Linear Multiuser Detectors for Quasi-Synchronous CDMA Systems *

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Abstract

A linear decorrelator detector is proposed for a quasi-synchronous *code-division multiple-access (QS-CDMA) cellular system. It is assumed that each of N users has a GPS generated local clock, and attempts to transmit in synchrony with the other users in its cell. The base station thus receives N signature waveforms which are approximately synchronized, but with a small timing error due to local oscillator drift and doppler shifts. The residual timing offset is discretized, and a decorrelator is constructed which attempts to reject the multiuser interference vectors corresponding to the set of possible offsets. Bounds and analytical results for the bit error rate (BER) are obtained for various interferer conditions.*

1 Introduction

Multiuser detection has received increasing attention since it was first shown capable of alleviating the near-far effect in CDMA communication systems [1]. However, the optimal multiuser detector of [1] requires knowledge of the signal times-of-arrival and amplitudes, which are in general difficult to estimate. Although various algorithms have been proposed for amplitude and time-delay estimation, in general these methods are quite complicated and not well suited to practical implementation. A much simpler multiuser detector was presented in [2], based on a coherent linear decorrelator, which did not require knowledge of signal amplitudes. However, the decorrelator method in [2] still required knowledge of the signature sequence arrival times.

We propose a quasi-synchronous CDMA cellular system, in which the mobile users attempt to transmit synchronously. It will be shown that QS-CDMA can then be implemented using a simple decorrelator receiver at the base station. Such systems have been considered in [3], for example, where conventional matched filter detection was employed, but the signature sequences were designed to have zero cross-correlation at small offsets. In contrast, we present a receiver design for QS-CDMA for arbitrary spreading codes, but which uses the decorrelation strategy to reject multiuser interference. The QS-CDMA system considered here is realized by providing every mobile user with a GPS receiver, with the user bit epochs triggered by the GPS clock waveform. At the base station (receiver), the incoming waveforms would still be asynchronous, due to oscillator drift and varying distances from the mobiles to the base. It will be shown that if the timing uncertainty occupies a sufficiently small region, it is possible to obtain excellent bit-error rate performance, without power control, using a relatively simple decorrelator detector.

The signal model and linear decorrelator for the QS-CDMA system are derived in Section 2. Bounds on the BER performance are obtained in Section 3. Results and conclusions are given in Sections 4 and 5, respectively.

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2 Derivation of the Decorrelator Detector

The signal model employed is similar to that in [4]. The n-th user transmits the waveform

$$s_n(t) = \sum_{m=-\infty}^{\infty} d_n(m) PN_n(t - mT), \qquad (1)$$

where $PN_n(t)$ is a random binary sequence formed by rectangular pulses of duration T_c sec., and amplitude $\{\pm 1\}$. All sequences $PN(t)$ are assumed to have common duration T sec., where $1/T$ is the bit rate. The information sequence is given by $\{d_n(m)\}$ for the n-th user, with binary signalling $d_n(m) \in \{-1, +1\}$

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assumed. We envision a digital implementation of the base station receiver, so that programmable correlators can be used to realize arbitrary decorrelator structures. Thus, a sampled signal model, as in [4] will be employed. The received multiuser signal is bandlimited to $1/T_c$ Hz., and then sampled at the Nyquist interval $T_s = T_c$ Hz, yielding the samples $r(mN_s + k)$ as defined below, during the m-th bit duration.

$$r(mN_s + k) = \sum_{n=1}^{N} a_n d_n(m) PN_n'(kT_s - T_n) + n(m), \tag{2}$$

for $k = 0, 1, \ldots, N_s - 1$, where $N_s = T/T_s$ is the number of Nyquist samples per bit. The waveform $PN_n'(t)$ represents the low-pass filtered version of the signature sequence. Initially, the amplitude $a_1 \in \mathcal{C}$ of user 1 (the desired user) is assumed known a-priori. However, the remaining amplitudes $a_n \in \mathcal{C}$, for $n = 2, 3, \ldots N$ are modeled as circular Gaussian with zero mean and variance J. The delays T_n are modeled as uniformly distributed in the interval $[-\alpha T, \alpha T]$, where $\alpha << 1$, as a consequence of the quasi-synchronous nature of the system. For example, in [5], a GPS-based QS-CDMA systems is analyzed, and it is shown that a maximum timing uncertainty of $\alpha T = 1.3 T_c$ can be achieved

