IIR Digital Filters with Peak-Constrained Least-Squared Errors

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Abstract

In this paper we show how to use extensions of Lawon's algorithm to design peak-constrained least-squares (PCLS) IIR digital filters. These extensions are based on the RWLS algorithm that was presented in [1].

1. Introduction

The most popular optimality criteria for digital filters is minimax (Chebyshev) in each band. In particular, the Parks-McClellan algorithm [2] is very popular for FIR digital filter design. Minimax (MM) and least-squares (LS) optimization problems are subsets of a more general class of optimization problems. We refer to problems in this general class as peak-constrained least-squares (PCLS) optimization problems [3]. In PCLS optimization we minimize the total squared error subject to constraints on the peak error.

In this paper we show how to use extensions of Lawon's algorithm to design PCLS IIR digital filters. We start by using Deczky's IIR filter design algorithm as an engine to design unconstrained least-squares IIR filters. The shapes of the weighting functions needed for PCLS filters are then determined by using extensions of Lawon's algorithm.

In [5] Lawson showed that minimax (Chebyshev) optimization problems can be reformulated in terms of equivalent weighted least-squares (WLS) optimization problems. As an example we consider the following minimax approximation problem where $D(x)$ is the desired function and $F(x)$ is the approximating function.

$$\text{Minimize: Maximum}\{\, | F(x) - D(x) |;\ \text{for } a \le x \le b \} \quad (1)$$

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Lawson showed that the problem in (1) can be reformulated as the following weighted least squares minimization problem.

$$\text{Minimize: } E = \int_a^b W(x) | F(x) - D(x) |^2 \, dx \quad (2)$$

The trick is to find the appropriate $W(x)$ that makes (1) equivalent to (2). The "Lawson algorithm" is an iterative procedure for determining $W(x)$.

Several researchers have studied the application of Lawson's algorithm to designing FIR digital filters, as in [6]-[7]. In this paper we will extend Lawson's algorithm and use piesewise-constant weighting functions to design IIR digital filters.

2. Deczky's Filter Design Algorithm

Deczky's filter design algorithm can minimize the following weighted squared error function as discussed in [4]:

$$F = \sum_{J=1}^{NP} WH(J) * \{X(N) * H(X,J) - FS(J)\} ** IP$$

X is a vector containing the zeros and poles in polar coordinates for elements 0, 1, ... $N-1$. Element $X(N)$ is a scale factor on the frequency response. $H(X,J)$ is the frequency response at the J-th frequency point. WH is the weighting function at the frequency point corresponding to J. IP is the exponent for the error measure. When IP equals 2 the corresponding IIR filter is designed according to the weigthed least-squares optimality criterion. In this paper we will use $IP = 2$.

Deczky's filter design algorithm can be used for PCLS optimization by finding the appropriate weighting function, as discussed in the next section.

3. Computation of Weighting Function

To quickly provide some intuition about the problem, we will briefly describe the general shape of the weighting function and the basic concept for updating it. We consider the computation of the stopband weighting function, $W_s(k+1, e^{j\omega})$, corresponding to Iteration $k+1$, based on the results from Iteration k. Fig. 1 shows the basic concepts.

At Iteration k we apply the weighting function $W_s(k, e^{j\omega})$, minimize the corresponding weighted squared error, and obtain the resulting $H(k, e^{j\omega})$. Assuming that the peak stopband gain specification is δ_s, we see that the gain at ω_s violates the specification in Fig. 1. Also, the first two sidelobe peaks violate the δ_s specification. Therefore, at Iteration $k+1$ we apply the updated weighting function, $W_s(k+1, e^{j\omega})$, which is larger than $W_s(k, e^{j\omega})$ at frequencies surrounding ω_s and the first two sidelobe peaks. $W_s(k+1, e^{j\omega})$ is equal to the nominal constant value, C_S, for the third sidelobe and beyond.

We denote the value of the i-th stopband stairstep in $W_s(k+1, e^{j\omega})$ at Iteration $k+1$ as $\gamma_s(k+1, i)$, as shown in Fig. 1. The stairstep boundaries are determined by the stopband null frequencies in $H(k, e^{j\omega})$. We let $H_s(k+1, i)$ denote the peak magnitude of the i-th stopband sidelobe.

