An Efficient Design of 2-D IIR Digital Filters Using Singular Value Decomposition

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Abstract

An efficient technique by using Singular Value Decomposition (SVD) method for the design of 2-D IIR digital filters is presented. It is shown that by assigning higher order filters to the sections with greater singular values (SVs), and lower order filters to the sections with lower SVs, a sizable reduction in the total number of required multiplications is achieved. An example is given to illustrate the computational efficiency of the proposed technique.

I. Introduction

In the past two decades many researchers have used SVD method for the design of 2-D filters [1-3]. The main advantage of implementing a 2-D digital filter via SVD technique is its lower requirements of multipliers compared to the general 2-D filter realization techniques. This is due to the fact that the singular values of the matrix representation of a digital filter usually decrease rapidly, therefore, matrix can be represented with less subsections than its order. This entails elimination of the subsections corresponding to the lower SVs. Hence, realization of these subsections will be more efficient than the original 2-D filter. For instance a direct realization of 2-D FIR filter of order $(N \times N)$ requires $(N+1)^2$ multipliers. The requirement for the same filter through SVD design is $2M \times (N+1)$, where M is the number of subsections and usually $M < r << N$ ("r" is the rank of the matrix). Yazdi, et al. [1] have shown that more reduction in realization complexity is possible by designing nonequal order 1-D FIR filters in different legs of the parallel structure. The order of the 1-D FIR subfilters have chosen directly proportional to their corresponding singular values. It has been shown that by using this technique about 30% reduction in the number of multipliers is achievable comparing to the conventional method with the same error performance.

In this paper, an improved method for SVD technique applicable to 2-D IIR filters is presented. Designed example shows that the method yields reduction in the number of multiplier coefficients, while it maintains the accuracy of the design.

II. Design methodology

Let matrix $A = \{a_{pq}\}$ be the desired sampled amplitude response of a 2-D filter with quaderantal symmetry, in the first quadrant

$$a_{pq} = \left| H(e^{j\pi\mu_p}, e^{j\pi\mu_q}) \right| \quad (1)$$

where μ_p and μ_q are the normalized frequencies as follows

$$\mu_p = \frac{p-1}{M-1}, \qquad \mu_q = \frac{q-1}{M-1} \quad (2)$$

for $1 \le p,q \le M$. The singular value decomposition of matrix A can be expressed as

$$A = \sum_{i=1}^{r} \sigma_i u_i v_i' \quad (3)$$

where $\sigma_1 \ge \sigma_2 \ge \ldots \ge \sigma_r$ are the singular values of matrix A, u_i and v_i are orthonormal column vectors. If we let $x_i = \sigma_i^{1/2} u$ and $y_i = \sigma_i^{1/2} v_i$, then (3) can be rewritten as follows

$$A = \sum_{i=1}^{r} x_i y_i' \quad (4)$$

where x_i and y_i are sets of orthogonal M-dimensional column vectors. These vectors can be considered as the sampled frequency responses of 1-D subfilters.

If A is a symmetric matrix, then equation (4) becomes as follows

$$A = \sum_{i=1}^{r} s_i x_i x_i' \quad (5)$$

where $s_1 = 1$ and $s_i = \pm 1$ for $2 \le i \le r$. Therefore, each section requires only one 1-D subfilter to be designed [2].

Let us assume that it is possible to design 1-D filters characterized by $\sigma_i^{1/2} \varphi_i$, therefore amplitude response of the designed filter in matrix form will be as follows

$$\overline{A} = \sum_{i=1}^{r} \sigma_i \varphi_i \varphi_i' \quad (6)$$

where $\overline{A}$ is approximately equal to A. The mean square error of the design can be expressed as

$$E = \left\| A - \overline{A} \right\| \quad (7)$$

where $\|Z\|$ is the norm of matrix Z as follows

$$\|Z\| = \left[\sum_{l} \sum_{m} z_{lm}^2 \right]^{1/2} \quad (8)$$

In the conventional method, subfilters characterized by φ_i were designed with equal order filters in all branches. In the proposed method [1], the order of subfilters are determined according to the significance of the singular values associated with each of them. To show the effectiveness of this method first let us assume that error in all branches but branch k are zero. Therefore, MSE will be

$$E_1 = \sigma_k \left\| \varphi_k \varphi_k' - u_k u_k' \right\| \quad (9)$$

In the next step, we assume that error in all branches expect branch j are zero, consequently it can be written as the following

$$E_2 = \sigma_j \left\| \varphi_j \varphi_j' - u_j u_j' \right\| \quad (10)$$

Assuming that $E_1 = E_2$ in equations (9) and (10), we can write

$$\frac{\sigma_j}{\sigma_k} = \frac{\left\| \varphi_k \varphi_k' - u_k u_k' \right\|}{\left\| \varphi_j \varphi_j' - u_j u_j' \right\|} \quad (11)$$

