A Closed-Loop Coherent PN Acquisition Scheme for DS/SS Systems Using an Auxiliary Sequence

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Abstract

We propose a closed-loop system for acquisition of the pseudo-noise (PN) sequence in direct-sequence spread-spectrum (DS/SS) systems. We introduce a novel idea of using an auxiliary sequence, as opposed to the PN sequence itself, for correlation with the incoming signal. The correlation function of the auxiliary sequence and the PN sequence has a triangle shape that covers essentially the entire period of the PN sequence. Consequently, their correlation provides the direction for the phase update of the local sequence generator in the acquisition scheme. With coherent demodulation, the average acquisition time of the proposed scheme is derived and is compared to that of the conventional serial-search acquisition receiver. Results suggest that the proposed system acquires PN phase at least twice faster.

1 Introduction

An important task of a DS/SS receiver is to generate a local replica of the transmitted sequence and to synchronize its phase to that of the incoming sequence. This synchronization process is typically accomplished in two stages : acquisition and tracking. In the first stage (acquisition), the phases of the two sequences are aligned to within a small range of one chip or less. Then in the second stage the tracking circuitry makes the fine adjustment and maintains the two codes in synchronism by means of a closed loop operation. Many types of DS/SS acquisition and tracking systems have been reported and analyzed [1].

In most DS/SS acquisition techniques the received signal and the local PN sequence are correlated to produce an output which is used for detecting whether the two codes are in synchronism or not. There are a variety of acquisition receivers depending on the type of the detector and the search algorithm [2]-[5]. PN acquisition systems can be coherent or noncoherent depending on whether or not the carrier phase is known during acquisition.

A direct technique for PN synchronization is the maximum-likelihood (ML) estimation of the phase. Practical implementations of the ML algorithm can be either parallel or serial. The receivers quantize the time uncertainty region into a number of discrete phases. Code waveforms (cells) having these phases are correlated with the received signal. In a parallel scheme the correlations are performed in parallel. The number of correlation branches is equal to the number of cells, which is proportional to the length of the uncertainty region. This receiver is impractical for PN sequences with long periods. In a serial scheme, each (discrete) phase of the local PN waveform is checked for phase alignment before stepping to the next discrete phase. When there is no a priori information on the received PN code phase, the phases in the uncertainty region are searched in a uniform (unidirectional) fashion from one end of the region to the other.

The simplest of the uniform serial-search techniques [2,3,4] is the single-dwell system, where a single correlator is used to examine each of the possible phases for a fixed period of time until a phase alignment is detected. When phase alignment is rejected, the phase is updated to the next one in a unidirectional fashion. A coherent single-dwell acquisition system is shown in Figure 1. The dwell time is nT_c seconds.

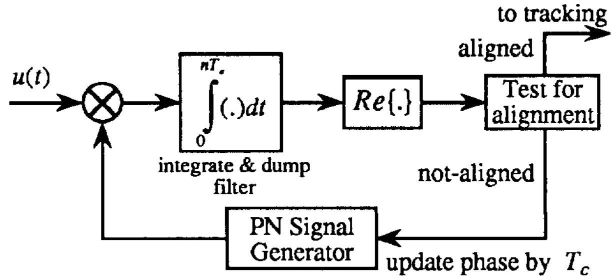

Figure 1. Coherent, fixed-dwell, serial-search acquisition receiver

In this paper we propose a closed-loop acquisition scheme which provides the direction for the local phase update. This is accomplished by using an auxiliary sequence, as opposed to the PN sequence itself, to correlate with the incoming signal.

2 The Proposed Acquisition System

The baseband representation of a received DS/SS signal during acquisition is of the following form:

$$u(t) = \sqrt{2P}c(t-\tau) + z(t) \tag{1}$$

where P is the average signal power, τ is the unknown code delay to be estimated by an acquisition system, $z(t)$ is the zero mean AWGN with two-sided power spectral density $2N_o$, and $c(t)$ is the code waveform

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expressed as

$$c(t) = \sum_{k=-\infty}^{\infty} c_k P_{T_c}(t - kT_c) \qquad (2)$$

In this expression, c_k is the k^{th} chip of a binary m-sequence which is periodic with period N, i.e., $c_k = c_{k+N}$ for all k, and $P_{T_c}(.)$ is the unit amplitude rectangular pulse shape in the interval $[0,T_c]$, where T_c is the chip duration. In (1), it is assumed that there is no data modulation during acquisition. Only single-user case is considered here.

