THEORETICAL STUDY OF A MULTISENSOR EQUALIZER USING THE MSE FOR THE RADIOMOBILE CHANNEL (GSM MODEL)

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ABSTRACT

The theoretical performance of a multisensor receiver using a Linear or Decision Feedback equalizer (DFE) associated to the MMSE criterion is analyzed as a function of the equivalent discrete Channel Impulse Response (CIR). ***The multisensor equivalent discrete CIR is derived, with some assumptions, from an extended single sensor channel model given by the GSM specifications for different environments.*** *It is then used to yield the optimum taps settings in the MMSE sense.* ***A comparative study of the performance (BER) of different structures of the single and multisensor equalizer is carried out.*** *The different configurations taken into account are* ***a linear or decision feedback for single or multisensor equalizer and a purely diversity combining scheme.*** *In all cases the comparison of the theoretical performance is done for different numbers of sensors.*

1. INTRODUCTION

The radiomobile channel is characterised by the presence of time-variant multipath-fading propagation. This phenomenon presents three main impairments to communication systems, namely time spread, multipath-fading, and Doppler spread. Time spread causes intersymbol interference, ISI, (interference between adjacent symbols), multipath-fading results in a very low received signal-to-noise ratio when the channel exhibits a deep fade and Doppler spread varies the fading at a rate which depends on the speed of the vehicle.

These problems have given rise to a major research effort to improve equalizers which can help in the correction of these propagation defaults. Generally these equalizers use the signal received by only one sensor.

New possibilities in the miniaturization of high frequency receivers allow the consideration of the use of several sensors and multipath-fading can be combatted by using diversity techniques where the receiver is provided with multiple independently faded replicas of the same information signal. Diversity is effective because the probability of simultaneously having two or more independently faded channels in a deep fade is small [3].

As described in [1] it is convenient to model the transmitter filter, channel and the received filter (matched filter and noise whitening filter) as an (2L+1)-transversal filter with tap spacing equal to T, the symbol duration. It is usually called the equivalent discrete-time white noise channel model.

In this paper this model is extended, with some assumptions, to a multisensor reception scheme. Then the theoretical performance of a multisensor-linear equalizer (M-LE) or multisensor-decision-feedback equalizer (M-DFE), is analyzed as a function of the equivalent discrete channel impulse response. The channel models are those given by the GSM specifications for different environments: Typical Urban (TU), Rural Area (RA) and Hilly Terrain (HT). Finally the results for different equalizer structures are comparatively analyzed to identify the most effective structure.

This paper is organized as follows. Section 2 describes the system structure and the proposed equivalent discrete channel model . In Section 3 the analytical formulation of the optimum coefficients, in the MMSE sense, of the different single or multisensor structures, as a function of the channel caracteristics, is presented. Also, the theoretical Bit Error Rate (BER) is calculated from the corrected channel impulse response (convolution of the equivalent discrete CIR and optimum impulse response of the equalizer) for a QPSK modulation scheme. Finally, in Section 4, the results are presented and the structure presenting the least BER is proposed.

2. SYSTEM AND EQUIVALENT DISCRETE CHANNEL MODEL

2.1. Single-sensor channel

The multipath-fading propagation channel for mobile communications systems using one sensor can be described by its time varying impulse response, given by

$$c(t,\tau) = \sum_n \alpha_n(t) e^{-2\pi f_c \tau_n(t)} \delta(\tau - \tau_n(t)) \qquad (1)$$

Where α_n is the attenuation, τ_n is the delay associated with the n^{th} path and f_c is the carrier frequency.

As the number of paths is usually large and unpredictable, $c(t,\tau)$ is modeled by a complex gaussian process with a Doppler spectrum whose bandwidth is a function of the vehicle speed.

