# Time-Frequency Distributions as Exploratory Tools in the Study of Biological Signals

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### Abstract

*Biological signals offer a special challenge to signal processing. Usually, a signal is assumed to be composed of complex exponentials which are the solutions to linear time-invariant (LTI) differential equations. Signals of biological origin often do not comply with these simplifying assumptions which are so useful in more conventional engineering application. Newly emerging techniques of time-frequency (t-f) analysis can provide new insights into the nature of biological signals so that new, more effective modeling and analysis techniques can be used. This paper describes some results using Reduced Interference Distributions (RID) as an exploratory tool in biological signal analysis, revealing new insights into the complex structure of these signals.*


## 1. Introduction

Conventional signal analysis often depends upon a number of techniques which depend on linearity and stationarity. Noise is assumed to be a white, stationary gaussian process, or at worse, a colored version of this. In biological systems, signals often result from time-varying systems which are appropriately described by time-varying differential equations. Extraneous biological activity which is not considered to be part of the signal is considered to be "noise".

Conventional spectral analysis based on the Fourier theory or parametric modeling approaches can yield inaccurate or completely misleading results in many biological applications. The most common t-f analysis technique, the spectrogram (SP), involves a moving time window. The signal is said to be stationary or at least quasi-stationary over this window, allowing time-invariant techniques. The spectrogram often presents serious difficulties when it is used to analyze rapidly varying signals, however. If the analysis window is made short enough to capture rapid changes in the signal it becomes impossible to resolve signal components which are close in frequency within the analysis window duration. If the window is made long enough to resolve the frequencies of sinusoids, the time of occurrence of sinusoidal segments becomes difficult to determine. This is the classical t-f uncertainty principle at work. Watkins [15] has addressed some of the problems of the spectrogram very well and provides examples of reasonable and unreasonable Spectrogram results in the analysis of marine mammal sounds.

Until recently, there was one alternative t-f analysis technique which was widely believed to avoid some of the problems of the spectrogram. The well known Wigner distribution avoids the problems of windowing and enjoys many useful properties, but often produces an unacceptable amount of interference or cross-term activity between signal components when the signal consists of many components. The effects of these cross-terms have come to be understood well[7, 8]. Despite its shortcomings, the Wigner distribution (WD) has been employed as an alternative to overcome the resolution shortcomings of the spectrogram. It also provides a high resolution representation in time and in frequency. The WD has many important and interesting properties. The WD has the important property of satisfying the time and frequency marginals in terms of the instantaneous power in time and energy spectrum in frequency. If the analytic form of the signal is used, proper instantaneous frequency and group delay are obtained as first moments of the WD. Also, time and frequency support properties result when using the WD. While the WD provides unambiguous high-resolution t-f representations of nonstationary monocomponent signals, such as chirps, its representations of multicomponent signals often are not useful. These representations contain interference (cross-terms) in different regions of the t-f plane resulting from interactions between signal components. The cross-terms are a hindrance to interpretation since they may obscure primary features of

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the signal.

Both the spectrogram and the WD are members of Cohen's Class of Distributions [4]. Cohen has provided a consistent set of definitions for a desirable set of t-f distributions which has been of great value in guiding and clarifying efforts in this area of research. Different members of Cohen's class can be obtained by using different kernels. The WD has a unity valued kernel. More recently, Choi and Williams introduced a new distribution, the Exponential Distribution (ED), with kernel $\phi_{ED}(\xi,\tau) = e^{-\xi^2\tau^2/\sigma}$, where $\sigma$ is a kernel parameter ($\sigma > 0$) [2]. The ED overcomes several drawbacks of the spectrogram and WD providing high resolution with suppressed interferences.

A recent comprehensive review by Cohen [5] provides an excellent overview of t-f distributions and recent results using them. This paper addresses a specific subset of t-f distributions belonging to Cohen's class. These are the time shift and frequency shift invariant t-f distributions. For these distributions a time shift in the signal is reflected as an equivalent time shift in the t-f distribution and a shift in the frequency of the signal is reflected as an equivalent frequency shift in the t-f distribution. The spectrogram, the WD and the RID all have this property.