We propose the acquisition system shown below.

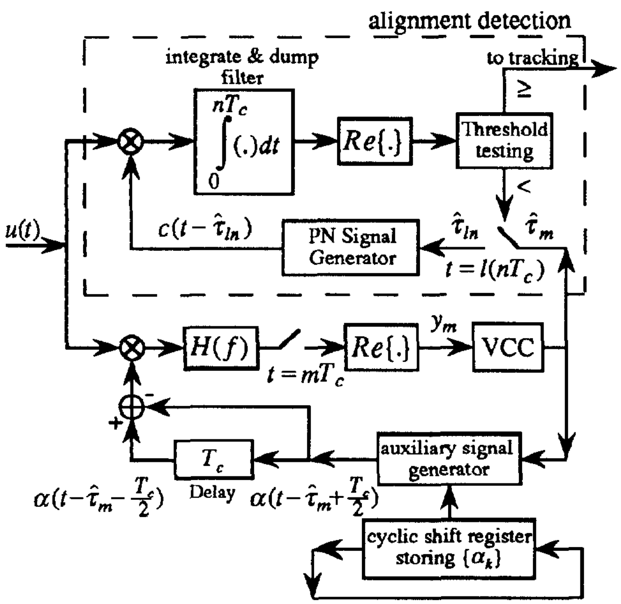

Figure 2. The proposed acquisition system

Note that we use a new local waveform $\alpha(t)$. It is defined as follows:

$$\begin{aligned} \alpha(t) &= \sum_{m=-\frac{N-3}{2}}^{\frac{N-3}{2}} \left[\frac{N-1}{2} - |m|\right] c(t - mT_c) \quad (3) \\ &= \sum_{k=-\infty}^{\infty} \alpha_k P_{T_c}(t - kT_c) \quad (4) \end{aligned}$$

where,

$$\alpha_k = \sum_{m=-\frac{N-3}{2}}^{\frac{N-3}{2}} \left[\frac{N-1}{2} - |m|\right] c_{k-m} \qquad (5)$$

The sequence $\{\alpha_k\}$ and the waveform $\alpha(t)$ are periodic with the respective periods N and NT_c, which are the same as the periods of $\{c_k\}$ and $c(t)$, respectively. As seen from (3), $\alpha(t)$ is a weighted sum of shifted versions of the m-sequence waveform $c(t)$.

The periodic cross-correlation function between $c(t)$ and $\alpha(t)$ is

$$\begin{aligned} R_{c\alpha}(\gamma) &= \frac{1}{NT_c}\int_0^{NT_c} c(t+\gamma)\alpha(t)dt \\ &= \sum_{m=-\frac{N-3}{2}}^{\frac{N-3}{2}} \left[\frac{N-1}{2} - |m|\right] R_{cc}(\gamma + mT_c) \end{aligned}$$

where, $R_{cc}(.)$ is the periodic autocorrelation function of a binary m-sequence, given by

$$R_{cc}(\gamma) = \begin{cases} 1 - \frac{N+1}{NT_c}|\gamma| , & |\gamma| \le T_c \\ -\frac{1}{N} , & T_c < |\gamma| \le \frac{NT_c}{2} \end{cases}$$

For $\gamma \in \left[-\frac{N-1}{2}T_c, \frac{N+1}{2}T_c\right]$, $R_{c\alpha}(\gamma)$ is given by

$$R_{c\alpha}(\gamma) = \begin{cases} \frac{(N-1)(N+3)}{4N} - \frac{N+1}{NT_c}|\gamma| , & |\gamma| \le \frac{(N-1)T_c}{2} \\ -\frac{(N-1)^2}{4N} , & \frac{(N-1)T_c}{2} < \gamma \le \frac{(N+1)T_c}{2} \end{cases}$$

$R_{cc}(.)$ and $R_{c\alpha}(.)$ have the same period NT_c. $R_{c\alpha}(\gamma)$ is plotted in Figure 3.