To avoid having to explicitly estimate the delays, the following approximate model for the received sequence is proposed. First, the delays are discretized to values $T_n(i)$, for $i = 1, 2, \ldots, N_T$. Next, approximate the received sequence as

$$\begin{aligned} r(mN_s + k) &= d_1(m) a_1 PN_1'(kT_s - T_1) \\ &+ \sum_{i=1}^{N_T} \sum_{n=2}^{N} a_n(i, m) PN_n'(kT_s - T_n(i)) + n(k). \end{aligned} \tag{3}$$

The $a_n(i, m)$ are dependent random variables, since only one $a_n(i, m)$, per user n can be nonzero. However, it will be shown that by approximating the $a_n(i, m)$ as independent, zero-mean, and circular Gaussian, a linear decorrelator receiver can be obtained. The additive noise sequence $n(k)$ is likewise zero-mean circular Gaussian, with covariance $\sigma_n^2 = 2N_0/T_s$ [4].

It is assumed that the receiver is synchronized to user $n = 1$ (a method for accomplishing such synchronization is described in [5].) The received samples during bit m are written in vector form as

$$\begin{aligned} \mathbf{r}(m) &= d_1(m) a_1(m) \mathbf{s}_1(T_1) \\ &+ \sum_{i=1}^{N_T} \sum_{n=2}^{N} a_n(i, m) \mathbf{s}_n(T_n(i)) + \mathbf{n}(m), \end{aligned} \tag{4}$$

where $\mathbf{r}(m)$ represents the N_s samples of $r(k)$, and $\mathbf{s}_n(T_n(i))$ represents the low-pass filtered sequence samples $PN_n'(kT_s)$. The additive noise vector is circular Gaussian with covariance matrix $\sigma_n^2 \mathbf{I}$, and the sum of multiuser interference and thermal noise can be modelled by a single circular Gaussian vector $\mathbf{n}'(m)$. Under the approximation that the $a_n(i)$ are independent and circular Gaussian, with variance J, it is readily seen that the total covariance function is given by

$$\begin{aligned} \mathbf{R}(m) &= E\{\mathbf{n}'\mathbf{n}'^H\} \\ &= \sum_{i=1}^{N_T} \sum_{n=2}^{N} J \mathbf{s}_n(T_n(i)) \mathbf{s}_n(T_n(i))^H + \sigma_n^2 \mathbf{I}. \end{aligned} \tag{5}$$

Having defined the statistics of $\mathbf{r}(m)$, the likelihood function required for demodulation is seen to be circular Gaussian, and the resulting optimal detector computes the decision variable

$$U = Re\{\mathbf{r}(m)^H \mathbf{R}(m)^{-1} e^{iarg\{a_1\}} \mathbf{s}_1(T_1)\} \underset{-1}{\overset{+1}{\gtrless}} 0. \tag{6}$$

Note that $\mathbf{R}(m)$ is positive definite and hence invertible as long as $\sigma_n^2 > 0$. The above detector is coherent, in that the phase of a_1 must be estimated – a noncoherent version of the detector is presented in [5].