If $\sigma_j > \sigma_k$, then to have an equal amount of error in both cases, error in subfilters of branch k can be greater than error in subfilter of section j by the factor of σ_j / σ_k. Therefore, it is concluded that overall error is more sensitive to sections with greater SVs, hence subfilters of such branches should be designed with more accuracy. Noting that the singular values of a matrix usually decrease rapidly, one can obtain a sizable reduction in the hardware by selecting the order of subfilters directly proportional to their corresponding SVs.

To decompose a 2-D magnitude samples of a recursive digital filter, one can either use *iterative singular value decomposition* method [3], or shift up the values of each decomposed vector with the absolute value of the most negative element of that vector to avoid negative elements in the magnitude specifications of each subfilter. In the design examples of this paper the later one is used. Let φ_i^- be the absolute value of the most negative component of vector φ_i. If e_φ is a vector whose elements are all equal to one and it has the same dimension as φ_i, then all components of the following vector

$$\tilde{\varphi}_i = \varphi_i + \varphi_i^- e_\varphi \qquad (12)$$

are nonnegative [2]. Let us assume that the shifted vectors, $\tilde{\varphi}_i$, with nonnegative elements can be realized as recursive digital filters, then by decreasing the same value in each subfilter, one can get the original specifications for each vector φ_i. If it is assumed that $\tilde{f}_i(z_j)$ for $1 \le i \le r$ and $j = 1,2$ are stable 1-D recursive filters such that

$$\left|\tilde{f}_i(e^{j\pi\mu_g})\right| \approx \tilde{\varphi}_i(g) \qquad (13)$$

where μ_g is similar to equation (2), then to process of an available record of data, the zero phase structure of Fig.1 can be employed [2]. In such a case the problem of different lag for each branch can be avoided.

The design can be completed by using any iterative nonlinear optimization technique for each filter [4]. To avoid the cumbersome process of stability test and stabilization, if necessary, one may generate 1-D stable polynomial as the denominator of each subfilter [4]. The following analog polynomial is used in the design example,

$$D(s_j) = a_1^2 s_j^2 + a_2^2 s_j + 1 \text{ for } j = 1,2 \qquad (14)$$

where its roots always have nonpositive real part. Therefore the following transfer function is used for each subfilter

$$H(s_j) = A \prod_{k=1}^{K} \frac{b_{1k}s_j^2 + b_{2k}s_j + 1}{a_{1k}s_j^2 + a_{2k}s_j + 1} \qquad (15)$$

where "K "is the cascaded number of filters of order two in each subfilter. Now, by using bilinear transformation, we can find the digital counterpart of (15). The coefficients of each cascaded filter are to be determined through optimization process.

III. Design example

To illustrate the usefulness of this technique, we have designed a low pass filter with the following amplitude specification by using both the equal order filters in different branches and the proposed method

$$H(e^{j\omega_1 T_1}, e^{j\omega_2 T_2}) = \begin{cases} 1 & 0 \le \omega_g \le \omega_p \\ \dfrac{(\omega_a - \omega_g)}{(\omega_a - \omega_p)} & \omega_p \le \omega_g \le \omega_a \\ 0 & otherwise \end{cases} \qquad (16)$$

where

$$\omega_g = \sqrt{\left[\tfrac{1}{2}(\omega_1 - \omega_2)^2 + \tfrac{1}{4}(\omega_1 + \omega_2)^2\right]} \qquad (17)$$

and

$$\omega_p = 0.35\pi \quad , \quad \omega_a = 0.5\pi$$

The sampled amplitude response is obtained by applying a grid of 41x41 to (16) in the first quadrant. This matrix is symmetric, therefore the structure shown in Fig.-1 is applicable to this case. Twelve nonzero singular values of the matrix are taken, therefore the design has twelve branches. The filter is designed in three different forms, i) all branches with the same filter orders equal to four, ii) all branches with the same order equal to two, iii) subfilters in the first branch with order six, second branch with order four and all others with order equal to two. Figures 2, 3 , and 4 correspond to these three cases. *Mean square error* (MSE) in the pass band and the stop band, and the number of necessary multiplications of the designed filters for these three cases are shown in Table-1. It is seen that for almost same error, designer can save up to 33% in computation complexity of the system, by using the proposed method comparing to the conventional technique.