It is seen from Figure 2 that the received signal is correlated with the difference of early and late versions of the local code waveform $\alpha(t)$. The correlation is performed by a loop filter with frequency response $H(f)$, a sampler, and a real-part operator denoted by the symbol Re. The impulse response of the loop filter, denoted by $h(t)$, is

$$h(t) = \begin{cases} 1/M , & 0 \le t \le MT_c \\ 0 , & \text{otherwise} \end{cases}$$

The correlator output is the error signal used to control the voltage controlled clock (VCC). As the loop action tends to drive the error signal to zero, the code delay estimate converges to the unknown code delay τ. The code delay estimate is updated once every T_c seconds with the following recursive equation:

$$\hat{\tau}_{m+1} = \hat{\tau}_m + K_{vcc} y_m \qquad (6)$$

where $\hat{\tau}_m$ is the estimate formed at time $t = mT_c$ and y_m is the input control signal of the VCC at time $t = mT_c$. y_m is as follows:

$$y_m = \frac{1}{M}\sum_{k=m-M}^{m-1} [s_k(\hat{\tau}_k, \tau) + \eta_k(\hat{\tau}_k)] \qquad (7)$$

where $s_k(\hat{\tau}_k, \tau)$ and $\eta_k(\hat{\tau}_k)$ are defined as

$$\begin{aligned} s_k(\hat{\tau}_k, \tau) &= \sqrt{2P}\int_{kT_c}^{(k+1)T_c} c(t-\tau)x(t-\hat{\tau}_k)dt \\ \eta_k(\hat{\tau}_k) &= Re\left\{\int_{kT_c}^{(k+1)T_c} z(t)x(t-\hat{\tau}_k)dt\right\} \quad (8) \end{aligned}$$

with $x(t) = \alpha(t - \frac{T_c}{2}) - \alpha(t + \frac{T_c}{2})$. K_{vcc} is the constant that determines the step size of the recursive equation. $K_{vcc} = 1/(n\sqrt{2P})$ is used for the receiver in Figure 2. In (7), M is not restricted to be an integer multiple of N. Therefore, the received signal is partially correlated with the early and late versions of $\alpha(t)$. The sequence $\{\alpha_k\}$ is stored in a cyclic shift register. The auxiliary signal generator produces the signal $\alpha(t - \hat{\tau} + T_c/2)$ from the sequence $\{\alpha_k\}$ and the phase estimate from the VCC.

The upper part in Figure 2 is the phase alignment detector. It has a PN signal generator. It correlates the incoming signal $u(t)$ with the PN code waveform $c(t - \hat{\tau}_{ln})$ for a fixed dwell-time of nT_c seconds. A new value of the code delay estimate $\hat{\tau}_m$ is fed to the detector every nT_c seconds, i.e., at $m = 0, n, 2n, ...$, and is tested for phase alignment. This continues until alignment is declared, which will initiate the tracking circuitry. Detection probability P_d is defined as the probability of declaring acquisition when the two phases are in fact aligned (H_1). Similarly, P_{fa} is defined as the probability of falsely declaring acquisition when the phases are not aligned (H_0). The hypotheses H_1 and H_0, for each integer l, are defined as

$$H_1 : |\hat{\tau}_{ln} - \tau| \leq \frac{T_c}{2} \qquad \text{(alignment)}$$

$$H_0 : T_c \leq |\hat{\tau}_{ln} - \tau| \leq (N-1)T_c \qquad \text{(no alignment)}$$