In order to simplify the detector structure, and facilitate the development of a BER bound, the inverse covariance matrix is approximated by an orthogonal projection matrix. As shown in [5], as the interferer power $J \to \infty$, the inverse covariance approaches

$$\begin{aligned} \mathbf{R}(m)^{-1} &\to \\ & \frac{1}{\sigma_n^2} \left[\mathbf{I} - \mathbf{S}_1' [\mathbf{S}_1'^H \mathbf{S}_1']^{-1} \mathbf{S}_1'^H \right] \\ &= \frac{1}{\sigma_n^2} \left[\mathbf{I} - \mathbf{P}_{S_1'} \right]. \end{aligned} \tag{7}$$

In the following, it is assumed that the signal vectors $\mathbf{s}_n(T(i))$ are linearly independent, so that the matrix $\mathbf{S}_1'^H \mathbf{S}_1'$ is invertible. The matrix $[\mathbf{I} - \mathbf{P}_{S_1'}]$ now spans a subspace including the vector $\mathbf{s}_1(T_1)$, but orthogonal to the multiuser interference.

The coherent multiuser detector can now be written as follows, using the orthogonal projection matrix.

$$U = Re\{\mathbf{r}(m)^H e^{iarg\{a_1\}} [\mathbf{I} - \mathbf{P}_{S_1'}] \mathbf{s}_1(T_1)\} \underset{-1}{\overset{+1}{\gtrless}} 0. \tag{8}$$

The decorrelator structure of the detector is evident from (8) – the matrix $[\mathbf{I} - \mathbf{P}_{S_1'}]$ rejects those components of $\mathbf{r}(m)$ corresponding to users $n = 2, \ldots, N$ at the N_T discretized delay values $\{T_n(i)\}$.

The bit-error rate is computed in a straightforward manner, once the mean and variance of the decision variable U are obtained. Without loss of generality, the phase of a_1 is taken to be zero, $d_1(m)$ is set to unity, and the remaining data symbols $d_n(m)$ are absorbed into the complex amplitudes a_n. The mean is then given by

$$E\{U\} = \quad (9)$$
$$|a_1|\mathbf{s}_1(T_1)^H[\mathbf{I} - \mathbf{P}_{S_1'}]\mathbf{s}_1(T_1) +$$
$$\sum_{n=2}^{N} |a_n| cos(arg\{a_n\})\mathbf{s}_n(T_n)^H[\mathbf{I} - \mathbf{P}_{S_1'}]\mathbf{s}_1(T_1),$$

and the variance is

$$\text{VAR}\ \{U\} = \frac{1}{2}\sigma_n^2 \mathbf{s}_1(T_1)^H[\mathbf{I} - \mathbf{P}_{S_1'}]\mathbf{s}_1(T_1). \quad (10)$$

The bit error rate is then equal to

$$P_e = \frac{1}{2}\text{erfc}\left(\frac{E\{U\}}{\sqrt{2\text{VAR}\{U\}}}\right). \quad (11)$$

If the residual multiuser interference due to discretization of the delays is neglected, the following simplified BER expression is obtained [5].

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}\mathbf{s}_1(T_1)^H[\mathbf{I} - \mathbf{P}_{S_1'}]\mathbf{s}_1(T_1)}\right). \quad (12)$$

3 Error Rate Bounds

An upper bound on the BER for the decorrelator detectors is next obtained, that is solely a function of the maximum cross-correlation of the sequences $PN_n(t)$ and number of users. The resulting expression is thus much simpler to evaluate than the exact BER, and provides insight into the relationship between the number of users that can be accommodated and the required length of the PN sequences. In deriving the bound, it is assumed that the undesired users are completely rejected ($T_n = T_n(i)$.) Although this assumption may at first seem unrealistic, as shown in Section 4, the undesired users are nearly rejected in the scenarios considered here, even when $T_n \neq T_n(i)$. In either coherent or noncoherent detection, the problem of upper bounding the BER (12) is equivalent to finding a lower bound, denoted by γ, for the effective normalized signal power, as follows.

$$\mathbf{s}_1^H[\mathbf{I} - \mathbf{P}_{S_1'}]\mathbf{s}_1 \geq \gamma, \quad (13)$$

where the dependence on the delays $\{T_n(i)\}$ has been suppressed for clarity. This lower bound can be obtained using a series of eigenvalue bounds as discussed in [5], with the final result given by

$$\mathbf{s}_1^H[\mathbf{I} - \mathbf{P}_{S_1'}]\mathbf{s}_1 \geq \quad (14)$$
$$\gamma = \max\left(1 - \frac{N_T(N-1)t_{max}^2}{1-(N_T(N-1)-1)t_{max}}, 0\right).$$

The error rate bound is obtained by replacing the normalized signal power in (12) by γ.