The weighting function starts at the frequency denoted ω_s'. ω_s' is slightly less than ω_s, where $\omega_s' = \omega_s - \Delta_s$. We have studied several approaches to determining ω_s' and then adjusted ω_s' iteratively. We have also fixed ω_s' at a constant value, and updated the weighting value centered at ω_s from iteration to iteration. In particular, we can let ω_n denote the frequency of the first null, and then define $\Delta_s = \omega_n - \omega_s$. In this case, the first stairstep of the piecewise-constant weighting function is applied symmetrically about ω_s.

We update the weighting function according to the following general rules in the stopband:

1) make $\gamma_s(k+1, i)$ larger than $\gamma_s(k, i)$ if $H_s(k, i) > \delta_s$
2) make $\gamma_s(k+1, i)$ smaller than $\gamma_s(k, i)$ if $H_s(k, i) < \delta_s$, but limit it to C_S.
3) make $\gamma_s(k+1, i)$ equal to $\gamma_s(k, i)$ if $H_s(k, i) = \delta_s$

The same rules apply for the passband:

1) make $\gamma_p(k+1, i)$ larger than $\gamma_p(k, i)$ if $H_p(k, i) - 1.0 > \delta_p$
2) make $\gamma_p(k+1, i)$ smaller than $\gamma_p(k, i)$ if $H_p(k, i) - 1.0 < \delta_p$, but limit to C_P.
3) make $\gamma_p(k+1, i)$ equal to $\gamma_p(k, i)$ if $H_p(k, i) - 1.0 = \delta_p$ We now consider the details for implementing the above rules. We first discuss the stopband updates of the weighting function.

$$\gamma_s(k+1, i) = B_s\{S_s(|H_s(k, i)|)\gamma_s(k, i)\}$$

$B_s\{x\}$ is a base clipping function that saturates at C_S., as follows:

$$B_S\{x\} = \begin{cases} x, x > C_S \\ C_S, x \le C_S \end{cases}$$

$S_s\{x\}$ is a symmetric saturating function. It is defined by

$$S_P(|H_P(k,i)|) = S\left\{1 + \beta_P \frac{|H_S(k,i) - G_P - \delta_P|}{\delta_P}\right\}$$

β_S is the stepsize factor. $S_S\{x\}$ saturates at limits denoted MIN and MAX. The typical values for MIN and MAX are 0.5 and 2.

S{x}=MAX if x>MAX

S{x}=X if MIN <x <MAX.

We now consider the passband updates of the weighting function.

$$\gamma_p(k+1, i) = B_p\{S_p(|H_p(k, i)|)\gamma_p(k, i)\}$$

$B_p\{x\}$ is a base clipping function that saturates at C_P , as follows:

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$$S_P(|H_P(k,i)|) = S\left\{1 + \beta_P \frac{|H_S(k,i) - G_P - \delta_P|}{\delta_P}\right\}$$

We now outline the basic steps for the algorithm.

step 1: Let $k = 0$

step 2: Let $W_k(e^{j\omega}) = C_p$ for the passband
Let $W_k(e^{j\omega}) = C_s$ for the stopband

step 3: Determine the frequency response of the filter

step 4: Fit a piecewise constant weighting function $W_{k+1}(e^{j\omega})$ to $H_k(e^{j\omega})$ using the updating method described later

step 5: If $W_{k+1}(e^{j\omega}) = W_k(e^{j\omega})$ stop, otherwise let $k = k+1$ and go to step 3.

4. IIR Filter Design Example

We now consider the design of a 40th order IIR filter to meet the following specifications: $F_p = 0.06$ cycles/sample, $F_s = 0.08$ cycles/sample, $DB_p = 2.0$ DB, $DB_s = -55.0$ DB. The frequency response of the IIR filter generated by the first iteration using uniform weighting is shown in Fig. 2. The frequency response and weighting function for Iteration 4 are shown in Fig. 3. The frequency response and weighting function for Iteration 5 are shown in Fig. 4. Finally, the frequency response and weighting function for Iteration 30 are shown in Fig. 5.