At time $t = mT_c$, code delay estimation error $e_{\tau,m}$ is defined as

$$e_{\tau,m} = \hat{\tau}_m - \tau \tag{9}$$

This error satisfies the recursion (6), i.e.,

$$e_{\tau,m+1} = e_{\tau,m} + K_{vcc} y_m \tag{10}$$

Convergence of the delay estimate to τ depends on the pull-in range of the receiver in Figure 2 and the initial error $e_{\tau,0}$ at time $t = 0$. The pull-in range is determined by the control signal y_m at the VCC input. We use the average value of this signal for $M = 1$ to find the pull-in range approximately. The average is taken with respect to both the noise and the code chips of the sequence c_k. Define $\underline{C} = [c_0, c_1, ..., c_{N-1}]$. Then for $M = 1$

$$\bar{y}_m = E_{\underline{C},\eta}\{y_m\} = E_{\underline{C}}\{s_{m-1}(\hat{\tau}_{m-1}, \tau)\} \tag{11}$$

since $E\{\eta_{m-1}(\hat{\tau}_{m-1})\} = 0$. In a binary m-sequence with period N, a randomly picked chip c_k is $+1$ or -1 with the probabilities $(N-1)/2N$ and $(N+1)/2N$, respectively. This yields $E\{c_k\} = -1/N$. It can be shown that $s_{m-1}(\hat{\tau}_{m-1}, \tau)$ contains terms of the form $c_{k+i}c_{k+j}$. For a binary m-sequence, we have $E\{c_{k+i}c_{k+j}\} = -1/N$ for $i \neq j$ and 1 for $i = j$. With this and Eqs. (2), (4), and (8), we obtain $\bar{y}_m$ as a function of $e_{\tau,m-1}$. It is periodic with period NT_c. For $e_{\tau,m-1} \in [-\frac{N}{2}T_c, \frac{N}{2}T_c]$,

$$\frac{\bar{y}_m}{\sqrt{2P}\frac{N+1}{N}} = \begin{cases} e_{\tau,m-1} + \frac{NT_c}{2}, & -\frac{NT_c}{2} \leq e_{\tau,m-1} < -\frac{(N-2)T_c}{2} \\ T_c, & -\frac{(N-2)T_c}{2} \leq e_{\tau,m-1} < -\frac{T_c}{2} \\ -2e_{\tau,m-1}, & |e_{\tau,m-1}| \leq \frac{T_c}{2} \\ -T_c, & \frac{T_c}{2} < e_{\tau,m-1} \leq \frac{(N-2)T_c}{2} \\ e_{\tau,m-1} - \frac{NT_c}{2}, & \frac{(N-2)T_c}{2} < e_{\tau,m-1} \leq \frac{NT_c}{2} \end{cases}$$

$\bar{y}_m$ is plotted in Figure 4. Note that the control signal spans the whole code period. Therefore, the pull-in range is NT_c seconds and the estimation error with the recursion (10) converges to zero regardless of the initial error $e_{\tau,0}$.

3 Performance Analysis

We evaluate the average acquisition time performance of the proposed receiver using the flow graph technique [2]. The flow graph for acquisition time is shown in Figure 5. We define the states as follows.

state 0, $\quad |e_{\tau,m}| \leq \frac{T_c}{2}$

state j, $\quad j = 1, 2, ..., \frac{N-1}{2}$, $|e_{\tau,m}| \leq |e_{\tau,m-1}|$ and $(j - 0.5)T_c < |e_{\tau,m}| \leq (j + 0.5)T_c$

state k', $\quad k = 1, 2, ..., \nu$, $|e_{\tau,m-1}| < |e_{\tau,m}|$ and $(k - 0.5)T_c < |e_{\tau,m}| \leq (k + 0.5)T_c$

state S, $\quad$ start (start acquisition)
state F, $\quad$ finish (achieve acquisition)

The starting state can be any of the states $0, 1, ..., \frac{N-1}{2}$ with the respective probabilities $P_0, P_1, ..., P_{\frac{N-1}{2}}$, where $P_0 = \frac{1}{N}$, $P_1 = P_2 = \cdots = P_{\frac{N-1}{2}} = \frac{2}{N}$. The value of ν is a parameter in the numerical results. The states $1', 2', ..., \nu'$, occur when phase alignment (state 0) goes undetected. They represent the case when the code delay estimation error magnitude is increasing from one iteration to the next, i.e., $|e_{\tau,m-1}| < |e_{\tau,m}|$.

