Determination of Effective Bandwidth in ATM Networks

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Abstract

As part of connection admission control, it is necessary to estimate the aggregate capacity required for both new and exisiting connections. Two techniques of capacity estimation are summarized in this article: large deviations theory (Chernoff bound) and eigenvalue/eigenvector technique. The large deviations theory provides an estimation of the tail distribution for the link saturation probability. For a bufferless multiplexer, the saturation probability can be used as a measure of cell losses. With a buffer, an asymptotic behavior of the multiplexer can be derived for a large buffer size. Usually, the dominant eigenvalue plays a major role and the aggregate capacity of connections can be approximated from the dominant eigenvalue. Numerical results of both techniques are compared in terms of the largest number of connections for homogeneous on/off sources.

1. Introduction

An ATM network is a connection-oriented network and requires that a connection be established between users prior to data transfer. For a connection request, a user presents network with a traffic descriptor and desired QoS (quality of service) requirements. The traffic descriptor provides information about the expected traffic characteristics from the user. ATM Forum UNI 3.0, for example, specifies that the traffic contract be given in terms of the source traffic descriptor (e.g. peak cell rate, the sustainable cell rate, the burst tolerance) and the CDV (cell delay variation) tolerance [ATM 93]. Given the source traffic descriptor, the network invokes the connection admission control (CAC) algorithm in order to determine whether the new connection request can be granted without disrupting QoS of existing connections. When the requirement of the new connection request exceeds the available resources, the connection request is denied.

Estimation of required resources for a connection is not trivial and has been the focus of many studies. Some factors known to affect the resource requirements are peak and average cell rates, the burstiness (the ratio of peak to mean rate), burst lengths, and the ratio of peak rate to the link capacity [Wood 90]. Most recent developments of the admission control algorithm center around the real-time determination of the 'effective bandwidth.' The effective bandwidth is defined as a per-source quantity associated with the bandwidth required for meeting QoS of the connections.

The purpose of this paper is to review recent developments of analytic techniques related to approximation of the effective bandwidth. Statistical multiplexers without and with a buffer are considered in sections 2 and 3, respectively. In a bufferless multiplexer, the focus is on the estimation of the probability that the aggregate load exceeds the multiplexer capacity. In a model with a buffer, the focus is on the estimation of the buffer overflow probability. In section 4, five analytic techniques are compared in terms of the maximum number of connections that can be accommodated in a multiplexer. A few concluding remarks are provided in section 5.

2. Bufferless Multiplexer

Consider n identical sources sharing a multiplexer with the capacity C. Let *i.i.d. r.v.* Λ_i $(i=1,2,\ldots,n)$ denote the load offered by a source. The *m.g.f.*,the mean, and the variance of source traffic are denoted by $\Lambda(\theta)=E[e^{\theta\Lambda}]$, $\overline{\Lambda}$, and σ^2, respectively. The aggregate input load from n sources is denoted by S_n so that $S_n = \sum_{i=1}^{n} \Lambda_i$ with the *m.g.f.* of $E[e^{\theta S_n}] = [\Lambda(\theta)]^n$. When the multiplexer has no buffering capability, cells are lost as soon as the total input load exceeds C. The saturation probability is expressed by $P[S_n > C]$. The connection admission control policy can be designed so that

$$P[S_n > C] < \epsilon, \tag{2.1}$$

where ϵ denotes the desired cell loss probability.

2.1. Stationary Model

It is well known that the *p.d.f.* of a sum of independent *r.v.*'s can be approximated by a Gaussian distribution (according to the central limit theorem). Given n *i.i.d.* sources, the capacity C which satisfies the saturation probability ($P[S_n > C]$) can be estimated by [Guér 91]

$$C = n\Lambda + \sqrt{n}\sigma\sqrt{-2\ln\epsilon - \ln 2\pi}. \tag{2.2}$$

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2.2. Chernoff Bound

Since the Gaussian approximation is good only near the mean of the sum distribution, computation of the saturation probability using the Gaussian approximation is suboptimal. To estimate the saturation probability, which is several standard deviations away from the mean, other approximations exist such as the Chernoff bound and the large deviations theory. For any nonnegative *r.v.*,

