PERFORMANCE ANALYSIS OF WAVELET TRANSFORM-BASED ADAPTIVE FILTERING

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Abstract

The Wavelet transform [2],[10],[12] has been introduced for quite some time now. In this paper, [1] *we examine the performance of wavelet transform-based adaptive filters [3]. The application of two typical wavelets (D4 and Haar wavelets) are studied extensively as indicators of wavelet transform-based adaptive filter performance in general.*

Experimental, results for a system identification application show an improvement in signal modelling and satisfactory convergence speed for a variety of conditions. equalization application.

We also compare the wavelet transform-based adaptive filters [9] to other transform-based adaptive filters such as discrete-cosine transform (DCT), discrete sine transform (DST), discrete Fourier transform (DFT), Walsh Hadamard transform (WHT) and discrete Hartley transform (DHT). The results of this comparison are mixed. We find that the performance depends on many factors but are consistent with the findings of [9]. The results are also largely dependent on the properties of wavelets which have been found to be well suited for analyzing non-stationary signals.

Finally, we present computational complexity considerations between these various transform-domain adaptive filters.

1 Introduction

The transform-domain LMS adaptive filter which was introduced in [11] is shown in Figure 1.

In this paper, we first review some analytical results presented in [4],[3]. We implement transform-domain adaptive finite impulse response (FIR) filters using wavelet transforms and compare their performance with transform-domain adaptive filters implemented using the variety of other transforms currently used. We present our simulation results and check their consistency with the analytical results. Conclusions are made based on the simulation results.

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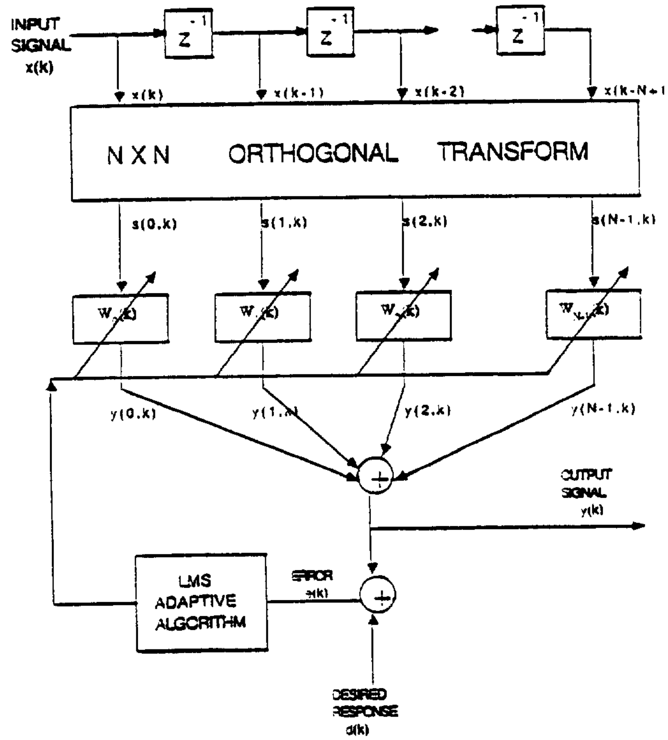

Figure 1: The transform-domain LMS adaptive filter

2 Wavelet-transform-based adaptive filtering

Some analysis of the convergence of the Wavelet-transform- domain LMS adaptive filter has been presented in [3], [4], [8].

From figure 1, at any time n, the transformed input vector is

$$S(n) = RX(n)$$

where R is the wavelet transform matrix and

$$X(n) = [x(n)x(n-1)x(n-2)....x(n-N+1)]^T$$

The auto-correlation covariance matrix of this input vector is given by

$$R_s = E[S(n)S(n)^T] = RR_xR^T$$

The transformed optimal weight vector is

$$W^w_{opt} = (R_s)^{-1}RR_xW_{opt}$$

where W_{opt} denotes the time-domain Wiener optimal weight vector. The minimum mean square error is

$$\begin{aligned} \xi^w_{min} &= E[|d(n)|^2] - P_s^T R_s^{-1} P_s & (1) \\ &= E[|d(n)|^2] - P_x^T R^T R_s^{-1} R P_s & (2) \\ &= \xi_{min} - P_x^T (R^T R_v^{-1} R - R_x^{-1}) P_x & (3) \end{aligned}$$