Table-1

Order of subfilters	MSE in passband	MSE in stopband	multiplie -rs
12x4	4.0e-4	1.5e-3	216
12x2	1.6e-3	3.1e-3	120
6-4-10x2	3.6e-4	1.5e-3	144

IV. Conclusion

A modified approach for the design of 2-D digital filters by using SVD method is applied to IIR filters. It is shown that the proposed method is an efficient way of designing 2-D IIR filters in the term of the complexity of realization.

References

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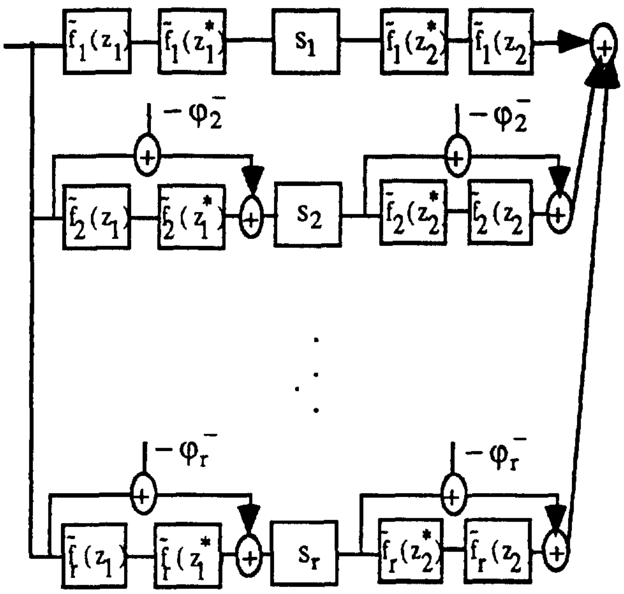

Fig.-1 Zero-Phase Realization of the SVD method for 2-D IIR digital filters.

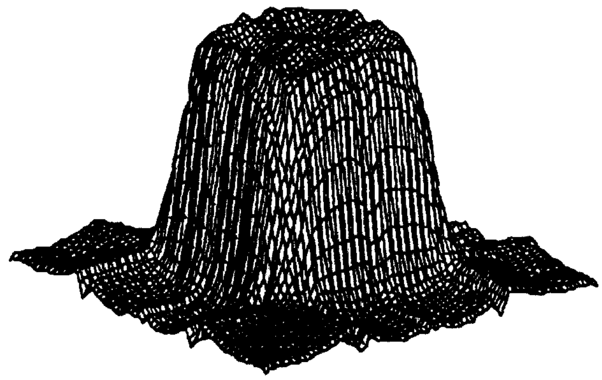

Fig.-2 Amplitude response of 2-D filter designed by the conventional, by using the same specifications as item (i)

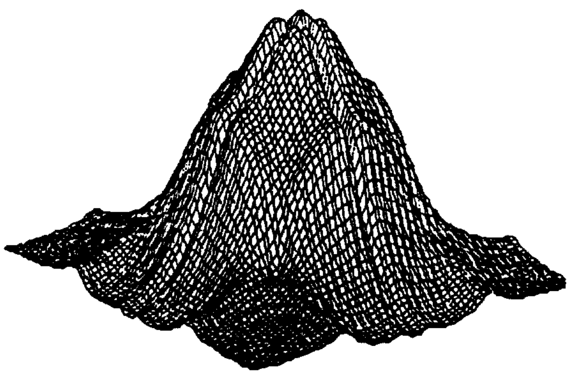

Fig.-3 Amplitude response of 2-D filter designed by the conventional method, by using the same specifications as item (ii)

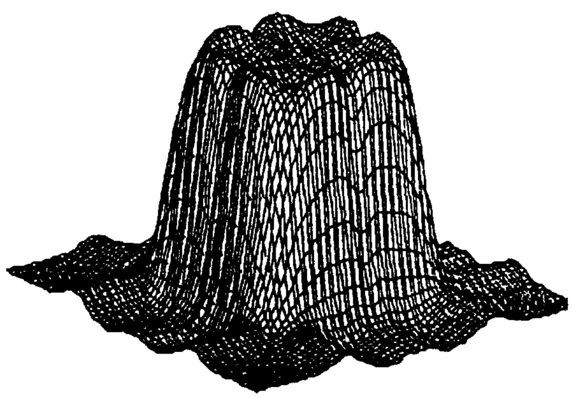

Fig.-4 Amplitude response of 2-D filter designed by the proposed method by using the same specifications as item (iii)