$$E[Y] = \int_0^\infty y f(y) dy \geq \int_1^\infty f(y) dy = P[Y > 1]$$

It is easily established that $P[Y > 0] = P[e^{\theta Y} > 1] \leq E[e^{\theta Y}]$ for positive θ. A slightly more general version of the bound can be written as

$$P[Y > y] = P[(Y - y) > 0] \leq E[e^{\theta(Y-y)}].$$

The bound for the saturation probability is then given by $P[S_n > C] \leq E[e^{\theta(S_n - C)}] = [\Lambda(\theta)]^n / e^{\theta C}$. Taking logarithms on both sides, one obtains

$$\ln P[S_n > C] \leq -(\theta C - n \ln \Lambda(\theta)),$$

which is valid for all $n \geq 1$ and $\theta > 0$. A tighter bound is obtained by selecting θ so that

$$\ln P[S_n > C] \leq -\sup_{\theta > 0}(\theta C - n \ln \Lambda(\theta)),$$

which is call the Chernoff bound. Cramér's theorem shows that the Chernoff bound is tight for large n [Buck 90]. Define $I(y) = \sup_{\theta > 0}(\theta y - \ln \Lambda(\theta))$, called the Cramér rate function. For $C = ny$, the approximation for finite n such that

$$\ln P[S_n > ny] \simeq -nI(y),$$

is called the large deviations approximation of $P[S_n/n > y]$ and be considerd a conservative estimate of the saturation probability [Kell 91].

For θ^* which yields the supremum in the Cramér rate function, the admission criterion can be written as

$$\ln P[S_n > C] \leq -(\theta^* C - n \ln \Lambda(\theta^*)) < \ln \epsilon. \quad (2.3)$$

When rearranged into

$$n\gamma - \ln \epsilon / \theta^* < C, \quad (2.4)$$

$\gamma = \ln \Lambda(\theta^*)/\theta^*$ can be regarded as a per-source quantity satisfying the capacity requirement. Thus, γ is called the effective bandwidth [Kell 91, Gibb 91]. As will be seen later, other definitions of the effective bandwidth are possible. The concept of the effective bandwidth is attractive since real-time computation is possible and cicuit-switched connection admission methods can be used.

The effective bandwidth can be extended to the multiplexer with multiple classes of sources. With K classes of sources, an expression similar to (2.3) can be derived as

$$\ln P[S > C] \simeq -\sup_{\theta}(\theta C - \sum_{k=1}^{K} n_k \ln \Lambda_k(\theta)),$$

where the subscript k is used for statistics for class-k sources. The same derivation steps yield $\sum_{k=1}^{K} n_k \gamma_k - \ln \epsilon / \theta^* < C$, where $\gamma_k = \ln \Lambda_k(\theta^*)/\theta^*$ represents the effective bandwidth of a class-k source.

3. Buffered Multiplexer

Consider a multiplexer with the output capacity C and a buffer with size B. Cells are lost when the buffer becomes full. Let the buffer contents be denoted by a *r.v.* X. The measure of interest becomes $P[X > B]$ such that

$$P[X > B] < \epsilon, \quad (3.1)$$

where ϵ again represents the quality of service in terms of the cell loss probability. Since many studies of the effective bandwidth are based on the exact analysis of the buffered multiplexer with Markov fluid sources, the analysis in [Anic 82] is summarized in the next subsection. The asymptotic values used for estimation of the effective bandwidth are derived in subsection 3.2, with some examples provided in 3.3.

3.1. Exact Fluid Flow Model

Consider a statistical multiplexer with an infinite-size buffer. Cell arrivals to the multiplexer are modeled by a Markov fluid source with $(n+1)$ states. In state i $(0 \leq i \leq n)$, cells are generated at a constant rate λ_i. The source stays in state i for an exponentially distributed period, and makes a transition to one of adjacent states ($(i-1)$ and $(i+1)$). The rate of transition from state i to state j $(j = i-1, i+1)$ is denoted by $\alpha_{i,j}$. When $\lambda_i = i\lambda$, this model can be used for a multiplexer with n *i.i.d.* on/off sources with appropriate state transition probabilities.