The excess mean square error is

$$\xi^w_{xss} = \mu tr(R_s)\xi^w_{min}$$

where

$$Tr(R_s) = C\sum_{j=0}^{J}\sigma_j^2 = C\sum_{j=0}^{J}\sum_k (a_k^j)^2$$

is the sum of the energies of the signal $x(n)$ at dilation levels $j = 0, 1, 2, ..., J$. This sum is monotone increasing and bounded above by the total energy of $x(n)$. This implies the more severe the truncation of the wavelet reconstruction sum, the less the excess mean square error for a given ξ^w_{min}. It only makes sense then to expect that ξ^w_{min} vary inversely proportional to J. In fact if J is increased so that $J + 1 = N$, then R is a square matrix and the second term of equation 3 vanishes.

Notice that when R_x can be approximated by a circulant matrix, then R_s is approximately diagonal [9], [6].

In [4], three algorithms were presented; one based on a constant convergence factor, another based on a time-varying convergence factor and another based on an exponentially-weighted convergence factor. These algorithms are respectively given by the following

$$\begin{aligned} W(n+1) &= W(n) + 2\mu\epsilon(n)S(n) & (4) \\ W(n+1) &= W(n) + 2\mu R_s^{-1}S(n)\epsilon(n) & (5) \\ W(n+1) &= W(n) + 2\mu D(n)S(n)\epsilon(n) & (6) \end{aligned}$$

where $D(n) = diag(2^{-j}), j = 0, 1, 2, 3 ..... J$ and usually the convergence condition is $0 < \mu < \frac{1}{\lambda_{V_{max}}}$ where $\lambda_{V_{max}}$ is the maximum eigenvalue of R_v.

In experiments on noise cancellation with no external reference signal, in [4], it was shown using Daubechies' Ψ_4 (which we call D4 here) wavelets that the best algorithm was the one based on an exponentially-weighted convergence factor. In our experiment here, we have used the exponentially-weighted convergence factor as well. This is also consistent with findings in [9].

3 Comparison with other transform-based adaptive filters

The wavelet transform adaptive filter LMS (WVTLMS) algorithms are implemented using two different types of wavelet constructions: the Daubechies wavelets of order 4 and the Haar wavelets. In both cases, the transformed signal is obtained by processing the input through alternative permutation and filtering transformations [12], [10], [2]. The procedure is similar to quadrature mirror filtering where each step of the filtering zeros into the details of the signal at different resolutions levels.

Apart from the usual performance measures used for adaptive filters [9] such as the eigenvalue spread, we also use the diagonality factors which gives us an idea of the weights of the diagonal relative to the weights of the off-diagonal terms. This might be used to better compare the rate of convergence. These results are presented in [1].

When R_x can be approximated by a circulant matrix, then R_s is approximately diagonal [9]. Therefore, the correlation of the input signal is reduced depending on how close to circulant the input signal autocorrelation matrix is.

Simulations Results

We considered two applications of transform-domain adaptive filters; system identification and channel equalization.

It is well-known that wavelet theory is better suited to the analysis of non-stationary signals [2], [10]. Therefore, we have used both time-invariant and time-varying system identification. In some adaptive channel equalization applications such as for fading channels as found in mobile communications, the channel

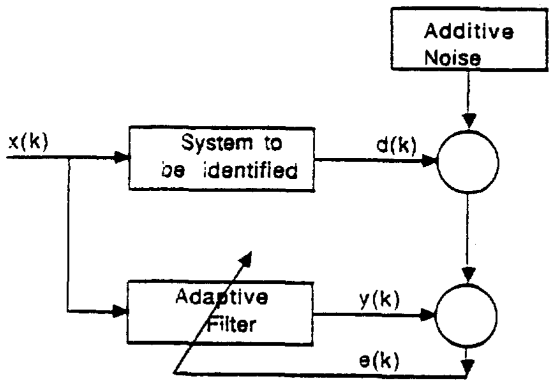

Figure 2: Setup for system identification simulati experiments

parameters are time-varying and can be considere non-stationary.

