| \neq |\zeta_i|$, $\forall k \neq i$, or, (ID$_b$) $\exists l$: $\arg(\zeta_i) + 2\pi l/P \neq \arg(\zeta_k)$, $\forall k \neq i$.

Instead of factorizing $S_{11x}(k;z)$ as in [2], [11] in order to identify the common zeros among $S_{11x}(k;z)$ for $k = 0, 1, \ldots, P-1$, we consider the Fourier transforms

$$S_{11x}(0;\omega) = \frac{c_{11s}(0)}{L} H(\omega) H^*(\omega), \qquad (5)$$

$$S_{11x}(k;\omega) = \frac{c_{11s}(0)}{L} H(\omega) H^*(\omega - \frac{2\pi}{P}k),\ 1 \leq k \leq P-1. \qquad (6)$$

Upon eliminating $H(\omega)$ from (5) and (6) we obtain

$$H^*(\omega) S_{11x}(k;\omega) = H^*(\omega - \frac{2\pi}{P}k) S_{11x}(0;\omega), \qquad (7)$$

which for $k \in [1, P-1]$ and $m \in [-L, L]$ yields, in the lag domain, cyclic correlation based linear equations of the Yule-Walker type:

$$\sum_{l=0}^{L} \left[C^*_{11x}(k; m+n) - e^{j\frac{2\pi}{P}kn}\, C^*_{11x}(0; m+n) \right] h(n) = 0. \quad (8)$$

Not all $(2L+1) \times (P-1)$ equations are linearly independent due to the symmetry property: $C_{11x}(k;-m) = \exp(j2\pi km/P)\ C^*_{11x}(P-k;m)$. However, if $H(z)$ is ID in the sense of [12], [11], [10], [7], [2], then $h(n)$ is uniquely identifiable from the solution of (8) with $k = 1, \ldots, P-1$ and $m = -L, \ldots, L$. For a proof (by contradiction) suppose that in addition to $h(n)$, there exists $\tilde{h}(n) \neq h(n)$ which satisfies (8). Then, Z−transforming (8) and reversing our earlier derivation we find that $\tilde{H}(z)$ also satisfies (4) which contradicts our ID condition.

To develop matrix forms of (8), define

$$\begin{aligned}
\mathbf{h} &:= [h(0)h(1)\ldots h(L)]', \\
\mathbf{h}_0 &:= [h(1)\ldots h(L)]', \\
\mathbf{c}_k &:= [\mathrm{Re}C_{11x}(k;m)\mathrm{Im}C_{11x}(k;m),]',\ m = 0, \ldots, L, \\
\mathbf{c} &:= [\mathbf{c}_0', \mathbf{c}_1', \ldots, \mathbf{c}_{P-1}'],
\end{aligned}$$

where prime denotes conjugate transpose. Concatenating equations like (8) we find

$$\bar{\mathbf{A}}(\mathbf{c})\ \mathbf{h} = \mathbf{0}. \qquad (9)$$

With output data only, $\mathbf{h}$ can be specified up to a scale ambiguity. As usual, one has two options: either set $\|\ \mathbf{h}\ \| = 1$, or fix w.l.o.g. one of the coefficients to a known value, say $h(0) = 1$. The former renders (9) equivalent to minimizing $\mathbf{h}'^*\ \bar{\mathbf{A}}'^*\ \bar{\mathbf{A}}\ \mathbf{h}$ subject to $\|\ \mathbf{h}\ \| = 1$, and hence the coefficient vector can be found (using SVD) as

$$\mathbf{h} = \text{min. e-vector}(\bar{\mathbf{A}}'\bar{\mathbf{A}}). \qquad (10)$$

The latter suggests moving the first column of $\bar{\mathbf{A}} := [-\mathbf{b}\ \mathbf{A}]$ to the r.h.s. of (9) to obtain the matrix equation

$$\mathbf{A}(\mathbf{c})\ \mathbf{h}_0 = \mathbf{b}(\mathbf{c}), \qquad h(0) = 1 \qquad (11)$$

which can be solved using least-squares to find

$$\mathbf{h}_0^{LS} = [\mathbf{A}'(\mathbf{c})\mathbf{A}'(\mathbf{c})]^{-1}\ \mathbf{A}'(\mathbf{c})\ \mathbf{b}(\mathbf{c}). \qquad (12)$$

Note that both $\bar{\mathbf{A}}$ and $\mathbf{A}$ have dimensionalities which are independent of the data length but dependent upon the FIR channel order L and the oversampling factor P.