5. Conclusion

In this paper we showed how to use extensions of Lawon's algorithm to design IIR digital filters according to the PCLS optimality criterion. A design example was included. We are currently implementing nonlinear programming algorithms to designing IIR and complex FIR filters according to this optimality criterion.

6. References

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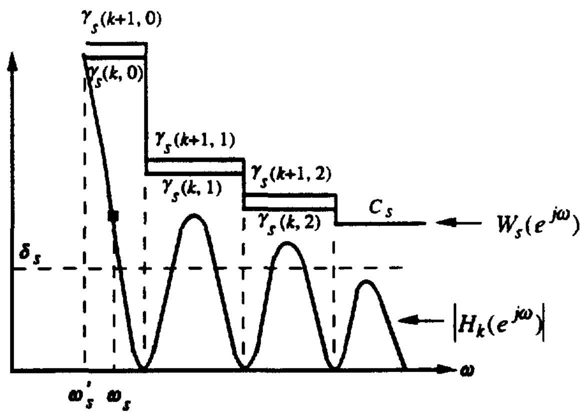

Fig. 1. Typical weighting functions in the stopband for iterations k and $k+1$.

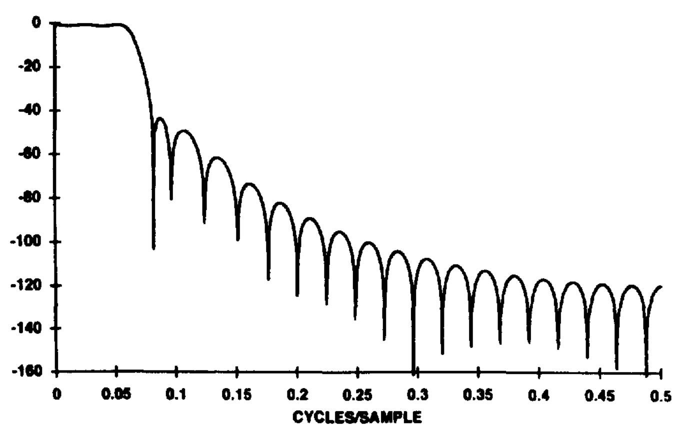

Fig. 2. Frequency response at the first iteration using uniform weighting.

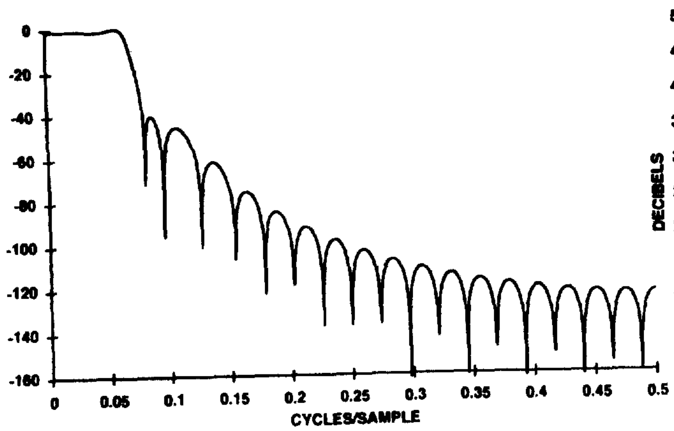

Fig. 3a. Frequency response at the fourth iteration.

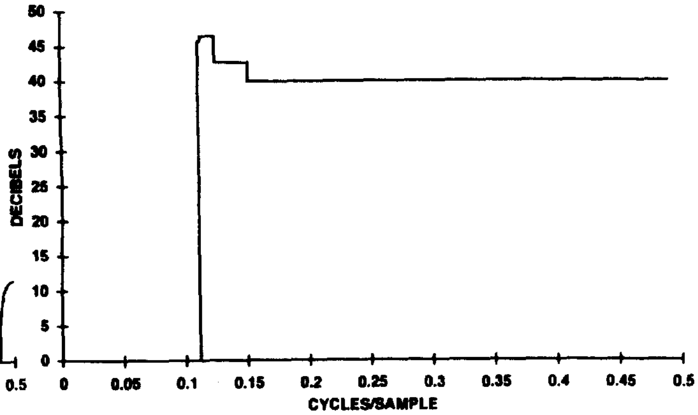

Fig. 4b. Weighting function at the fifth iteration.