P_d and P_{fa} are defined as before. In Figure 5, z indicates the unit delay operator. Unit delay for the proposed receiver is nT_c seconds. At any state, except state 0, there is either a false alarm or no false alarm. In the case of a false alarm, it takes one unit of time to declare phase alignment, followed by K_p units of time needed to realize that there is no phase alignment. K_p is the penalty time due to a false alarm. When there is no false alarm, it takes one unit of time to declare no phase alignment. If there is phase alignment (state 0) and is not detected, then the magnitude of the error increases until it is equal to νT_c. Then it decreases until it is within half a chip. This cycle goes on until acquisition is detected. Note that the false alarm probability for states 1 and $1'$, denoted by P'_{fa}, is different from P_{fa} for the other states. For states 1 and $1'$, we may have $\frac{T_c}{2} < |e_{\tau,m}| < T_c$, which is neither H_1

nor H_0. In this case $P'_{fa} \geq P_{fa}$. However, for large N such as $N = 511$, assuming $P'_{fa} = P_{fa}$ has negligible effect on the following analysis.

We use the generating function technique of [2] in our analysis. The generating function $U(z)$ is

$$U(z) = \sum_{m=0}^{\infty} P_{SF}(m) z^m \tag{12}$$

where, $P_{SF}(m)$ is the probability of going from state S to state F in m steps. $U(z)$ is also the transformed signal at node F when a unit input is applied to node S. Applying flow graph reduction methods, we obtained

$$U(z) = \frac{\frac{2}{N} P_d z \left[\left(\sum_{l=0}^{\frac{N-1}{2}} H^l(z) \right) - 0.5 \right]}{1 - (1 - P_d) z H^{2\nu - 1}(z)} \tag{13}$$

where $H(z) = P_{fa} z^{K_p+1} + (1 - P_{fa}) z$. Note that the probability of reaching acquisition (state F) once the acquisition process has started is 1, which is equal to $U(1)$. $U(z)$ contains statistical information about the acquisition process. The average acquisition time $\bar{T}_{acq}$ is derived from $U(z)$ as follows:

$$\bar{T}_{acq} = \left[\sum_{m=0}^{\infty} m P_{SF}(m) \right] nT_c = \left. \frac{dU(z)}{dz} \right|_{z=1} nT_c \tag{14}$$

Substituting (13) into (14) we obtain,

$$\bar{T}_{acq} = \left[(P_{fa} K_p + 1) \frac{N^2 - 1}{4N} + \frac{1 + (1 - P_d)(2\nu - 1)(P_{fa} K_p + 1)}{P_d} \right] nT_c \tag{15}$$

Note that, the first term in (15) is the average time to reach state 0 from state S. The second term is the average time to reach state F from state 0. We compare this average acquisition time with that of the coherent, serial-search single-dwell acquisition receiver shown in Figure 1. Using formulae in [2], the average acquisition time for Figure 1 is

$$\bar{T}_{acq,serial} = \frac{2 + (2 - P_d)(N - 1)(P_{fa} K_p + 1)}{2 P_d} nT_c \tag{16}$$

For $\frac{1}{P_d} \ll \frac{N^2-1}{4N}$, which is the case if P_d is not too small, the ratio of (16) and (15) is

$$R_1 = \frac{\bar{T}_{acq,serial}}{\bar{T}_{acq}} \approx \frac{\left(1 - \frac{P_d}{2}\right) N}{(1 - P_d)(2\nu - 1) + \frac{N^2-1}{4N} P_d} \tag{17}$$

This is plotted in Figure 6 for different values of ν. $N = 511$ is used in this plot. It is observed that the proposed system acquires the incoming sequence phase at least twice as fast as the serial search system. For fixed P_d, as ν gets smaller acquisition time superiority of the proposed system over the single-dwell serial-search system is further enhanced.