It is possible to gain (at least in theory) tolerance to additive stationary noise by avoiding the $k = 0$ set of equations. In this case one starts with $y(n) := x(n) + v(n)$, writes (6) for $k = k_1 \neq 0$ and $k = k_2 \neq k_1 \neq 0$, eliminates $H(\omega)$, and uses inverse Fourier transform to obtain

$$\sum_{l=0}^{L} \left[C^*_{11y}(k_1; m+n) - e^{j\frac{2\pi}{P}(k_1-k_2)n}\, C^*_{11y}(k_2; m+n) \right] \times h(n) = 0. \qquad (13)$$

The importance of oversampling is evident from (13) or (8). In addition to the $k = 0$ cycle which is also present in symbol-spaced sampling, the periodicity introduces cycles at $k \neq 0$. As a result, extra degrees of freedom are created and more equations become available.

In practice, we use the natural sample cyclic correlation estimator (e.g, [1])

$$\hat{C}_{11y}(k;m) = \frac{1}{T} \sum_{n=0}^{T-1} y(n) y^*(n+m)\, e^{-j\frac{2\pi}{P}kn}, \qquad (14)$$

which is m.s.s. consistent, $\lim_{T\to\infty} \hat{C}_{11y}(k;m) \overset{m.s.s.}{=} C_{11y}(k;m)$, provided that the input variance is finite, $|c_{11s}(0)| < \infty$, and the output noise $v(n)$ has absolutely summable cumulants up to order four. Consistency of $\hat{c}_{11x}$ and identifiability guarantee that [1] our LS estimator $\mathbf{h}_0^{LS}$ obtained as in (12) with $\hat{\mathbf{c}}$ replacing $\mathbf{c}$ will be m.s.s. consistent: $\lim_{T\to\infty} \hat{\mathbf{h}}_0^{LS} \overset{m.s.s.}{=} \mathbf{h}_0$.

It is now of interest to look into optimality and performance analysis issues.

3. ASYMPTOTICALLY BLUE

Consider the class of linear estimators formed by using weight matrix $\mathbf{W}$ in the sampled version of (12)

$$\hat{\mathbf{h}}_0^{WLS} = [\mathbf{A}'(\hat{\mathbf{c}})\mathbf{W}\mathbf{A}(\hat{\mathbf{c}})]^{-1}\mathbf{A}'(\hat{\mathbf{c}})\mathbf{W}\ \mathbf{b}(\hat{\mathbf{c}}), \qquad (15)$$

where in addition to replacing $\mathbf{c}$ by $\hat{\mathbf{c}}$, we have also dropped for notational convenience the conjugation (the latter is possible if one rewrites each of the "complex" equations (8) as a pair of "real" ones corresponding to the Re-Im parts).

We wish to find the optimum $\mathbf{W}$ so that the asymptotic covariance matrix $\mathbf{\Sigma}_0^{WLS} := \lim_{T\to\infty} T\mathrm{cov}\ \{\hat{\mathbf{h}}_0^{WLS}\}$ is minimized. According to [9], this is achieved if one selects

$$\mathbf{W}_o(\mathbf{h}_0) = [\mathbf{D}(\mathbf{h}_0)\mathbf{\Sigma}_c(\mathbf{h}_0)\mathbf{D}'(\mathbf{h}_0)]^{-1}, \qquad (16)$$

