Quadratic Filters for Image Contrast Enhancement

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Abstract

The classical unsharp masking method of image contrast enhancement is based on the addition of an amplitude scaled linear highpass filtered version of the image to itself. However, the highpass filter also enhances noise components present in the image. Some improvement in the quality of the enhancement has been obtained using a fairly simple quadratic filter in place of the linear highpass filter. In this paper, we first explain why the linear unsharp masking performs well for contrast enhancement of noise free images. A further modification to make the overall approach considerably less sensitive to noise is then proposed. A general class of quadratic filters utilizing Weber's law is also introduced and their important properties are discussed. The modified unsharp masking scheme together with the proposed class of quadratic filters is shown to provide significant improvement in the quality of enhanced images compared to that obtained using similar approaches.

1 Introduction

Image enhancement seeks to improve the visual quality of images. However, an inherent difficulty is to define a mathematical criterion for visual quality. As a result, many algorithms remain to a large extent empirical and a final assessment can only be performed by the human observer.

In this paper we consider image *contrast* enhancement based on the unsharp masking method. This classical scheme is conceptually simple and yields pleasant results by utilizing an effect called *simultaneous contrast.* Simultaneous contrast describes the visual phenomenon that the difference in the *perceived* brightness of neighboring regions depends on the sharpness of the transition.

Unsharp masking is implemented by adding back to an image a scaled signal which primarily contains high frequency (edge) information of the original. Traditionally, linear high-pass filters have been used to extract the edges. While these filters are capable of detecting strong contours they fall short of picking up details. Moreover, they tend to amplify noise noise components present in the image without taking into consideration properties of the human visual system.

Recently Yu et. al [1] proposed quadratic filters which alleviated that problem by achieving excellent detail resolution. They extended Teager's algorithm originally proposed by Kaiser [2] into two dimensions. The resulting quadratic filters were further investigated by Mitra et al. [3] who showed that these filters perform similar to local-mean-weighted adaptive high- or bandpass filters, by assuming random inputs.

The nonlinear filter output depends on the local background brightness, and as a result, it follows another property of human vision called *Weber's law.* This quality makes the new filters very attractive for image enhancement. However, although their output is proportional to the local brightness average, some visible noise is still added in the unsharp masking process.

In Section 2 we suggest a further improvement of this image enhancement scheme which keeps the conceptual advantage of unsharp masking based on the superior properties of quadratic filters while being less sensitive to noise. In the following section we demonstrate that the 2-D quadratic operators proposed in [3] fall into a general filter class which performs like linear mean-weighted high- or bandpass filters. In Section 4 we relate these filters to the gradient operators. Section 5 provides some filter examples and some enhancement results are provided. Concluding remarks are included in Section 6.

2 Conventional Unsharp Masking and Its Modifications

Unsharp masking for edge enhancement, commonly used in the printing industry, is equivalent to adding back the scaled gradient magnitude to the original signal. Mathematically, unsharp masking can be formulated as

$$x_e[m,n] = x[m,n] + \gamma \nabla x[m,n]. \quad (1)$$

where $x_e[.]$ denotes the enhanced image; $x[.]$ represents the original input image; $\nabla x[.]$ refers to the gradient of $x[.]$, and γ is the enhancement factor. A commonly used gradient function is the discrete Laplacian operator.

Our proposed modification is based on replacing the gradient operator $\nabla x[.]$ in Eq. (1) by an *enhancement fraction* $\Delta x[m,n]$, derived from quadratic filters where

$$\Delta x[m,n] := f\left(\frac{y[m,n]}{\max(|y(m,n)|)}\right) \times x[m,n] \quad (2)$$

with $y[m,n]$ denoting the output of the quadratic filter. An example of the quadratic is the Type 1B quadratic filter defined by [3]

$$\begin{aligned} y[m,n] \;=\; & 2x^2[m,n] \\ & - x[m-1,n+1]x[m+1,n-1] \\ & - x[m-1,n-1]x[m+1,n+1]. \quad (3) \end{aligned}$$

As the filter response $y[m,n]$ in Eqs. (2) and (3) can take both positive and negative values, the function $\max(|y[m,n]|)$ selects the maximum absolute value. The (normalized) quotient $y[m,n]/\max(|y(m,n)|)$ is referred to as the *enhancement map*, and the overall product results in an enhancement fraction whose magnitude is bounded by the maximum gray level of the display device. The proposed block diagram of the proposed modification is shown in Fig. 1.