Suppose the multiplexer is in state i $(0 \leq i \leq n)$ at time t. From the Markovian transition, the probability that the state changes from i to $(i-1)$ in the interval δt is given by $\alpha_{i,i-1}\delta t$. Similarly, the state change to $(i+1)$ occurs with the probability of $\alpha_{i,i+1}\delta t$. Let $P_i(t, x)$ $(0 \leq i \leq n,\ t \geq 0,\ x \geq 0)$ denote the probability that at time t, the source is in state i and the buffer content does not exceed x. Ignoring terms with $O(\delta t^2)$ and higher, we obtain

$$P_i(t + \delta t, x) =$$

$$\alpha_{i-1,i}\delta t P_{i-1}(t, x) + \alpha_{i+1,i}\delta t P_{i+1}(t, x)$$

$$+[1 - (\alpha_{i,i-1} + \alpha_{i,i+1})\delta t] P_i(t, x - (\lambda_i - C)\delta t).$$

On the right hand side, the first two terms represent transition probabilities from states $(i-1)$ and $(i+1)$ to i, respectively. The last term represents the changes in buffer lengths in state i, when the input to and the output from the buffer are expressed as $\lambda_i \delta t$ and $C\delta t$, respectively. Dividing both sides by δt and letting δt approach 0, the following simplified expression is obtained after removing transient terms and time-dependent variables:

$$(\lambda_i - C)\frac{dP_i(x)}{dx} = \alpha_{i-1,i}P_{i-1}(x) - (\alpha_{i,i-1}+\alpha_{i,i+1})P_i(x) + \alpha_{i+1,i}P_{i+1}(x), \quad (3.2)$$

where $P_i(x)$ denotes the probability that the source is in state i and that the buffer content does not exceed x. $P_i(x) = 0$ when i is not in the defined range of $(0 \leq i \leq n)$.

Let $\vec{P}(x) = [P_0(x), \ldots, P_n(x)]^T$. The input rates at different states are written in either a matrix form as $\mathbf{\Lambda} = diag\{\lambda_0, \lambda_1, \ldots, \lambda_n\}$ or a vector notation as $\vec{\lambda} = [\lambda_0, \ldots, \lambda_n]$. Also, let $\mathbf{I}$ denote unit matrix. Eq. (3.2) can be written in a matrix form as

$$(\mathbf{\Lambda} - C\mathbf{I})\frac{d\vec{P}(x)}{dx} = \mathbf{M}\vec{P}(x), \qquad (x \geq 0) \quad (3.3)$$

where the (i,j)-th element of the matrix $\mathbf{M}$ is given by $-(\alpha_{i,i-1}+\alpha_{i,i+1})$, $\alpha_{i+1,i}$ and $\alpha_{i-1,i}$ for $j=i$, $j=i+1$, and $j=i-1$, respectively. All other elements are 0's.

Eq. (3.3) provides the general framework of the model. The i-th diagonal element of $(\mathbf{\Lambda} - C\mathbf{I})$ is given by $(\lambda_i - C)$ and represents the rate of change in the buffer content when the source is in state i. This rate of change is typically referred to as a drift in the context of a diffusion model.

It can be recognized that the framework is a linear first-order ordinary differential equation whose solution is known to have the form of

$$\vec{P}(x) = \sum_{i=0}^{n-\lfloor C \rfloor -1} a_i e^{z_i x}\vec{\phi}_i + \vec{\pi}, \quad (3.4)$$

for some constants a_i and $\vec{\pi}$. The constant vector $\vec{\pi}$ is the stationary probability vector such that $\vec{\pi} = \vec{P}(\infty)$. Hence, $\mathbf{M}\vec{\pi} = \vec{0}$ and $\vec{1}^T\vec{\pi} = 1$ ($\vec{1} = [1, \ldots, 1]$). Here, z_i's and $\vec{\phi}_i$'s are eigenvalues and corresponding eigenvectors of $(\mathbf{\Lambda} - C\mathbf{I})^{-1} \cdot \mathbf{M}$. The coefficients $\{a_i\}$ are obtained from a system of linear equations derived from the boundary condition. In fact, the solution above represents a spectral expansion (projection, decomposition) of $\vec{P}(x)$ in terms of eigenvectors.