4 Results

Figure 1 shows the real-valued noiseless decorrelator output for each individual user as a function of received code delay, when the assumed uncertainty region is ± 1 chip. The output $y_n(\tau)$ due solely to user n is thus defined by

$$y_n(\tau) = Re\{a_n^* \mathbf{s}_n(\tau)^H[\mathbf{I} - \mathbf{P}_{S_1'}]\mathbf{s}_1(T_1)\}. \quad (15)$$

The users have zero phase, and are assigned signature sequences corresponding to length-31 Gold codes. The power ratio $J/S = |a_n|^2/|a_1|^2$ of user n to user 1 is set to 20 dB in Figure 1. The actual decorrelator output would be a sum of these individual responses. The decorrelating vectors defining the interference subspace includes the nominal, $\pm\frac{1}{2}$, and ± 1 chip shifted versions of the signal vectors from the undesired users. The figure shows clear nulls in the decorrelator output for users 2 and 3 at those delays. Between these discrete delays, we find that the undesired users are "nearly" decorrelated. Outside this ± 1 chip uncertainty region, the decorrelator output for users 2 and 3 may be quite large.

Figures 2-3 show the analytical BER of the quasi-synchronous detector together with the performance bounds for BPSK. For all cases, the number of users $N = 3$. In determining the decorrelator response we have assumed the best-case delay for the desired user (i.e. 0.0 chips), and the worst-case delay for the unwanted users. The worst-case delays were determined by individual brute-force searches over the uncertainty region in steps of $\frac{1}{20}$-th of a chip, in order to minimize the expected value of the decision variable (9). Also, all user amplitudes were taken to be real to allow the greatest degree of signal cancellation. The matched filter error rate is also shown and is computed via the best-case/worst-case strategy, above. As the decorrelator uncertainty region was expanded, the matched filter response changed, but only very slightly (as we searched for a worst-case over a larger time region). Only a single representative curve is shown. For further comparisons, the ideal BPSK result is shown, as is the decorrelator output bound computed using the SNR bound (14) which bounds the signal suppression effect assuming perfect decorrelation of the undesired users.

In Figure 2, the interferer/desired user power ratio is +20 dB, with length-31 Gold codes. The matched filter is essentially useless throughout the SNR range due to the strong near-far effect. The quasi-synchronous detector error rate, however, is only mildly effected by the increased interference. Thus, the quasi-synchronous detector is able to function reliably in a difficult near-far environment.

Figure 3 repeats the +20 dB interference scenario for BPSK, but the user codes are increased to length-63 Gold codes. The performance is superior to that in Figure 2 indicating that longer codes will give better interference immunity. Most importantly, longer codes allow efficient operation even with large delay uncertainties. This is clearly due to a reduction in the signal suppression effect, as the desired and undesired signals have lower cross-correlations.

5 Conclusions

A quasi-synchronous CDMA system has been proposed, in which the mobile users in a cellular network attempt to transmit in synchrony, using a common GPS-derived clock. Although the received signal at the base station is not exactly synchronous, the greatly reduced time uncertainty region allow the implementation of a simple linear decorrelator-based multiuser detector. Both exact BER expressions and bounds on the BER were derived, and it was shown that effective multiuser interference suppression could be achieved, even with a relatively coarse ($\pm 1/2$ chip) discretization of the delay uncertainty region.

Although the proposed system requires that each mobile user have its own GPS receiver, the rapidly declining cost of GPS equipment may make implementation practical in the near future. Furthermore, the increased network capacity and improved BER performance obtained through multiuser detection may offset the disadvantage of increased complexity of the mobile transmitters. Most importantly, the base station implementation complexity is not greatly increased, since the conventional matched filters need only be replaced by appropriate linear decorrelators.