System Identification

Several simulations were run to test the suitability of this algorithm for tracking non-stationary or time-varying systems. The set-up for the simulations is shown in figure 2. We used three coloring filters which are designed respectively to be lowpass, highpass and bandpass to color the input signal x_k to the adaptive filter and the system to be identified. This gives us three different types of input signal autocorrelation matrix which are close to circulant to different degrees. The transforms used in the simulations presented here are Haar, D4, Discrete Cosine Transform (DCT) and Walsh Hadamard Transform (WHT).

Some examples of simulation results for a system identification application are shown in figures 3, 4, 5 and 6. In figure 6, there was no difference between the convergence curves using different transforms. There was little difference in the final weights of the system to be identified as shown.

Channel Equalization

The model used in this simulation for the non-stationary channel is similar to the ones found in [7] with the channel parameters varying with time as a first-order Markov process [5].

Using D4 wavelets in the transform-domain LMS adaptive filter, a time-varying channel was simulated. We used a step-size of $\mu = 0.05$ and time-varying channel parameter of $\alpha = 0.8$. The results are shown in figures 7 and 8.

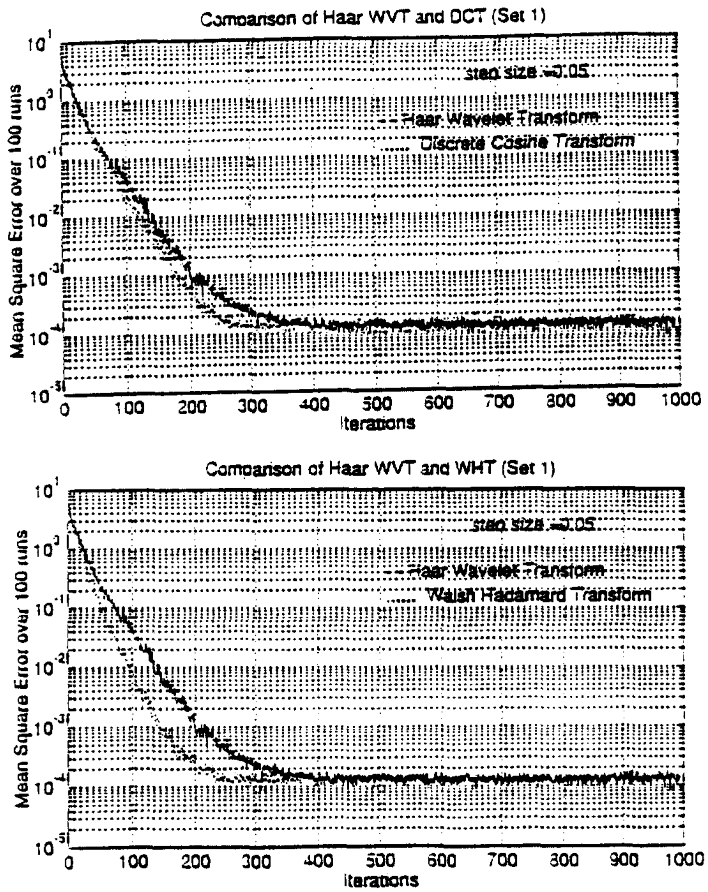

Figure 3: Comaprison of Haar with DCT and WHT for Lowpass input

From these figures, we see that the WVTLMS adaptive filter using D4 wavelets converges faster than the regular LMS algorithm for a time-varying channel. More extensive comparisons are being done by the to comapare different transforms for this application.

4 Conclusions and Future Work

Wavelet-transform-domain adaptive filtering allows better modelling of the possibly non-stationary input signal (e.g. in speech processing applications)

For colored noise input for system identification, the WVTLMS algorithms do not converge as fast as the others DCTLMS, DFTLMS and WHTLMS. This is because, the transformed input autocorrelation matrix does not produce a better eigenvalue spread.

WVTLMS algorithms are very suitable for cases where the model order of the system to be identified is insufficient.