2.2. Multisensor-channel

Consider the receiver built up of **M sensors**. The model given in 2.1 must be extended. Using the same notation as in 2.1. The received baseband signal on channel 1, considered as the reference one, is the sum of the different replicas of the transmitted signal arriving from different directions, and can be written as follows:

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$$x_1(t) = \sum_n \alpha_n(t) e^{-2\pi f_c \tau_n(t)} s_n(t-\tau_n(t)) \quad (2)$$

The signal received on the second sensor can be written:

$$x_2(t) = \sum_n \alpha_n(t) s(t-\tau_n(t) - \Delta_{n,2}) v_{n,2} \quad (3)$$

where $\Delta_{n,2}$ is the delay due to the direction of arrival of the n^{th} path, and $v_{n,2}$ is the carrier phase shift.

Suppose the distance between each sensor is 10 cm, which is the case in practical situations. The $MAX(\Delta_{n,2})$ is 0.3 ns which is largely less than the inverse of the bandwidth of the signals considered here. As a result the following approximation can be made:

$$s_n(t-\Delta_{n,2}) \approx s_n(t) \quad (4)$$

Thus, in this case only the α_n are different on each sensor.

In order to allow practical simulation, the proposed model of the channel impulse response is the following:

$$c_j(\tau) = \sum_{i=1}^{6} \tilde{\alpha}_{i,j} \times \delta(\tau - \tau_i) \quad (5)$$

$$\text{with} \quad \tilde{\alpha}_{i,j} = \sqrt{p_i} \cdot e^{j\theta_{i,j}} \quad (6)$$

and p_i the power of the i^{th} path given by the GSM model and $\theta_{i,j}$ a random angle, uniformly distributed between 0 and 2π, computed for each channel j.

The GSM recommendations [6] present propagation models for three different environments: Typical Urban areas (TU), Rural areas (RA), and Hilly terrains (HT). They are given in terms of

- discrete number of taps, each determined by their time delay and average power;

- the Rayleigh distributed amplitude of each tap, varying according to a Doppler spectrum S(f).

The reduced configuration of 6 taps is defined for the three cases in table 1.

n	Ra		Ht		Tu	
	τ_n (μs)	(dB)	τ_n (μs)	(dB)	τ_n (μs)	(dB)
1	0.0	0.0	0.0	0.0	0.0	-3.0
2	0.1	-4.0	0.1	-1.5	0.2	0.0
3	0.2	-8.0	0.3	-4.5	0.5	-2.0
4	0.3	-12.0	0.5	-7.5	1.6	-6.0
5	0.4	-16.0	15.0	-8.0	2.3	-8.0
6	0.5	-20.0	17.2	-17.7	5.0	-10.0

Table 1 : GSM Models

2.3. System structure and Equivalent discrete-time white noise channel model

The system stucture (Figure 1) is an M-diversity decision feedback equalizer (M-DFE) associated to the Minimum Mean Square Error (MMSE) criterion. Using the following notations

$$y_{p,n} = s_{R_p}(nT) \quad b_{1p,n} = b_{1p}(nT) \quad r_{p,n} = r_p(nT) \quad (7)$$

and supposing that the channel is bandlimited, i.e $r_{p,n} = 0$ for $|n| > L$, the received signal, $s_{R_p}(nt)$, on the p^{th} sensor filtered by the reception filter $h_r(t)$ is given by:

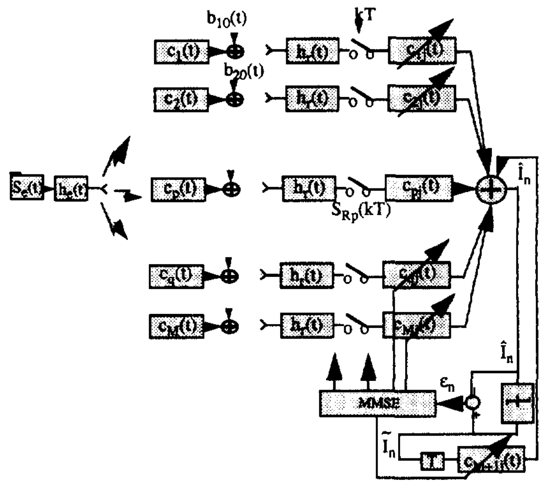

Figure 1 - M-Diversity Decision Feedback Equalizer.