## 2. Time-Frequency Distributions

Some theoretical underpinnings for understanding t-f distributions will be presented in this section. We will show that the important issues can be readily understood in a manner similar to the relationships between a signal, its Fourier transform and a filter applied to that signal.

### 2.1. Ambiguity Function Relationships

The key to understanding t-f relationships and manipulations is a thorough understanding of the ambiguity domain. Let $Z(f)$ be the Fourier Transform (FT) of the signal $z(t)$[1]:

$$Z(\omega) = F[z(t)] = \int f(t)e^{-j\omega t}dt \qquad (1)$$

and

$$z(t) = F^{-1}[Z(f)] = \frac{1}{2\pi}\int Z(\omega)e^{j\omega t}d\omega \qquad (2)$$

Let $R_z(t,\tau)$ be the ***instantaneous autocorrelation*** of a complex signal $z(t)$, defined as:

$$R_z(t,\tau) = z(t+\tau/2)z^*(t-\tau/2) \qquad (3)$$

where $z^*$ denotes the complex conjugate of z. The Wigner distribution of $z(t)$ is defined as the FT of $R_f(t,\tau)$ with respect to the lag variable $\tau$.

$$W_z(t,\omega) = F_\tau[z(t+\tau/2)z^*(t-\tau/2)] = F_\tau[R_z(t,\tau)] \quad (4)$$

Similarly, but with a different physical meaning, the symmetrical ambiguity function (AF) is defined as the Inverse Fourier transform of $R_z(t,\tau)$ with respect to the first variable.

$$A_z(\theta,\tau) = F_t^{-1}[z(t+\tau/2)z^*(t-\tau/2)] = F_t^{-1}[R_z(t,\tau)] \quad (5)$$

Thus, $W_z(t,\omega)$ and $A_z(v,\tau)$ are related by the two-dimensional (2-D) FT.

$$W_z(t,\omega) = \iint A_z(\theta,\tau)e^{-j(t\theta+\omega\tau)}d\theta\, d\tau \qquad (6)$$

These relationships may be combined with Eq. (1) to show that $C_z(t,\omega;\phi)$ may be found by

$$C_z(t,\omega;\phi) = \iint \phi(\theta,\tau)A_z(\theta,\tau)e^{-j(t\theta+\omega\tau)}d\theta\, d\tau \quad (7)$$

Thus, while $W_z(t,\omega)$ may be found from the symmetric ambiguity function by means of a double Fourier transform, any member of Cohen's Class of Distributions may be found by first multiplying the kernel, $\phi(\theta,\tau)$ by the symmetric ambiguity function and then carrying out the double FT. The generalized ambiguity function, $\phi(\theta,\tau)A_z(\theta,\tau)$ [6] is a key concept in t-f which aids one in clearly seeing the effect of the kernel in determining $C_z(t,\omega;\phi)$ .

The kernel for the WD is unity,as previously noted, so the generalized ambiguity function is identical to the ambiguity function and its t-f representation (the double Fourier transform) preserves both the auto-terms and the cross- terms. The kernels of the spectrogram and the RID emphasize the auto-terms and deemphasize the cross-terms, but in very different ways. The spectrogram is commonly computed by moving a fixed length window along the signal. At each window position the portion of the signal selected by the window is Fourier transformed and a spectrum is computed by taking the modulus squared value of this Fourier transform. Thus one has a time-varying spectrum where the time index is referenced to the center of the window. An alternative view is to view the spectrogram as a member of Cohen's class. As such, it has a kernel which is the ambiguity function of the window.

