Perceptual Wiener Filtering for Image Restoration

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Abstract

We propose a perceptual Wiener filter for image restoration, a linear space-variant filter which accounts for the human visual system response to image details and noise in the vicinity of an edge. This filter provides a reduction in the ringing artifact observed in the vicinity of edges, when compared to the response of the widely used linear space-invariant classical Wiener filter. A fast, approximate implementation of the filter is discussed.

1 Introduction

Image restoration is the process of recovering an original image from a degraded version. The degradation is often due to the characteristics of an image formation system, producing blurring distortion and additive noise. This is modeled as the convolution of the original image or scene $u(i,j)$ with a blurring point spread function $d(i,j)$, followed by the addition of noise $n(i,j)$ to produce the degraded image $z(i,j)$

$$z(i,j) = u(i,j) * d(i,j) + n(i,j) \qquad (1)$$

The noise $n(i,j)$ is often assumed to have a white Gaussian distribution with zero mean and variance σ_N^2. The blurring kernel has frequency response given by $D(f_1, f_2)$.

Methods for the restoration of degraded images include inverse (or pseudo-inverse) filtering and Wiener filtering. Due to the presence of the noise, inverse or pseudo-inverse filtering may cause noise amplification which can obscure the desired image information. It is well known that this noise amplification can be avoided to a great extent by using a Wiener filter. However, Wiener filters are known to introduce other artifacts, such as ringing in response to image edges [1], which are objectionable to human observers. In the past decade, efforts have been devoted to the development of alternative methods which do not introduce these artifacts. Recursive Kalman filter approaches [1, 2] and an iterative Miller regularization-based technique [3] are quite successful in suppressing ringing and preserving edges while providing greater noise reduction in nonedge regions. However, their heavy computational requirements makes them impractical for real time applications. An adaptive approach to Wiener filtering [4] suppresses noise with decreased edge ringing, but does not address the problem of blur removal.

In this paper, we incorporate properties of human visual perception in the development of a Wiener filter based approach and show that the ringing artifacts caused by traditional Wiener filtering can be suppressed. The advantage of our Wiener filter based approach in comparison with the adaptive Kalman filter and iterative approaches is in its greatly reduced computational requirements and potential for parallel implementation. These merits make it a good candidate for real time applications.

2 Perceptual Wiener Filter

Masking is the well-known property of human perception in which the relative visibility of image details and noise decreases in the vicinity of an edge. Human vision is quite sensitive to noise in a flat region, but is able to tolerate a large amount of noise near an edge. The noise visibility increases as the distance from the edge increases. Anderson and Netravali [5] incorporated this property in their approach to the restoration of noisy images, but this approach does not account for the blurring due to the image formation system. We use a similar approach for our perceptual Wiener filter design, but we account for both the blurring distortion and the additive noise.

The "masking function" M_{ij} at coordinate (i,j) is

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defined [5] as a local measure of spatial detail

$$M_{ij} = \sum_{p=i-k}^{i+k} \sum_{q=j-l}^{j+l} C^{\sqrt{(i-p)^2+(j-q)^2}} \left[|m_{pq}^H| + |m_{pq}^V|\right] \quad (2)$$

where m_{pq}^H and m_{pq}^V are the vertical and horizontal slopes of the image intensity at (p,q), and C is a constant controlling the rate of exponential decay of the masking effect.

The relationship between the visibility of noise and the masking value has been studied experimentally [5, 6] and has been modeled as

$$\begin{aligned} V_{ij} &= V(M_{ij}) \\ &= \frac{\beta}{\alpha M_{ij} + 1} \end{aligned} \quad (3)$$

where α and β are tuning parameters which must be adjusted experimentally for each class of images.

We use the visibility function of (3) in our perceptual approach to the design of a Wiener filter. The frequency response of the classical Wiener filter is given by

$$W(f_1, f_2) = \frac{D(f_1,f_2)S_{uu}(f_1,f_2)}{D(f_1,f_2)D^*(f_1,f_2)S_{uu}(f_1,f_2) + \sigma_N^2} \quad (4)$$

where $S_{uu}(f_1, f_2)$ is the power spectral density of the original image. We alter the effect of the noise variance by multiplying it by the local visibility V_{ij}, producing the frequency response

$$Pij(f_1, f_2) = \frac{D(f_1,f_2)S_{uu}(f_1,f_2)}{D(f_1,f_2)D^*(f_1,f_2)S_{uu}(f_1,f_2) + V_{ij}\sigma_N^2} \quad (5)$$

In areas of no spatial activity, i.e., where M_{ij} is zero and V_{ij} is equal to β, the noise variance component of the modified frequency response is $\beta\sigma_N^2$. In such a region, human vision is most sensitive to noise, and therefore, as much noise as possible is removed. In a region of high spatial activity where V_{ij} is small, the equivalent noise variance in the perceptual Wiener filter, $V_{ij}\sigma_N^2$, is also relatively small, and therefore the filter closely approximates an inverse filter, preserving image structure.