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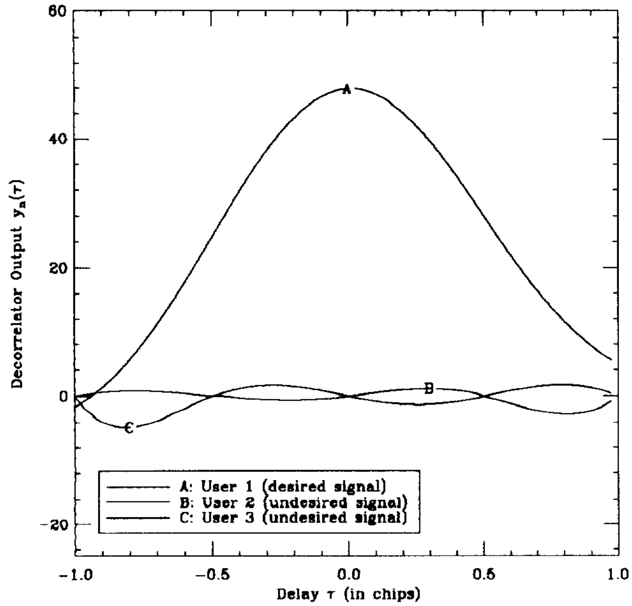

Figure 1: Decorrelator outputs due to individual users – delay uncertainty of ± 1 chip, $J/S = 20$ dB.

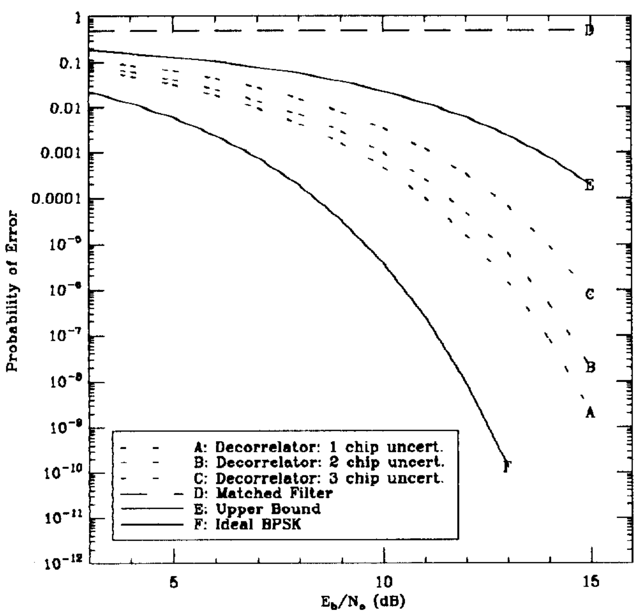

Figure 2: Bit-error rates for 31 chip sequences, $J/S =$ 20 dB.

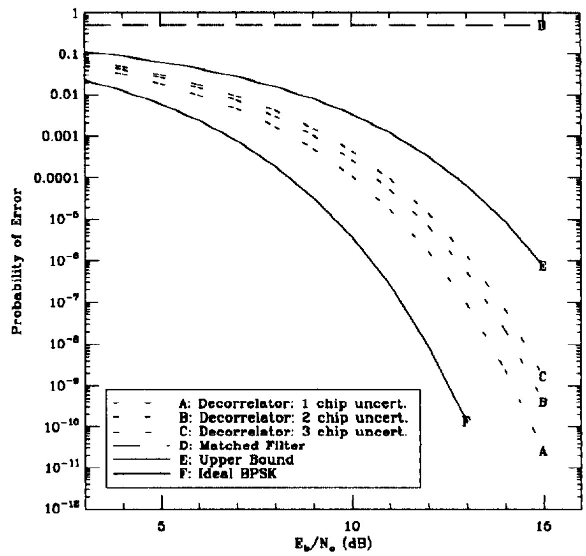

Figure 3: Bit-error rates for 63 chip sequences, $J/S =$ 20 dB.