### 2.2. The Exponential Distribution

The ED is an attempt [2] to improve on the WD. Its performance has been compared with those of the spectrogram and the WD in a variety of environments [13,14]. The $\sigma$ parameter may be varied over a range of values to obtain different trade-offs between cross-term suppression and high auto-term t-f resolution. In

[1]the integral limits are $-\infty$ to $\infty$ in this paper

fact, as $\sigma$ becomes very large the ED kernel approaches the WD kernel. This provides the best resolution but the cross-terms become large and approach WD cross-terms in size. The ED maintains a unity value along the $\tau$ and $\theta$ axes, which is sufficient to maintain almost all of the desirable properties of the WD. The RID, which is a generalization of the ED shares many of the desirable properties of the WD, but also has the important reduced interference property as well. A comprehensive discussion and comparison of RID and other distributions is carried out in a recent book chapter [18] and a design procedure for RID kernels has been developed [9]. One may start with a primitive function, $h(t)$, which has certain simple constraints, and evolve a full-fledged RID kernel from it.

## 3. Biological Signal Structure

Biological signals often consist of a mixture of short sinusoidal segments, chirps and impulses. The phase relationships of these components are highly variable, resulting in very different appearing composite signals, even though the basic signal components may be the same. The brief duration of most of these components precludes good spectral estimates using conventional window based techniques. A time-frequency-energy representation is often very useful as a means of distinctly separating the signal components. The spectrogram usually fails in this task due to its window requirement. The RID often dramatically reveals the underlying structure of these signals and leads to further useful characterization of the signal in terms of its fundamental components. In order to proceed with this argument, several examples are presented.

### 3.1. Brain Signals

RID development has been driven to a large degree by the difficulty of characterizing brain biopotentials. These biopotentials can be roughly divided into two categories, the segments of the ongoing electroencephalogram (EEG) or segments of the EEG which are time-locked to some event such as a stimulus. The latter are called event-related potentials or ERPs.

The first example to be discussed is a segment of the EEG obtained during an epileptic seizure, using direct cortical electrodes[19]. Figure 1. shows the results. There are several signal components which are distinct in time-frequency, particularly for the RID. One can readily see that 1) the signal is highly non-stationary and time-varying and that it consists of three or four distinct components. EEGs are often described in terms of frequency components (e.g. $\delta, \theta, \alpha, \beta$), and ERPs are described in terms of time components (e.g. $P_{300}$, the peak at 300 msec. after the stimulus). This terminology does not adequately characterize the true nature of these signals which often exhibit compact t-f energy features as shown in Figure 1.

Similar features have observed in ERPs evoked by word stimuli flashed to patients with phobias[12]. Figure 2(a) shows the contours at $\frac{1}{2}$ the maximum energy of five features from such an ERP. These features are described by five Gabor logons varied in amplitude, time, frequency, spread and chirp rate in order to account for a large portion of the energy in the RID of the ERP[1]. Figure 2(b) shows the fit to the ERP achieved by these five Gabor logons. The use of these features in a pattern classification scheme has yielded a better classification of the ERPs in terms of the class membership of the words than did previous efforts which utilized raw t-f features based on peaks in the ED[3, 12].

Figure 3 illustrates the effect of SP windows on an EEG segment during an epileptic seizure and its scaled and frequency shifted analogs. These results were produced by the Binomial distribution which is a convenient discrete realization of the RID principles[10, 18]. Both the long and short SP windows smear the t-f representation, but the RID shows complex signal structure. Further, the spectrogram is invariant to time and frequency shift, but not scale. The RID is also scale invariant in that the t-f representation is compressed in time and expanded in frequency as required by the scale property of the FT. The scalogram, based on the wavelet transform[13] would be scale and time-shift invariant but not frequency shift invariant.

### 3.2. Joint Clicks

Temporomandibular joint problems plague many people and cause a number of difficulties. Joint pathologies often result in clicks or creaking (crepitation). The RID appears to be a very useful tool in characterizing these signals [16]. Gabor logons provide very good fits to several types of joint clicks[1]. Uncertainty measures on the RID[17] provide good quantitative assessments of crepitation.