$$y_{p,n} = \sum_{k=-L}^{L} I_k r_{p,n-k} + b_{1p,n} = \sum_{k=-L}^{L} I_{n-k} r_{p,k} + b_{1p,n}$$

$$\text{where} \quad r_p(t) = h_e(t) * c_p(t) * h_r(t) \quad (8)$$

* denotes the convolution product and b_{1p} the noise filtered by the reception filter. I_n is the n^{th} transmitted symbol, towards the emission filter $h_e(t)$, of duration T seconds.

If $h_e(t)*h_r(t)$ is specified to be the Nyquist filter (Square Root Raised Cosine), $b_{1p}(nT)$ are independent [2]. Furthermore if the noise at the input of the reception filter is gaussian then $b_{1p,n}$ are independent, identically distributed, gaussian random variables [2].

Thus the equivalent discrete-time white noise model is an (2L+1)-tap transversal filter with $r_{p,k}$ as its' coefficients and tap spacing equal to T.

3. THE OPTIMUM MULTISENSOR DFE IN A MMSE SENSE

The equivalent discrete-time channel model defined in the last section can be used to yield the optimum coefficients of the multisensor DFE in an MMSE sense. In order to do so, the following definitions should be considered.

The structure of the multisensor DFE, illustrated in fig.1, consists of a Finite Impulse Response (FIR) filter with (K_1+1) coefficients on each sensor and (K_2+1) coefficients in the decision-feedback loop.

Thus according to this structure the following vectors can be defined:

- the vector of the input signal

$$Y_n = \left[y^t_{0,n} \; y^t_{1,n} \cdots \cdots y^t_{p,n} \cdots \cdots y^t_{M,n} \; y^t_{M+1,n} \right]^t \quad (9)$$

where for $0 \le p \le M$, $y^t_{p,n}$ is the input vector of the FIRs on the M sensors and for $p = M+1$, $y^t_{M+1,n}$ is the input vector of the decision feedback filter, formed by the K_2 previous symbols detected.

- the vector of equalizer coefficients

$$C = \left[c^t_0\ c^t_1\ \dots\ c^t_p\ \dots\ c^t_M\ c^t_{M+1}\right]^t \quad (10)$$

where $c_{p,j}$ is the jth coefficient of the pth sensor and $c_{M+1,j}$ is the jth coefficient of the decision feedback filter.

The nth estimated symbol at the output of the equalizer is given by:

$$\hat{I}_n = \sum_{p=1}^{M}\sum_{j=0}^{K_1} c_{p,j}y_{p,n-j} + \sum_{j=1}^{K_2} c_{M+1,j}\tilde{I}_{n-j} \quad (11)$$

which can be written in vectorial form as $\hat{I} = C^tY_n$,

The error to be minimised is the difference between the detected symbol and estimated one, given by

$$\varepsilon_n = \tilde{I}_n - C^tY_n \quad (12)$$

To derive the optimum equalizer the orthogonality relationship [1] must be satisfied , giving

$$E[\varepsilon_n Y^H_n] = 0^H \quad (13)$$

H being the Hermitian transpose. The optimum coefficients are deduced after some calculations, giving

$$c_{opt} = \Gamma^{-1}\xi \quad (14)$$

where $\Gamma = E[Y_nY^H_n]$ is the auto and cross correlations matrix between equalizer inputs and $\xi = E[Y_n\tilde{I}^*_n]$ the cross correlation vector between the equalizer input and the detected sequence.

As shown in Section 2, the equalizer inputs are given as functions of the equivalent discrete-time impulse response of the channel. Therefore they can be used to calculate the elements of Γ and ξ.