where matrix $\mathbf{D}$ has columns expressible in terms of $\mathbf{h}_0$ and derivatives of the $\mathbf{A}$, $\mathbf{b}$ in (10) w.r.t. the ith entry c_i of $\mathbf{c}$:

$$\mathbf{d}_i = \frac{\partial \mathbf{b}(\mathbf{c})}{\partial c_i} - \frac{\partial}{\partial c_i}\mathbf{A}(\mathbf{c})\mathbf{h}_0, \qquad (17)$$

and $\mathbf{\Sigma}_c$ is the asymptotic covariance matrix of $\hat{\mathbf{c}}$ defined as $\mathbf{\Sigma}_c := \lim_{T\to\infty} T\mathrm{cov}\ \{\hat{\mathbf{c}}, \hat{\mathbf{c}}\}$. Because $C_{11x}(k;m)$ is generally complex (c.f. (14)), the entry of $\mathbf{\Sigma}_c$ can be found if we know the asymptotic second-order cumulant

$$\lim_{T\to\infty} T\mathrm{cum}\{\hat{C}_{11x}(k_1;m_1), \hat{C}_{11x}(k_2;m_2)\} := \sigma_c(k_1+k_2;k_2;m_1;m_2), \qquad (18)$$

$$\lim_{T\to\infty} T\mathrm{cum}\{\hat{C}_{11x}(k_1;m_1), \hat{C}^*_{11x}(k_2;m_2)\} := \bar{\sigma}_c(k_1-k_2;k_2;m_1;m_2). \qquad (19)$$

It turns out that both σ_c and $\bar{\sigma}_c$ can be determined in terms of $h(n)$ [3]. Let us define $H_{11}(k;m) := \sum_{n=0}^{L} h(n)\, h^*(n+m)\exp\{-j2\pi kn/P\}$ and $H_{20}(k;m) := \sum_{n=0}^{L} h(n)\, h(n+m)\exp\{-j2\pi kn/P\} = H_{02}^*(-k;m)$, and the $(k+l)$th order cumulant of s_l with $k(l)$ (un)conjugated copies of s_l at the 0th lag as c_{kls}. We then find [3],

$$\begin{aligned}\sigma_c(k_1;k_2;m_1;m_2) &= c_{40s}H_{11}(k_1;m_1)H_{11}(k_2;m_2)\\ &+\frac{c_{02s}c_{20s}}{P^2}\sum_{\zeta}\sum_{l=0}^{P-1} e^{-j\frac{2\pi}{P}k_2\zeta}\\ &\Big[H_{20}(l;\zeta)H_{02}(k_1+k_2-l;\zeta+m_2-m_1)e^{-j\frac{2\pi}{P}(k_1+k_2-l)m_1}\\ &+H_{11}(l;\zeta+m_2)H_{11}(k_1+k_2-l;\zeta-m_1)e^{j\frac{2\pi}{P}(k_1+k_2-l)\zeta}\Big]\end{aligned} \qquad (20)$$

and a similar expression for $\bar{\sigma}_c$ of (19). Using the above closed form expression for $\sigma_c(\mathbf{h}_0)$ and $\bar{\sigma}_c(\mathbf{h}_0)$, we can express $\mathbf{\Sigma}_c$ and hence $\mathbf{W}_{opt}$ in (16) as a function of $\mathbf{h}_0$.

The performance of the asymptotically Best Linear Unbiased Estimator (BLUE) can also be evaluated [9] using the asymptotic covariance matrix

$$\mathbf{\Sigma}_0^{WLS} = (\mathbf{A}'\mathbf{W}_o\mathbf{A})^{-1}\mathbf{W}_o\mathbf{A}'\mathbf{D}'\mathbf{\Sigma}\mathbf{D}'\mathbf{A}\mathbf{W}_o(\mathbf{A}'\mathbf{W}_o\mathbf{A}). \qquad (21)$$

The LS estimator's asymptotic covariance matrix $\mathbf{\Sigma}_0^{LS}$ can also be obtained by setting $\mathbf{W} = \mathbf{I}$ in (21).