### 3.3. Marine Mammal Sounds

Marine mammal sounds are well characterized using the RID and overcome some of the shortcomings of the SP as described by Watkins[15]. Toward this end, RID is in regular use in Watkins' lab at Woods Hole Oceanographic Institution. It has been used to characterize the fine structure of dolphin whistles and clicks[14]. RID clearly reveals both the tonal structure in the whistles and the temporal structure of clicks which are simultaneously produced by these animals. It appears that the clicks of marine mammals such as whales and dolphins may have a distinctive structure based on the individual animal and may be useful in nonintrusive tagging and tracking of these animals.

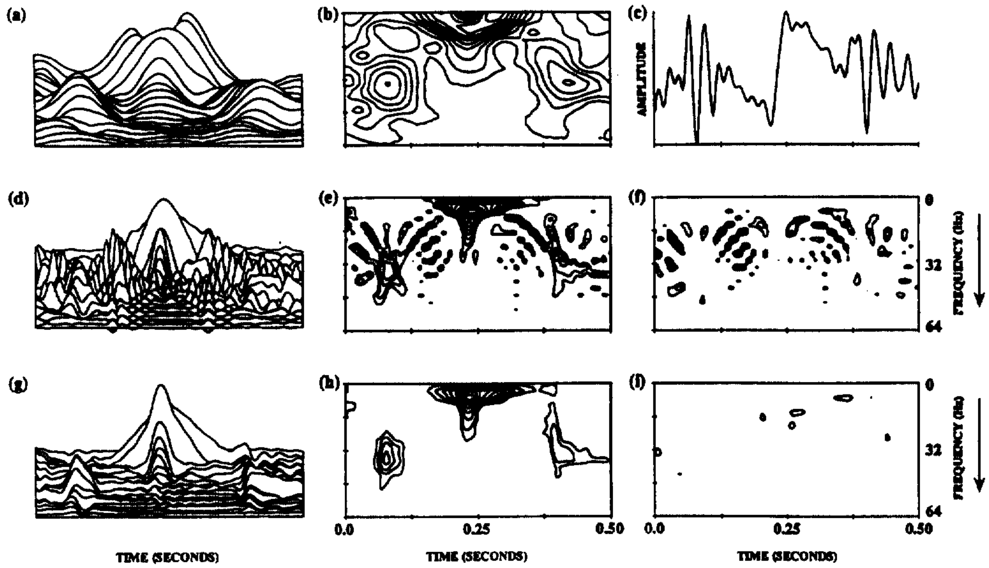


Figure 1. Comparison of SP, WD and RID analysis of an EEG segment during an epileptic seizure. (a) and (b) are mesh and contour plots of the SP, (c) is the time series, (d) and (e) are the mesh and contour plots of the WD, (f) is the contour plot of the negative energy portion of the WD, (g) and (h) are the mesh and contour plots of the RID and (i) is the contour plot of the negative energy portion of the RID.