Thus, the basic idea behind the modified unsharp masking method is to weigh large changes in adjacent pixel values more strongly than small image changes. In fact, it is equivalent to traditional unsharp masking of Eq. (1) with an enhancement factor γ which is no longer a constant, but varies with each pixel location.

Note that the division in Eq. (2) could be implemented with little hardware requirements. If we slightly modify our original scheme and choose $\max(|y[m,n]|)$ as a power of two, then division merely becomes a shift operation. In the next step we can correct the mismatch by multiplying Eq. (2) with an appropriate scaling factor which can be incorporated into the enhancement factor γ.

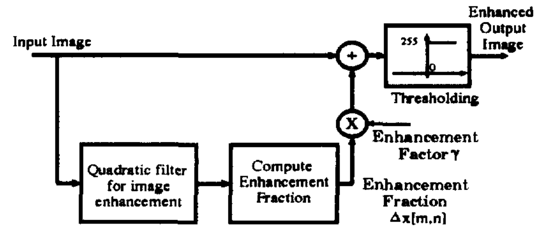

Figure 1: The block diagram depicts the proposed modification of the unsharp masking technique.

The modified unsharp masking technique enhances edges proportional to their original character, i.e., strong edges are more emphasized than weaker borders. Furthermore, it proves to be robust to input images with very different characteristics, i.e, it is not necessary to tune the enhancement factor γ precisely to each input image to achieve a pleasant enhancement effect. The final thresholding operation reduces distortions around high contrast edges without affecting the dynamic range of the original image. High (step) edges are only slightly enhanced to avoid overshoots. Small and medium intensity changes, on the other hand, are additionally outlined proportionally to their original heights.

The proposed scheme is thus capable of producing subtle and pleasant image enhancement, if appropriate filter operations are employed in the feed-forward path as described in the next section.

3 Quadratic Filters for Image Enhancement

A useful class of nonlinear filters for image enhancement can be defined by [5]

$$\begin{aligned} y[m,n] \;=\; & \sum_{i=-M}^{M} \sum_{j=-M}^{M} h[i,j,-i,-j] \\ & \times x[m-i,n-j]x[m+i,n+j], \quad (4) \end{aligned}$$

with a kernel condition

$$\sum_{i=-M}^{M} \sum_{j=-M}^{M} h[i,j,-i,-j] = 0. \quad (5)$$

The filters defined by Eqs. (4) and (5) yield constant outputs for sinusoids of arbitrary frequency. This can be shown by taking a 2-D sinusoidal input which when applied to Eq. (4) and after making use of Eq. (5) results in

$$y[m,n] = \frac{A^2}{2} \sum_{m_1=-M}^{M} \sum_{n_1=-M}^{M} h[m_1, n_1, -m_1, -n_1] \times \cos(4\pi(m_1 f_1 + n_1 f_2)).$$

The sums for computing $y[m,n]$ are independent of pixel locations m, n, thus resulting in a constant output.

It has also been shown in [5] that with an input of the form

$$x[m,n] = A \sin(\omega(m+n))$$

the resulting output $y[m,n]$ is also constant. This value is even proportional to the square values of (discrete) frequency ω and amplitude A, if ω is small.

More importantly, under certain conditions, quadratic operators defined by Eq. (4), can be approximated by linear mean-weighted high- or band-pass filters.

We arrive at the linear mean-weighted approximation assuming input signals $x[m,n]$ which can be expressed as a sum of local mean $\mu_x[m,n]$ (dc offset) and a slowly alternating (ac) part $x_1[m,n]$.

More mathematically, after substituting $x[m,n] = \mu_x[m,n] + x_1[m,n]$ into Eq. (4), simple calculations yield:

$$\begin{aligned} y[m,n] &= \sum_{i=-M}^{M} \sum_{j=-M}^{M} h[i,j,-i,-j] \\ &\quad \times x_1[m-i, n-j] x_1[m+i, n+j] \\ &\quad + \mu_x[m,n] \sum_{i=-M}^{M} \sum_{j=-M}^{M} h[i,j,-i,-j] \\ &\quad \times (x_1[m-i,n-j] + x_1[m+i,n+j]) \end{aligned}$$

From this and Eq. (??) we conclude that for slowly varying input signals with magnitude significantly lower than the local mean $\mu_x[m,n]$, the first (nonlinear) term can be expected to be much smaller than the second describing a linear filtering operation. This assumption is based on properly designed quadratic filters.