The variance of the acquisition time can also be derived from $U(z)$.

$$\sigma^2_{T_{acq}} = \left[\frac{d^2U(z)}{dz^2} + \frac{dU(z)}{dz} - \left(\frac{dU(z)}{dz} \right)^2 \right]_{z=1} (nT_c)^2 \tag{18}$$

Substituting (13) into (18) yields

$$\begin{aligned}\sigma^2_{T_{acq}} = (nT_c)^2 \Bigg[& \frac{1 - P_d}{P_d^2} + \\ & P_{fa} K_p (K_p + 1) \left(\tfrac{N^2-1}{4N} + (2\nu - 1) \tfrac{1-P_d}{P_d} \right) + \\ & (P_{fa} K_p + 1) \left(\tfrac{N^2-1}{4N} + (2\nu - 1)(1 - P_d) \tfrac{2+P_d}{P_d^2} \right) + \\ & (P_{fa} K_p + 1)^2 \Bigg(\tfrac{(N^4 - 12N^3 + 2N^2 + 12N - 3)}{48N^2} + \\ & \tfrac{1-P_d}{P_d^2} (2\nu - 1 - P_d)(2\nu - 1) \Bigg) \Bigg]\end{aligned} \tag{19}$$

The variance of the acquisition time for the receiver in Fig 1 can be shown to be

$$\begin{aligned}\sigma^2_{T_{acq,serial}} = (nT_c)^2 \Bigg[& \frac{1 - P_d}{P_d^2} + \\ & (N - 1) \Big\{ (P_{fa} K_p + 1) \tfrac{4 - 2P_d - P_d^2}{2P_d^2} + \\ & P_{fa} K_p (K_p + 1) \tfrac{2 - P_d}{2P_d} + \\ & (P_{fa} K_p + 1)^2 \left(\tfrac{(1-P_d)(N-1-P_d)}{P_d^2} + \tfrac{N-5}{12} \right) \Big\} \Bigg]\end{aligned} \tag{20}$$

The ratio $R_2 = \frac{\sigma^2_{T_{acq,serial}}}{\sigma^2_{T_{acq}}}$ is plotted versus ν in Figure 7 for different values of P_{fa}, P_d and K_p. It is observed from the plot that, the acquisition time variance of the serial-search system is at least four times the acquisition time variance of the proposed system. For fixed ν and K_p, R_2 increases as P_{fa} gets larger and P_d gets smaller. For fixed P_{fa}, P_d and K_p, R_2 decreases as ν increases. When all other parameters are fixed, R_2 decreases as K_p increases.

The correlation filter parameter M and the VCC step size K_{vcc} affect the value of ν. For a fixed K_{vcc}, ν increases as M increases. The correlation filter reduces the fluctuations on the VCC control signal caused by noise and different code chips. Setting M large will reduce the fluctuations at the expense of a high ν. On the other hand, the fluctuations can be detrimental when M is too small. This fact was not represented in the above derivations and results.

4 Conclusion

A closed-loop coherent acquisition system for DS/SS systems is proposed. Unlike the uniform serial-search technique, this system provides the direction for the local phase update by using an auxiliary sequence. The average and variance of the acquisition time of the proposed system are obtained using the flow graph technique. These results are then compared with those of the single-dwell serial-search receiver. On the average, the proposed system acquires the incoming phase at least twice faster. Moreover, acquisition time variance of the proposed system is at most one fourth of that of the single-dwell system.

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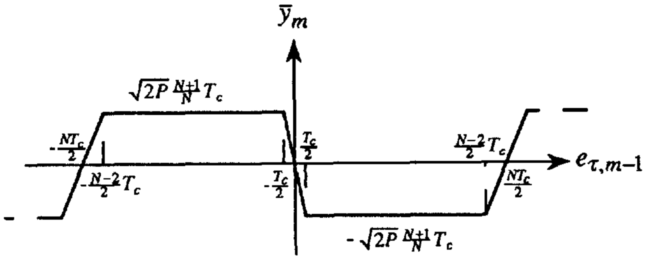