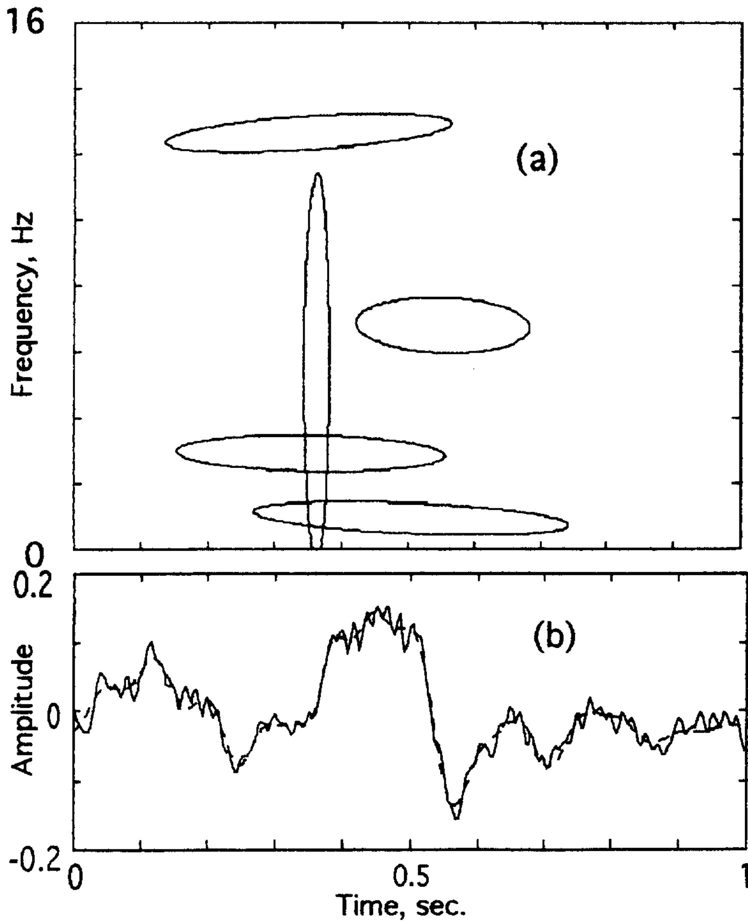


Figure 2. (a) Gabor logon features of an ERP. (b) Resulting fit (dashed line) to the time series (solid) line.

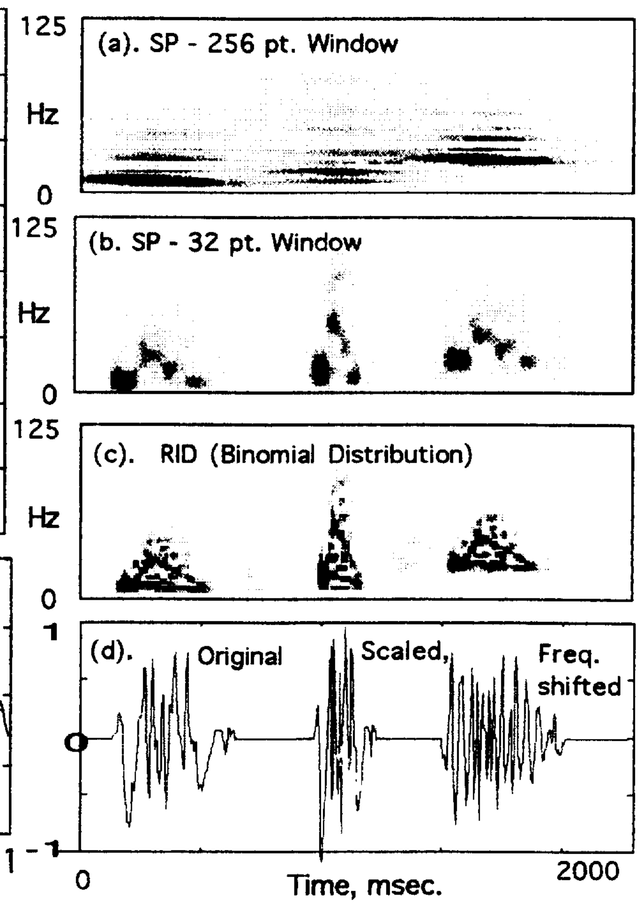


Figure 3. (a) SP - 256 pt. window, (b) SP - 32 pt. window, (c), RID and (d) Original, scaled and frequency shifted EEG segments.

## 4. Conclusions

RID analysis is useful in identifying complex features of biological signals so that one can understand the underlying structure. This understanding can lead to effective strategies for modeling and feature selection. We have been able to use the RID results to devise time-varying differential equations to describe epileptic discharges. Much more work needs to be done to capitalize on the often dramatic visual presentations provided by RID and other emerging time-frequency analysis techniques. In addition, improved direct measures on these TFDs need to be to provide detection and discrimination of signals based on their t-f structure. A number of colleagues are using RID based techniques on a variety of problems with considerable success at present.

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