As a result, we can denote filters belonging to the class of Eqs. (4) and (5) as mean-weighted high- or band-pass filters when above assumptions are met. Our underlying (coarse) image model assumes images composed of smooth areas with slightly varying reflectances under uniform lighting conditions. In these regions the proposed filter class responds to small surface changes with outputs proportional to the local mean brightness. At locations where different image areas meet, i.e., at edges, large output values result as explained in the following section.

4 Quadratic Filters and Gradient Techniques

We show now that filters defined by Eqs. (4) and (5) when used to detect edges in an image, lead to results resembling those of (squared) gradients. In fact, gradient and quadratic operators are analytically related. To demonstrate this, we start with the 1-D Teager algorithm given by Kaiser [2]

$$y[n] = x^2[n] - x[n+1]x[n-1]. \tag{6}$$

Equation (6) is a one-dimensional example of Eq. (4). If we approximate the gradient of an input sequence $x[n]$ by

$$\nabla x[n] \approx x[n+1] - x[n-1], \tag{7}$$

and estimate the center pixel of Eq. (6) by the arithmetic mean of its neighbors as

$$x[n] \approx \frac{1}{2}(x[n+1] + x[n-1]),$$

then simple manipulations yield

$$(\nabla x[n])^2 \approx 4x^2[n] - 4x[n+1]x[n-1]. \tag{8}$$

Equation (8) reveals that the discrete 1-D approximation of Teager's algorithm performs similarly to a squared gradient. The squaring operation effectively emphasizes changes in the input sequence.

For the two-dimensional case, a similar relation between quadratic filters and gradient approximations can be derived. To illustrate this consider, for example, the extended Robert's Cross Operator [6] and compare its operations with that of the Type 1B quadratic filter of Eq. (3). Manipulations performed analogously to the one-dimensional case yield

$$\begin{aligned} (\nabla x[m,n])^2 &\approx 8x^2[m,n] \\ &\quad - 4x[m-1,n-1]x[m+1,n+1] \\ &\quad - 4x[m-1,n+1]x[m+1,n-1]. \end{aligned}$$

While the left hand side denotes a squared gradient approximation, the right hand side equals the Type 1B filter up to a multiplicative constant. This result shows that 2-D gradient operators and quadratic filters are indeed related.

5 Illustrative Examples

We compare the performances of three types of quadratic filters for image enhancement applications.

The first filter considered is the Type 1B quadratic filter proposed in of Eq. (3) defined by

$$\begin{aligned} y[m,n] &= 2x^2[m,n] \\ &\quad - x[m-1,n+1]x[m+1,n-1] \\ &\quad - x[m-1,n-1]x[m+1,n+1]. \quad (9) \end{aligned}$$

We compare its performance with a new quadratic filter

$$\begin{aligned} y[m,n] &= x[m,n-1]x[m,n+1] \\ &\quad + x[m-1,n]x[m+1,n] \\ &\quad - x[m-1,n-1]x[m+1,n+1] \\ &\quad - x[m-1,n+1]x[m+1,n-1]. \end{aligned}$$

The third filter considered was derived by [7] when optimizing with respect to isotropic edge responses:

$$\begin{aligned} y[m,n] &= 3x^2[m,n] \\ &\quad - \frac{1}{2}x[m+1,n+1]x[m-1,n-1] \\ &\quad - \frac{1}{2}x[m+1,n-1]x[m-1,n+1] \\ &\quad - x[m+1,n]x[m-1,n] \\ &\quad - x[m,n+1]x[m,n-1]. \end{aligned}$$

All operators have the structure of Eqs. (4) and (5). The first two filters yield similar enhancement effects in the absence of noise. However, when noise is present the second example proves to be more robust as the (squared) center pixel is not used for computing the output. Thurnhofer's filter approximates an isotropic squared gradient. Note that a first derivative operator (gradient) is in general not isotropic.

First, Fig. 2 shows the original eye region of "Womanhat". In Figs. 3 and 4 we compare the enhancement results obtained using a (linear) gradient with that obtained using Type based on the traditional unsharp masking scheme. Note that after addition of a scaled gradient the noticeable image enhancement still remains small. With the quadratic filter we observe enhanced edges along the brim of the hat but also note some noise in smooth regions. The inherent noise problem of the simple unsharp masking scheme can be alleviated by utilizing the proposed modified approach. In particular visually pleasing results were obtained when the square function was used as a point transformation in Eq. (2). Figures 5 and 6 show that we can retain image enhancement without introducing disturbing noise artifacts. The noticable difference between the Type 1B filter and the isotropic operator remains small due to the limited printer resolution. For superior detail enhancement, however, the latter is recomended.