In practice, given noisy data $\{y(n)\}_{n=0}^{T-1}$, we will first estimate $\hat{C}_{11y}(k;m)$ as in (14) and then solve the $\hat{\mathbf{c}}$-based LS problem in (12) to obtain $\hat{\mathbf{h}}_0^{LS}$. Substituting $\hat{\mathbf{h}}_0^{LS}$ first in (17) and then in (18)-(20) we can estimate $\mathbf{D}(\hat{\mathbf{h}}_0^{LS})$ and $\mathbf{\Sigma}_c(\hat{\mathbf{h}}_0^{LS})$ and thus $\mathbf{W}_0(\hat{\mathbf{h}}_0^{LS})$ using (16). Plugging the estimated weight matrix in (15) we obtain the asymptotically BLUE $\hat{\mathbf{h}}_0^{WLS}$, the consistency of which is guaranteed when ID conditions hold and the input/noise processes are mixing as we mentioned before.

The overall asymptotically optimal estimator based on cyclic correlations was reported in [3] and entails nonlinear minimization of the quadratic functional $(\hat{\mathbf{c}}-\mathbf{c})'\ \mathbf{\Sigma}_c^{-1}\ (\hat{\mathbf{c}}-\mathbf{c})$. Because the latter is non-convex w.r.t. $\mathbf{h}_0$ it is likely to possess local minima. Good initialization with the linear methods of this paper not only accelerates but also prevents convergence to local minima.

4. "DETERMINISTIC" METHOD – COMPARISONS

The Fourier transform of (1) yields $X(\omega) = H(\omega)S(P\omega)$. But it also holds that $S(P\omega) = S[P(\omega - 2\pi k/P)]$ which allows us to "eliminate" the input $S(P\omega)$ from $X(\omega)$ and $X(\omega - 2\pi k/P)$, and arrive at

$$H(\omega)X(\omega - \frac{2\pi}{P}k) = H(\omega - \frac{2\pi}{P}k)X(\omega), \quad k = 1,\ldots,P-1. \qquad (22)$$

We have in principle turned the output-only system identification problem into an input-output one, and similar to [6], no whiteness assumption was used in deriving (22). Specifically, it can be shown that under standard persistent of excitation (PE) conditions for the input spectrum $S(\omega)$, the FIR channel $H(\omega)$ can be recovered exactly in the noise-free case. Although our PE conditions coincide with those in [6], our cyclic formulation appears to be computationally simpler than the matrix formulation of [6].

Upon recasting (22) in the time-domain, it follows after elementary manipulations that

$$\sum_{l=0}^{L} h(l)x(n-l)\sin\left[\frac{\pi}{P}k(n-2l)\right] = 0, \qquad (23)$$

which for $k = 1,\ldots,P-1$ and $n = 0,1,\ldots,T-1$ yields, similar to (9)-(12), two matrix equations, namely, $\bar{\mathbf{X}}_s\mathbf{h} = \mathbf{0}$, or, $\mathbf{X}_s\mathbf{h}_0 = \mathbf{x}$ with $\bar{\mathbf{X}}_s := [-\mathbf{x}\ \mathbf{X}_s]$ and $h(0) := 1$. Correspondingly, we obtain (subject to $\|\mathbf{h}\| = 1$) the solution $\mathbf{h}$ as the eigenvector associated with the minimum eigenvalue of $\bar{\mathbf{X}}_s'\bar{\mathbf{X}}_s$, or, the LS solution

$$\hat{\mathbf{h}}_0^{LS} = (\mathbf{X}_s'\mathbf{X}_s)^{-1}\mathbf{X}_s'\mathbf{x}, \qquad h(0) = 1. \qquad (24)$$

Similar to the methods of Section 3, weighted alternatives are also possible.