3 Implementation

Theoretically, the perceptual Wiener filter has an infinite impulse response. However, the effective response of these filters is often much smaller. Therefore, an approximate FIR filter can often achieve the performance of an IIR filter but with lower computational complexity. There are several methods for determining the approximate FIR filter, including methods described in [7].

We use a method for obtaining an FIR filter similar to that introduced in [8], by minimizing the criterion function

$$I_2 = \int_{-0.5}^{+0.5} \int_{-0.5}^{+0.5} [H(f_1,f_2) - P(f_1,f_2)]^2 df_1 df_2 \quad (6)$$

where $P(f_1, f_2)$ is the desired perceptual Wiener filter (PWF) frequency response and $H(f_1, f_2)$ is the frequency response of the FIR filter approximation of the PWF. In general, a two dimensional solution to this minimization problem is required. However, when $P(f_1, f_2)$ has radial symmetry, a one dimensional FIR filter can be found more easily by minimizing the simplified one dimensional criterion

$$I_1 = \int_{-0.5}^{+0.5} [H_1(f) - P(f,0)]^2 df \quad (7)$$

The desired two dimensional filter can then be generated from the one dimensional result using the McClellan transform.

Rewriting (7) in the discrete domain and assuming that the FIR filter has $2N+1$ samples, we have

$$\begin{aligned} I_1 &= \int_{-0.5}^{0.5} \left[\sum_{n=-N}^{N} h(n)e^{-j2\pi fn} - P(f,0)\right]^2 df \\ &= \int_{-0.5}^{0.5} \left[\sum_{n=0}^{N} 2\tilde{h}(n)cos(2\pi fn) - P(f,0)\right]^2 df \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{h}(i)\tilde{h}(j)a_{ij} + \sum_{i=1,odd}^{N} \tilde{h}(i)b_i + C \\ &= \tilde{H}^T A \tilde{H} - B^T \tilde{H} + C \end{aligned} \quad (8)$$

where

$$\tilde{h}(i) = \begin{cases} 0.5 & i = 0 \\ h(i) & \text{otherwise} \end{cases}$$

$$a_{ij} = 4\int_{-0.5}^{0.5} cos(2\pi fi)cos(2\pi fj)df \quad (9)$$

$$A = [a_{ij}] \quad i,j = 1,3,\ldots,N \quad (10)$$

$$b_i = 4\int_{-0.5}^{0.5} cos(2\pi fi)P(f,0)df \quad (11)$$

$$B^T = [b_1, b_2, \ldots, b_N] \quad (12)$$

$$C = \int_{-0.5}^{0.5} [P(f,0)]^2 df \quad (13)$$

The necessary condition for $\tilde{h}(i)$ to be an optimal solution is

$$\frac{\partial I_2}{\partial \tilde{h}(i)} = 0, \quad i = 0, 1, \ldots, N \tag{14}$$

Taking partial derivatives, we obtain

$$\frac{\partial I_2}{\partial \tilde{H}} = 2A\tilde{H} - B \tag{15}$$

The necessary condition becomes

$$\tilde{H} = [2A]^{-1}B \tag{16}$$

4 Signal Equivalent Approach

A fast, approximate, implementation of the space-variant perceptual Wiener filter can be achieved by using signal equivalent approach proposed in [4]. In this approach, the space-variant filter is approximated by a space-varying weighted sum of two fixed filters

$$Pij(f_1, f_2) = f_{ij}P1(f_1, f_2) + (1 - f_{ij})P2(f_1, f_2) \tag{17}$$

where

$$f_{ij} = \frac{1}{\alpha M_{ij} + 1} \tag{18}$$

$$P1(f_1, f_2) = \frac{D(f_1, f_2)S_{uu}(f_1, f_2)}{D(f_1, f_2)D^*(f_1, f_2)S_{uu}(f_1, f_2) + \beta\sigma_N^2} \tag{19}$$

$$P2(f_1, f_2) = \frac{D(f_1, f_2)S_{uu}(f_1, f_2)}{D(f_1, f_2)D^*(f_1, f_2)S_{uu}(f_1, f_2) + V_m\sigma_N^2} \tag{20}$$

and V_m is the minimum visibility, corresponding to the maximum value of masking M_{ij}. In areas of low spatial activity, f_{ij} approaches one and the noise reducing filter of (19) dominates in (17). In areas of high spatial activity, f_{ij} approaches zero, visibility approaches V_m and the approximate inverse filter of (20) dominates in (17). Thus, the filter chosen is a compromise between the two extreme cases (minimum and maximum visibility) for the space-varying filter of (5).