3.1. Calculation of Γ and ξ

Assuming that the additive noise and signal input are independent and zero mean and also that they are independent from channel to channel, the elements of $\Gamma = E[Y_nY^H_n]$ are given by the following expresssions,

for $0 \le p \le M$,

$$\Gamma_{p,q}(i,j) = 2\sum_{k=-L}^{L} r_{p,k}r^*_{q,k+i-j} + \sigma_B\delta(p-q)(i-j) \quad (15)$$

$$\Gamma_{p,M+1}(i,j) = r_{p,D+1+j-i} = (\Gamma_{M+1,p}(i,j))^H \quad (16)$$

$$\xi_p(j) = 2r_{p,j} \quad (17)$$

and if the symbols are supposed to form an independent identically distributed (i.i.d) sequence, with the previous decisions assumed to be correct:

$$\Gamma_{M+1,M+1}(i,j) = 2\delta(i-j) \quad (18)$$

$$\xi_{M+1}(j) = 0 \quad (19)$$

3.2. The Bit Error Rate (BER) for a QPSK modulation

After equalization the estimate of the n^{th} symbol is given by

$$\hat{I}_n = \sum_{p=1}^{M}\sum_{j=0}^{K_1} c_{p,j}y_{p,n-j} + \sum_{j=1}^{K_2} c_{M+1,j}\tilde{I}_{n-j} \quad (20)$$

The estimated symbol can also be expressed in terms of the emitted symbol and intersymbol interference, giving:

$$\hat{I}_n = I_nR(0) + \underbrace{\sum_{k\ne n} I_nR((n-k)T)}_{IIS(m_i)} + B_{1,n} \quad (21)$$

where $R(t) = \sum_{p=1}^{M} r_p(t) * c_p(t) + c_{M+1}(t)$, called the Corrected Channel Impulse Response (CCIR), is the convolution of the M channels with the M forward filters situated on the M sensors added to the impulse response of the feedback filter of the decision loop. $B_I(t)$ is the noise at the output of the equalizer. The probability of error is composed of the conditionnal probabilities to the messages m_i formed by the $2L+1$ symbols interfering with I_n, i.e $m_i = \{I_{n-L},\dots,I_{n-1},I_{n+1},\dots,I_{n+L}\}$. For the QPSK the symbol is formed by 2 bits $I_n = a_n + jb_n$. and Let M = card($\{m_i\}$)= 2^{L+1}. The BER is given by

$$P_e = \frac{1}{M}\frac{1}{4}\sum_{i=1}^{M}\left\{\begin{array}{c} P(\hat{a}_n = 1/a_n = -1, m_i) + \\ P(\hat{a}_n = 1/a_n = -1, m_i) + \\ P(\hat{b}_n = -1/a_n = 1, m_i) + \\ P(\hat{b}_n = 1/a_n = -1, m_i) \end{array}\right\} \quad (22)$$

with

$$P(\hat{a} = -1/a = 1, m_i) = \frac{1}{2}erfc\left[\frac{-R(0) + IIS(m_i)}{\sqrt{2\sigma^2_b}}\right] \quad (23)$$

It should be noted that in 23 , σ^2_b is the variance of the noise at the output of the equalizer, therefore 23 becomes:

$$P(\hat{a} = -1/a = 1, m_i) = \frac{1}{2}erfc\left[\frac{-R(0) + IIS(m_i)}{\sqrt{2\sum_{p=1}^{M}\sum_{i=0}^{K_1}\left|c_{opt_{p,i}}\right|^2\sigma_{B_p}}^{2}}\right] \quad (24)$$

with σ_{B_p} the noise at the input of each sensor which are assumed to be identical

4. RESULTS

The theoretical performance of different structures of the equalizer is calculated using the analytical expressions given in section 3 for systems using a QPSK modulation, with a bit rate of 1.2 Mbits/s, and the $IIS(m_i)$ formed by 6 adjacent symbols to the symbol of interest: I_n.

The average performance is calculated for several different generated random phases $\theta_{i,j}$ of the channel given by equations 5 and 6.