Choice of wavelet bases is dependent on the types of signal inputs. Daubechies and Haar WVTLMS al-

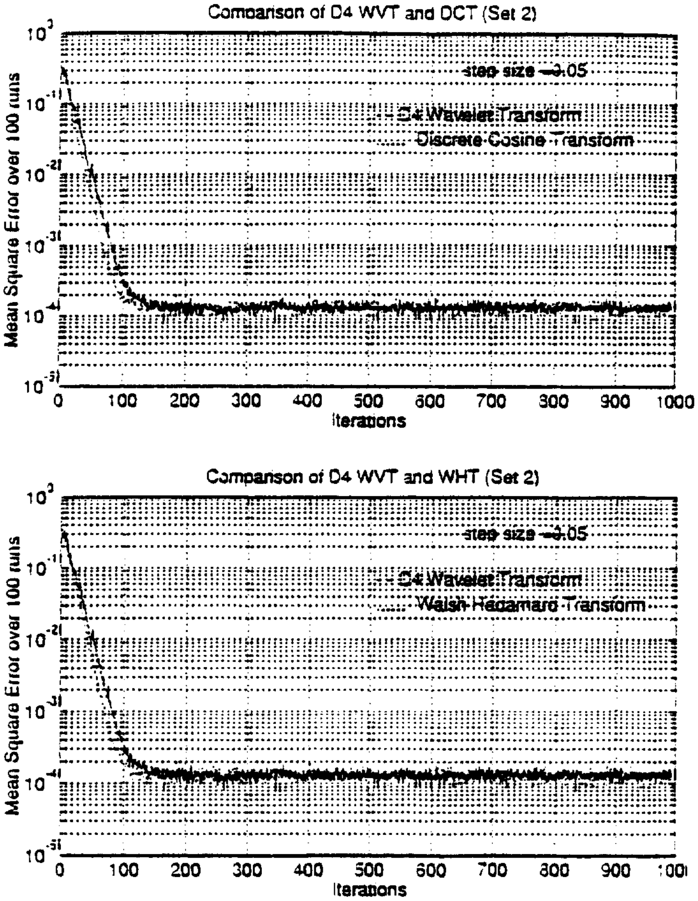

Figure 4: Comparison of D4 with DCT and WHT for Lowpass input

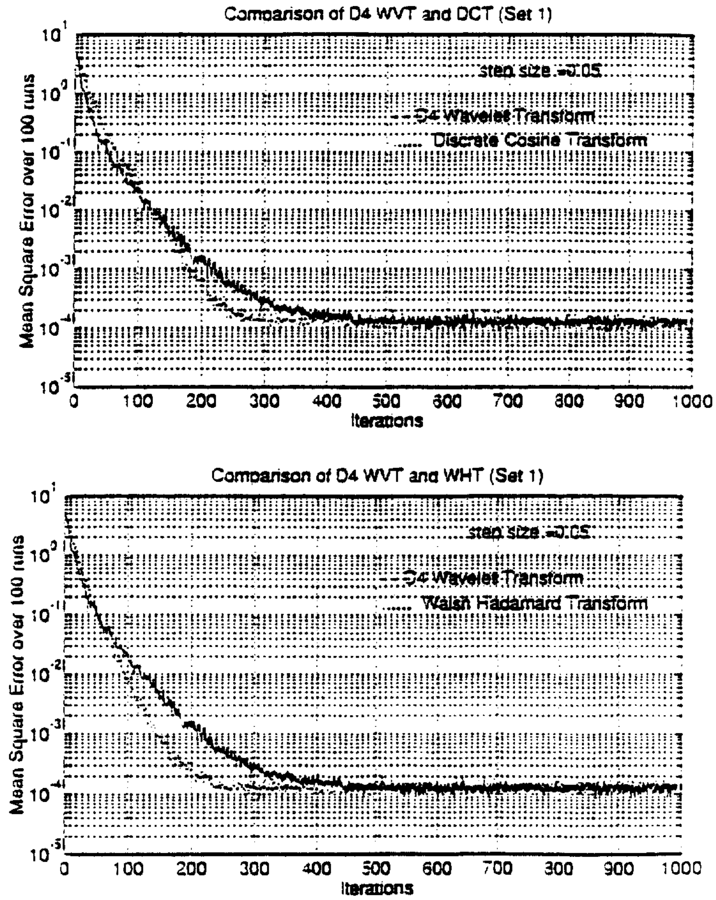

Figure 5: Comparison of D4 with DCT and WHT for Highpass input

gorithms perform differently for different situations.