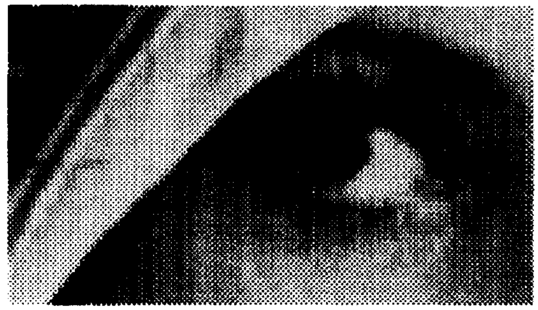

Figure 2: "Womanhat's" eye region. Note that the brim of her hat and the eye appear slightly blurred and could benefit from image enhancement.

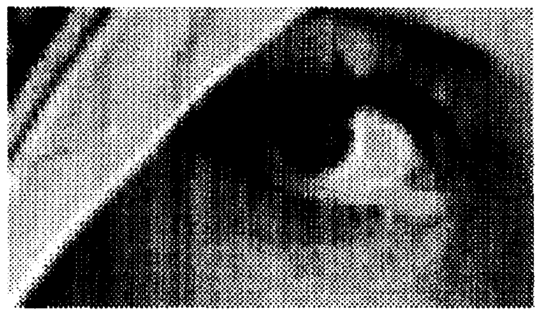

Figure 3: Enhanced and zoomed version of "Womanhat's" eye region after a scaled gradient output was added back to the original. The noticable difference to the original image is small.

6 Concluding Remarks

We introduced a modified version of unsharp masking which is flexible enough to accommodate different filter types in the feed-forward loop. By computing an enhancement map and final thresholding we enhance edges according to their original characteristics.

Furthermore, we suggested a particular class of quadratic filters. Due to their behavior which can be approximated by mean-weighted high or band-pass filtering and their gradient-like edge detection property, they are very good canditates for unsharp masking.

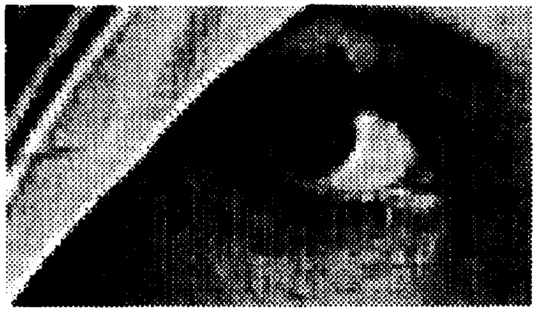

Figure 4: Result after a scaled Type 1B quadratic filter output was added. We see enhanced edges along the brim of the hat but also note some noise in smooth regions.

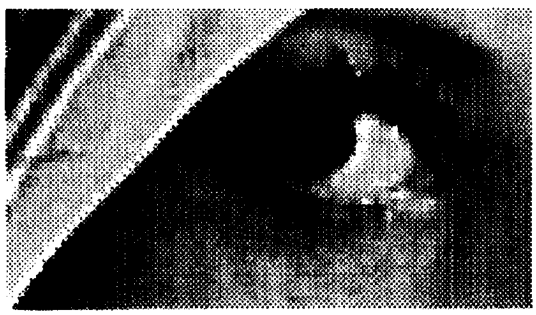

Figure 5: Image enhancement result when using the Type 1B filter combined with modified unsharp masking. Note that very little background noise has been introduced and a noticable enhancement result been kept.

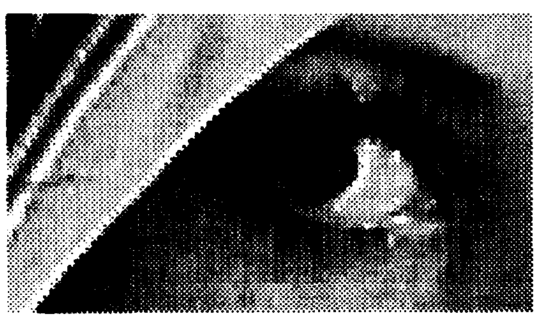

Figure 6: Image enhancement result when using the isotropic quadratic operator together with modified unsharp masking. Note that the isotropic operator enhances details superiorly.

Finally, combination of modifications have resulted in an uncomplicated and useful enhancement scheme.

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