Figure 4. Average VCC input signal for M=1

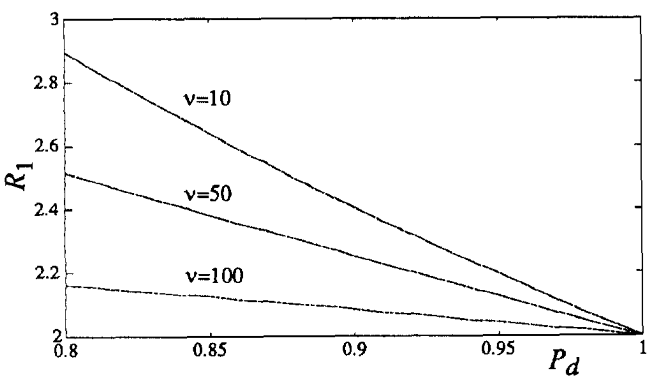

Figure 6. Acquisition time comparison for N=511, v=10,50,100

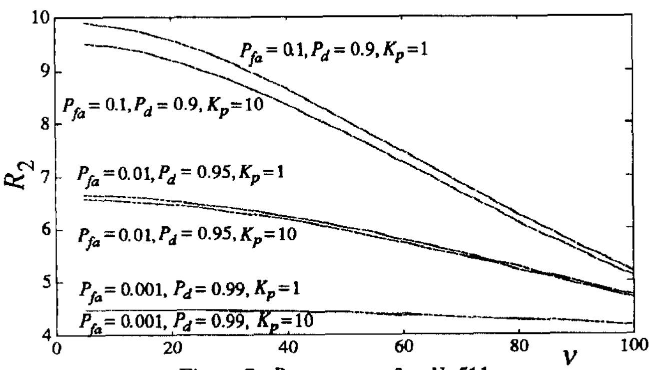

Figure 7. R_2 vs. v for N=511

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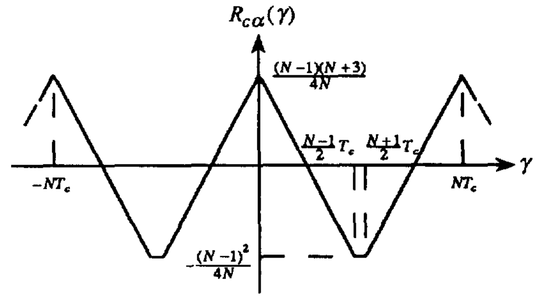

Figure 3. Periodic cross-correlation function

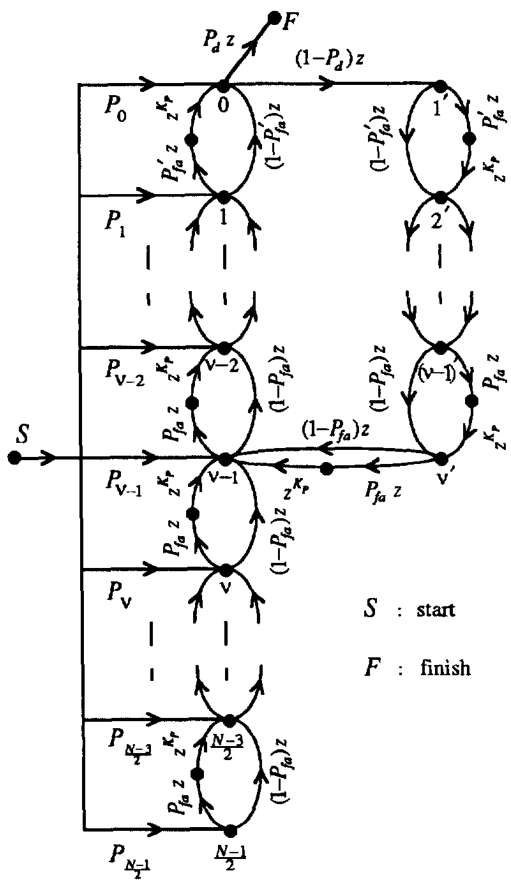

Figure 5. System Flow Graph