With $x(0) \neq 0$ and $h(0) = 1$, an iterative solution is possible using

$$h(n) = \frac{\sum_{l=0}^{n-1} h(l)x(n-l)\sin\left[\frac{\pi}{P}k(2l-n)\right]}{x(0)\sin(\frac{\pi}{P}kn)}. \qquad (25)$$

It has been claimed (e.g., [6]) that "deterministic" approaches such as the ones in (24) and [6] have an edge over the "stochastic" ones such as those in Section 3 and [12], because in the absence of noise and with finite data the former yield the exact system while the latter do not. Apart from ignoring model mismatch issues the statement is not true at least for the approaches of this paper.

We wish to show that in the noise-free case (7) is equivalent to (22) and thus "deterministic" and "stochastic" approaches coincide even with finite data. Since noise $v(n) = 0$, let us adopt in (7) the natural cyclic cross-periodogram estimator

$$\hat{S}_{11x}(k;\omega) = \frac{1}{T}X(\omega)X(\frac{2\pi}{P}k - \omega), \qquad (26)$$

eliminate $X(\omega)$, and conjugate the resulting equation to make it identical to (22).

When noise is present, the problem is challenging because it is analogous to errors-in-variables I/O system identification where both input and output records are contaminated by mutually correlated and colored noises. In the presence of noise, identifiability is not guaranteed with deterministic methods. "Stochastic" approaches in this context have more potential because they possess some form of (at least asymptotic) optimality, even under model mismatch.

Remark: Although our methods were developed for single channel cyclostationary time series which arise as a result of fractionally sampling, using the MIMO stationary ⇔ SISO cyclostationary equivalence, they are obviously applicable to general FIR multichannel identification scenarios which may arise due to multipath and/or multivariate data collected by an array of sensors.

5. SIMULATIONS

For all simulations, the true discrete-time channel, $\mathbf{h}_0$, is a nonminimum phase channel of order $L = 8$ and is given by:

$$\mathbf{h}_0 := [0.1764, -0.1296, 0.1680, 0.6152, 0.9679\ 1.0000, 0.6689, 0.1545, -0.2634].$$

In addition, the symbols were i.i.d. from a BPSK alphabet, the noise was additive white Gaussian, and the oversampling factor was $P = 3$. Figures 1 and 2 show the mean and standard deviation over 100 Monte Carlo runs of linear least squares method (12). Figure 1 displays the results for $T = 300$ while Figure 2 uses $T = 1,000$. The mean and standard deviation of the eigenbased linear method (10) are shown for 100 Monte Carlo runs in Figures 3 and 4 for $T = 300$ and $T = 1,000$ respectively. Tables 1 and 2 compare the performance in terms of variance for the multichannel eigenbased approach of [12] with the linear least-squares solution of (12) and the eigenbased solution of (10). Both Table 1 and Table 2 were computed using 100 Monte Carlo runs with $T = 300$ and $T = 1,000$ respectively.

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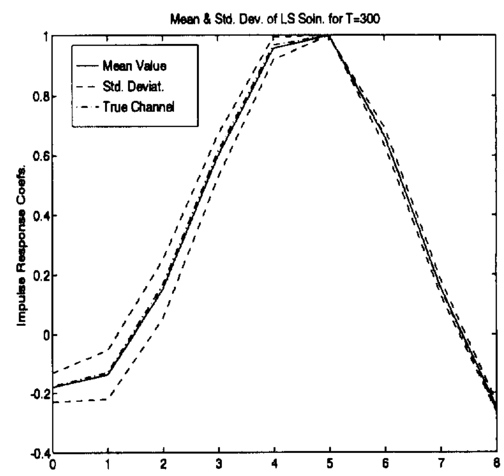

Figure 1. Least Sqr. Method: SNR = 30 dB and T=300

IR Coefficient	Eigen. Method [TXK-94] Variance $\times 10^{-1}$	Linear Cyclic Methods: Linear LS Variance $\times 10^{-1}$	Linear Cyclic Methods: Linear Eigen. Variance $\times 10^{-1}$
h(0)	0.028	0.024	0.013
h(1)	0.032	0.077	0.035
h(2)	0.032	0.097	0.038
h(3)	0.105	0.052	0.012
h(4)	0.074	0.014	0.001
h(6)	0.831	0.009	0.001
h(7)	0.182	0.070	0.001
h(8)	0.194	0.030	0.002