The buffer overflow probability in a multiplexer with the buffer size B is approximated from the analysis above for an infinite-buffer model by $P[X > B]$. Since $P[X \leq B] = \vec{1}^T\vec{P}(B)$, the overflow probability can be written as $P[X > B] = 1 - P[X \leq B] = 1 - \vec{1}^T\vec{P}(B)$. Furthermore, using the fact that $1 = \vec{1}^T\vec{\pi} = \vec{1}^T\vec{P}(\infty)$, (3.4) can be rearranged into $\vec{P}(\infty) - \vec{P}(x)$, so that

$$P[X > B] = \sum_{i=0}^{n-\lfloor C \rfloor -1} a_i e^{z_i B}(\vec{1}^T\vec{\phi}_i). \quad (3.5)$$

This basic construct was formulated by Kosten for an infinite number of Markov fluid sources [Kost 74]. Anick *et al.* considered a finite number of sources and provided not only a thorough investigation of properties of eigenvalues but a set of formulae for eigenvalues and coefficients $\{a_i\}$ [Anic 82]. The model was renamed as UAS (Uniform Arrival and Service) model by Daigle and Langford [Daig 86], and the finite buffer case was studied by Tucker [Tuck 88]. Although exact expressions are available for eigenvalues and coefficients, the real-time application of the solution may be limited due to the computational complexity of (3.5).

3.2. Asymptotic Model

When a new connection request is made from a user, the user provides a traffic descriptor which best characterizes the expected traffic behavior. If one is to utilize the solution technique for the Markov fluid source above, one has to repeatedly solve for the differential equation (3.3) at different values of C in order to determine the smallest capacity that satisfies the buffer overflow probability. For example, the multiplexer capacity C may be approximated by a logarithmic interpolation technique after a few values are computed [Mont 91]. This process is time-consuming and may not be suitable for the admission control, which requires a real-time decision.

When capacity C in (3.3) is considered as a variable, eigenvalues can be regarded as functions of C and can be written as $z(C)$ [Elwa 93]. Thus, one now considers a problem of finding C for a given z. By writing $C = g(z)$, (3.3) is arranged into

$$g(z)\vec{P} = \mathbf{A}(z)\vec{P}, \quad (3.6)$$

where $\mathbf{A}(z) = \mathbf{\Lambda} - \mathbf{M}/z$. This shows that $g(z)$ is an eigenvalue of the matrix $\mathbf{A}(z)$ where z is a parameter. Eq. (3.6) becomes then another eigenvalue equation, which has the same degree of complexity as the original equation (3.3). Instead of solving for the eigenvalue equation, an asymptotic approximation may be used.

Let the eigenvalues be indexed in an decreasing order of the negative real parts,

$$0 > Re(z_0) > Re(z_1) > \ldots$$

Since the overflow probability is given as a sum of exponential terms with exponents of $z_i B$, it is dominated by the term with the eigenvalue z_0 when B is large. Thus, z_0 is called the dominant eigenvalue. The asymptotic expression for the overflow probability in (3.5) becomes

$$P[X > B] \rightarrow a_0 e^{z_0 B}(\vec{1}^T\vec{\phi}_0), \quad (3.7)$$

which is directly from the exact analysis by Anick *et al.*

By assuming that $a_0 \simeq 1$, a further simplication can be made such that $P[X > B] \simeq e^{z_0 B} < \epsilon$. From the inequality, the dominant eigenvalue z_0 is approximated as

$$z_0 \simeq \ln \epsilon / B. \tag{3.8}$$

Given the approximated dominant eigenvalue, z_0, the eigenvalue of $\mathbf{A}(z_0)$ in (3.6), $g(z_0)$, can be computed. which is the desired effective bandwidth for n sources (*i.e.*, $C = g(z_0)$). Note that the effective bandwidth defined here is different from the one in subsection 2.2. It has to be pointed out, however, that the assumption that a_0 is near the unity may not be valid for some arrival processes. In fact, values of a_0 may be considerably different from unity, as reported in [Chou 94].