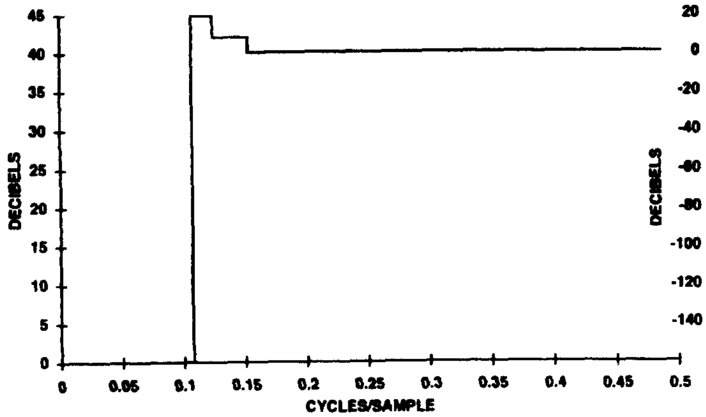

Fig. 3b. Weighting function at the fourth iteration.

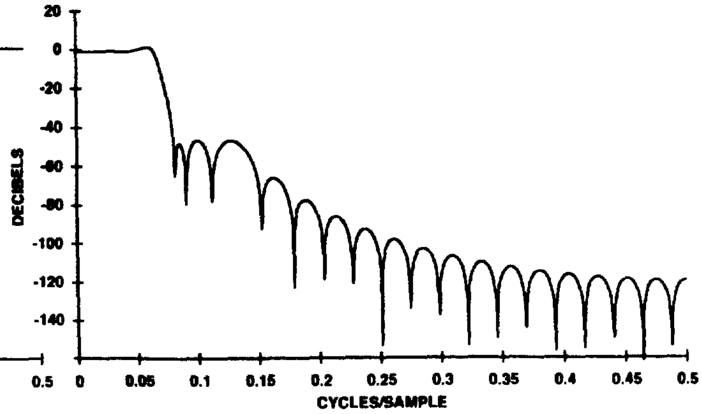

Fig. 5a. Frequency response at the 30th iteration.

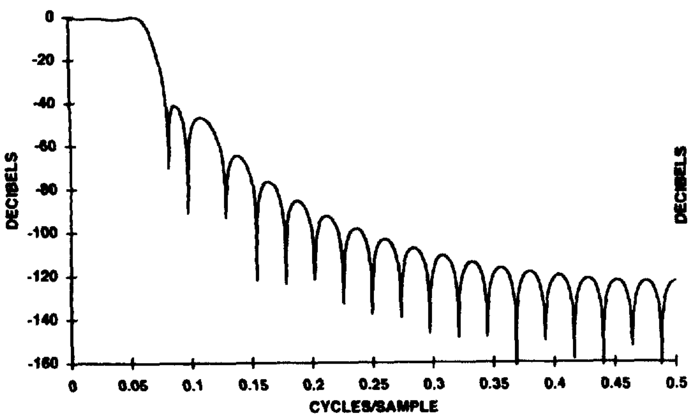

Fig. 4a. Frequency response at the fifth iteration.

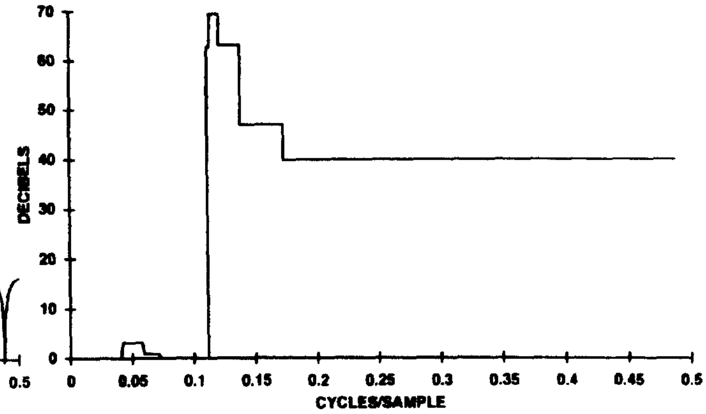

Fig. 5b. Weighting function at the 30th iteration.