Table 1. Comparisons for SNR = 30dB & T = 300

IR Coefficient	Eigen. Method [TXK-94] Variance $\times 10^{-2}$	Linear Cyclo. Methods: Linear LS Variance $\times 10^{-2}$	Linear Cyclo. Methods: Linear Eigen. Variance $\times 10^{-2}$
h(0)	0.030	0.090	0.040
h(1)	0.030	0.286	0.100
h(2)	0.063	0.380	0.120
h(3)	0.160	0.212	0.041
h(4)	0.080	0.069	0.002
h(6)	0.177	0.043	0.010
h(7)	0.096	0.020	0.003
h(8)	0.020	0.008	0.001

Table 2. Comparisons for SNR = 30dB & T = 1,000

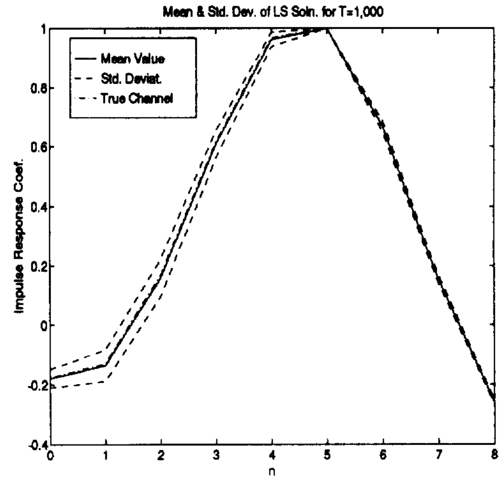

Figure 2. Least Sqr. Method: SNR = 30 dB and T = 1,000

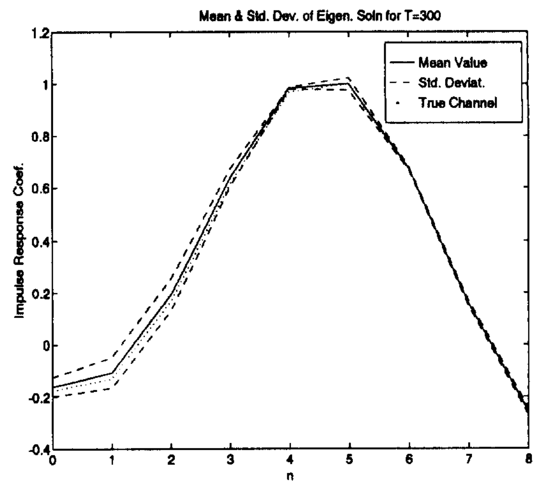

Figure 3. Eigenbased Method: SNR = 30 dB and T = 300

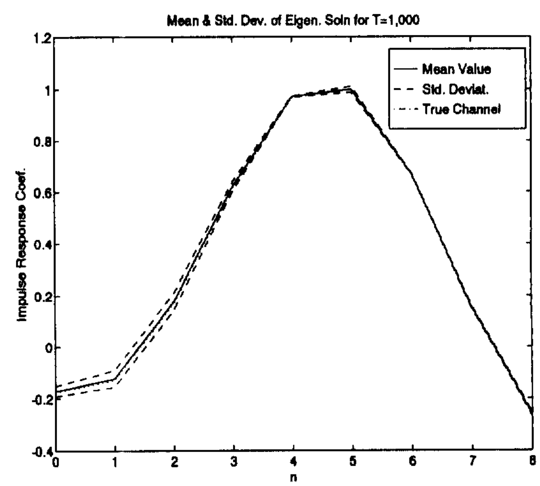

Figure 4. Eigenbased Method: SNR = 30 dB and T = 1,000