4. Example with On/Off Sources

In order to determine the effective bandwidth, the *m.g.f.* of the input process has to be characterized. We only consider on/off sources, which alternate between *on* and *off* periods. The lengths of the time that a source stays in *on* and *off* periods are exponentially distributed with the means of $1/\alpha$ and $1/\beta$, respectively. In an *on* period, unit capacity is consumed by the source. No load is imposed when in an *off* period. The *m.g.f.* of the arrival process is then given by $\Lambda(\theta) = E[e^{\theta \Lambda}] = (\beta + \alpha e^{\theta})/(\alpha + \beta)$. The value of θ^* which produces the supremum of the rate function is obtained by setting the first order derivative to 0: $\partial(\theta C - n \ln \Lambda(\theta))/\partial\theta = 0$. This results in

$$\theta^* = \ln[\beta C / \alpha(n - C)]. \tag{4.1}$$

Substitution into (2.4) yields the effective bandwidth.

A source behavior can be represented by a generator matrix and a load vector as

$$\mathbf{M} = \begin{bmatrix} -\alpha & \beta \\ \alpha & -\beta \end{bmatrix}$$

and $\vec{\lambda} = [0 \ \ 1]$, respectively. From (3.8), let $\eta = \ln \epsilon / B$. The effective bandwidth γ then becomes $g_0(\eta)$, which is the eigenvalue of the matrix $(\mathbf{\Lambda} - \mathbf{M}/\eta)$ with the largest negative real part. From $\mathbf{M}$ and $\mathbf{\Lambda}$ above, $g_0(\eta)$ can be evaluated as

$$\gamma = g_0(\eta) = \left[a(\eta) - \sqrt{a^2(\eta) - 4b(\eta)}\right] / 2\eta, \tag{4.2}$$

where $a(z) = z + \alpha + \beta$ and $b(z) = \alpha z$.

Numerical results of five techniques are compared for homogeneous n on/off sources. From the bufferless model, the aggregate bandwidth obtained from the Gaussian stationary approximation in (2.2) and the effective bandwidth by the large deviations technique in (2.4) are considered. Three techniques from the buffered model are the exact analysis of the the Markov fluid source in (3.5), its asymptotic result in (3.7), and the effective bandwidth from the eigenvalues in (3.9). (When numerical results are plotted in figures later, they are labeled as Stationary, Kelly, Anick Exact, Anick Asymptotic, and Elwalid, respectively.) All numerical evaluations are performed for the cell loss probability of $\epsilon = e^{-20} = 2.06 \times 10^{-9}$. Also, the average *on* period is set to unity ($\beta = 1$). The smaller the value of α is, therefore, the less frequently such bursts will occur. Consequently, the input load at a smaller value of α is lower and can result in more sources to be multiplexed.

At a given capacity C, the maximum number of connections which can be permitted without exceeding the desired cell loss probability can be evaluated at various buffer sizes. They are plotted in Figures 1 to 3 for different source traffic, characterized by different values of α. As pointed out above, the maximum number of connections is larger at smaller value of α. Also, the maximum number of connections from the bufferless model (stationary and Kelly's) are independent of buffer sizes, as expected. Since the stationary approximation is based on the average load from a source, it is expected to overestimate the number of connections. At $\alpha = 1$ and $C = 16.66$, the number of connections according to the stationary approximation is actually smaller than those from other techniques. This is from the fact that the number of connections (11) is small to allow for sufficient statistical multiplexing effect for the approximation to be valid [Guér 91].