With this approach, FIR approximations are only needed for the two filters of (19) and (20). The space-varying weighted sum of (17) is applied to combine the two filter outputs.

5 High Quality Masking Function

Since the approximate PWF is controlled by the masking function M_{ij}, the performance of the filter is highly dependent on the quality of this measure of spatial detail. In a noisy and blurred image, it is difficult to obtain a good estimate of spatial detail. Particularly when the noise level is high, the masking function M_{ij} mistakes noise as an active region in the image, leading to very poor restoration. Local averaging was used in [2] and [3] to overcome this problem. However, this leads to blurring of high frequency features and these features then do not contribute as they should to the measure of spatial detail.

In our work, anisotropic diffusion [9] and multiresolution adaptive image smoothing [10] techniques are applied to reduce noise while maintaining structure in the noisy and blurred image. Anisotropic diffusion has the adaptive characteristic that it smoothes the noise in flat regions of an image, while preserving or even sharpening edges. Therefore, anisotropic diffusion provides a better preprocessor than local averaging in estimating spatial detail. Specifically, the mean curvature diffusion technique introduced in [9] was used for lower level noise cases (noise variance σ_N^2 less than 100). For higher level noise, the multiresolution adaptive image smoothing technique introduced in [10] was found to provide better performance.

Noisy, blurred images with noise variances of 100 and 200 are shown in Figures 1a and 1b, respectively. The image of Figure 1a processed with mean curvature diffusion is shown in Figure 2a and the image of Figure 1b processed with the multiresolution adaptive image smoothing technique is shown in Figure 2b. The masking function computed from Figure 2a is shown as an image in Figure 3a. The masking function corresponding to Figure 2b is shown in Figure 3b.

Figure 1: Noisy, blurred images. (a) Left: noise variance 100. (b) Right: noise variance 200

Figure 2: Processed images. (a) Left: 2a processed by mean curvature diffusion. (b) Right: 2b processed by multiresolution adaptive image smoothing method.

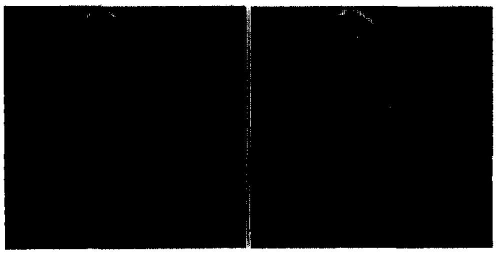

Figure 3: Masking function images. (a) Left: 2a after anisotropic diffusion. (b) Right: 2b after multiresolution adaptive image smoothing.

6 Experimental Result and Comparison

The perceptual Wiener filter has been tested by applying it to a noisy, blurred rendition of the camera man image. The original image in Figure 4(a) is blurred using a Gaussian defocus blurring kernel and then subjected to additive white Gaussian noise with zero mean and variance 400. The variance of the defocus blur is 4.0 and the size of the blurring kernel is 21×21. The blurred, noisy image in Figure 4(b) restored with the perceptual Wiener filter as equation (5) is shown in Figure 4(d). For comparison, the result from the classical Wiener filter is shown in Figure 4(c). Note that both the sharpness is improved and the noise is reduced in the PWF result.

We have also demonstrated the results of the signal equivalent approach. The original image in Figure 5(a) is blurred by same kind of blurring as Figure 4(b) and subjected to additive white Gaussian noise with zero mean and variance 4. The blurred, noisy image is shown in Figure 5(b). The image restored with the signal equivalent approach is shown in Figure 5(d). For comparison, the result from the classical Wiener filter is shown in Figure 5(c). Although the signal equivalent approach can not achieve the performance of the formal PWF, it still clearly outperforms the classical Wiener filter.

Figure 4: (a) Top left: original image. (b) Top right: noisy and blurred image. (c) Bottom left: restored with classical Wiener filter. (d) Bottom right: restored by PWF

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Figure 5: (a) Top left: original image. (b) Top right: noisy and blurred image. (c) Bottom left: restored with classical Wiener filter. (d) Bottom right: restored by PWF

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