WVTLMS is computationally efficient because the wavelet transform can be computed in $O(N)$

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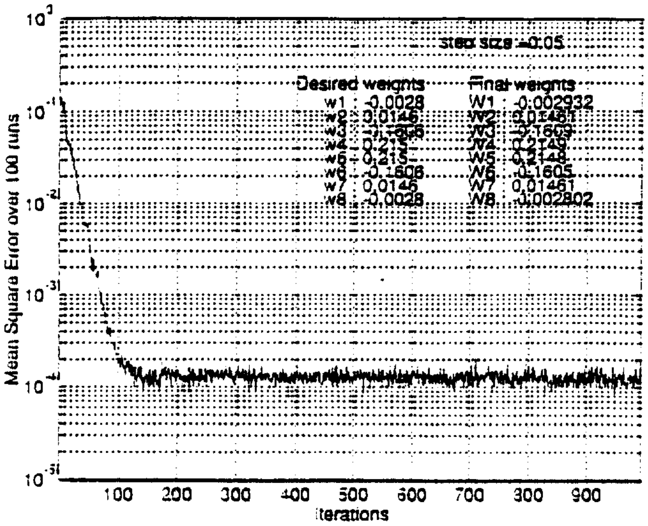

Figure 6: Comparison of D4 with DCT and WHT for Bandpass input

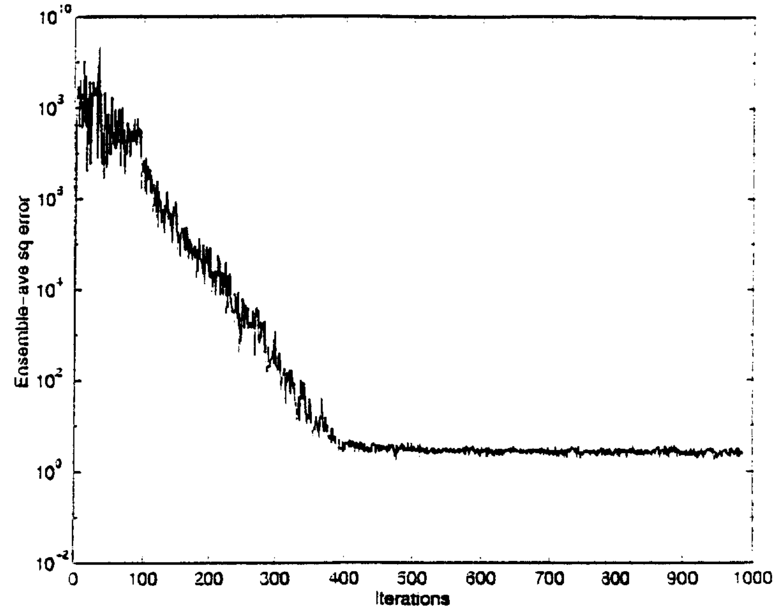

Figure 7: Channel equalization simulation result (LMS)

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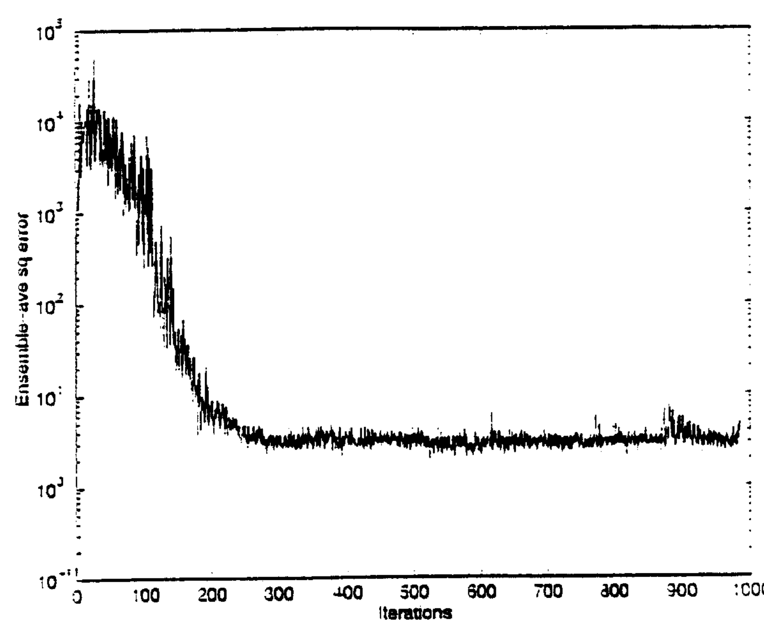

Figure 8: Channel equalization simulation result (WVTLMS)