Three results from the buffered model converge to each other near a relatively small buffer size of 100. Note that the buffer size here is given in units of the average *on* period ($\beta = 1$). Depending on the number of cells expected in an *on* period, the buffer size in units of cells can be significant. When the number of connections from some buffered models is smaller than the one from the stationary model at a small buffer size, it eventually becomes larger as the buffer size increases. Compared to the exact result, Elwalid's asymptotic approximation provides a lower bound of the number of connections, whereas the asymptotic result from Anick shows an upper bound. Recall that the Elwalid's approach derives the effective bandwidth of a single source by approximating the dominant eigenvalue by $z_0 = \ln \epsilon / B$. Namely, the approximation of the effective bandwidth is made as if the single source alone is able to use the entire buffer size B without considering the sharing of the buffer among many sources. Still, the Elwalid's approximation results in fewer connections than the exact result.

5. Conclusions

The effective bandwidth is shown to be affected by variance of burst and silence lengths in addition to the conventional traffic parameters (such as the peak rate and the burst lengths) [Sohr 92]. Furthermore, the bandwidth required for a new connection is known to be dependent upon the ratio of the peak rate to the link capacity. Clearly, if the peak rate approaches the link capacity, bandwidth close to the peak rate should be allocated almost independent of the average rate [Wood 90]. This is due to the fact that the statistical multiplexing cannot be expected at the peak rate close to the link capacity. Despite recent advances in approximations of the effective bandwidth, more studies are necessary to deal with realistic ATM environments.

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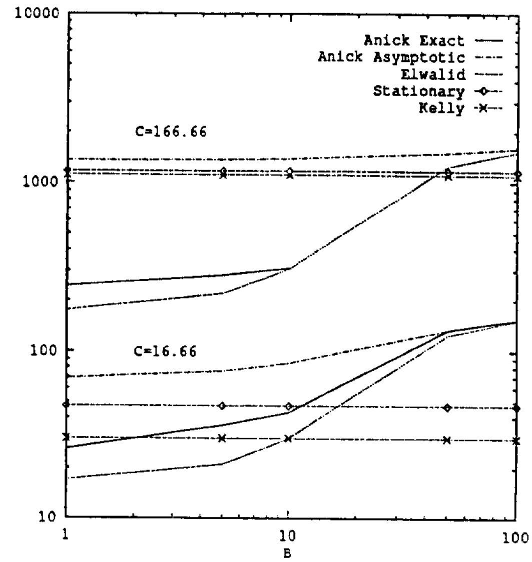

Figure 1: The maximum number of connections at $\alpha = 0.1$

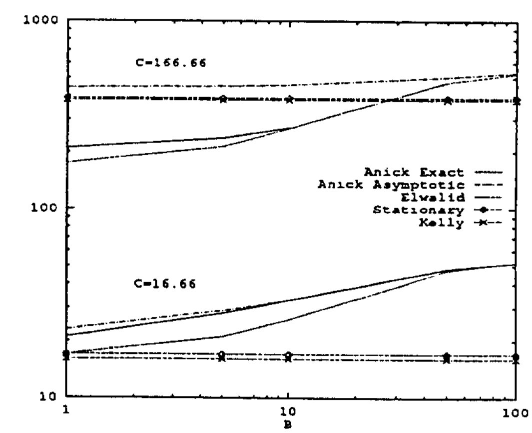

Figure 2: The maximum number of connections at $\alpha = 0.4$

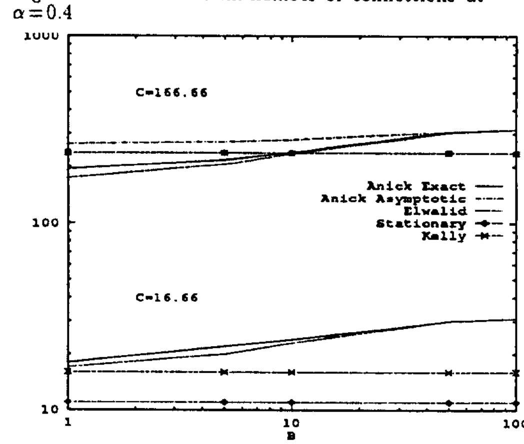

Figure 3: The maximum number of connections at $\alpha = 1.0$