4.1. AWGN Channel

Consider first of all, an AWGN channel, i.e $IIS(m_i) = 0$ and $R(0) = 1$. The results are illustrated in Fig. 2

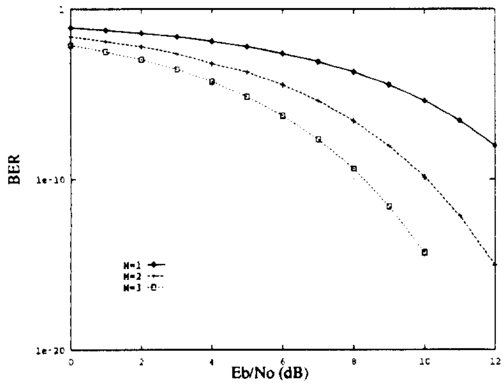

Figure 2 -BER for an AWGN for 1 to 4 sensors

The BER decreases when the number of sensors increases. This dependence is due to the fact that, in the case of an AWGN channel (no IIS), we have $\sum_{p=0}^{M}\sum_{i=0}^{K_1}\left|c_{optp,i}\right|^2 = \frac{1}{M}$, i.e the signal to noise ratio is M times larger than at each sensor's input. In fact at the output of the M-LE or M-DFE the signal is in-phase combined in contrary to the noise which is summed in an incoherent manner.

4.2. Equalization test

Consider the following two ray channel shown in Table 2 :

Coefficients	Delay (us)	Power (dB)
1	0.0	0.0
2	3.2	0.0

Table 2 : Two path channel model

In this model the delayed signal has the same power as the direct one and the delay is about 4 times the symbol duration.

The results illustrated in Figure 3 are obtained for three different equalization schemes, namely single sensor equalization with 5 and 17 coefficients, a purely spatial filter, i.e only one coefficient on each sensor, and finally a 2 sensor equalizer with 9 coefficients on each sensor. The $IIS(m_i)$ is formed by 8 symbols adjacent to the current one.

The performance of a purely spatial filter and a 2 sensor equalizer are better than that of a single sensor equalizer.

Together with their capability to introduce a gain in the signal to noise ratio, they show interesting performance in eliminating ISI, particularly the M-LE .

It should also be noted that a large number of coefficients is needed for the single sensor LE to obtain a satisfactory BER, in opposition to the M-LE; even with 1 coefficient on each sensor, it shows better performance

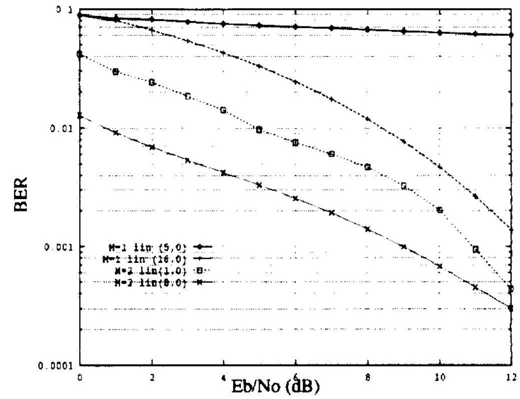

Figure 3 -Two path channel model for equalization test

4.3. TU Channel

Consider the TU channel given in Table 1. Figure 4 illustrates the BER obtained for a single sensor equalizer and a purely spatial filter (i.e only 1 coefficient on each sensor). The BER obtained when a decision loop of 4 coefficients is added to these structures is also shown.

The performance of the purely spatial filter becomes interesting for more than 2 sensors compared to that for a single sensor, in both the linear and the DFE scheme. In addition the decision loop improves the performance. The M sensor spatial filter tries to combine the paths strongly correlated to the direct path (small delays, less than T) and to null the paths presenting low correlation to the direct path (large delays, greater than T). However the decision feedback loop eliminates the ISI introduced by the previously detected symbols on the present estimated symbol.

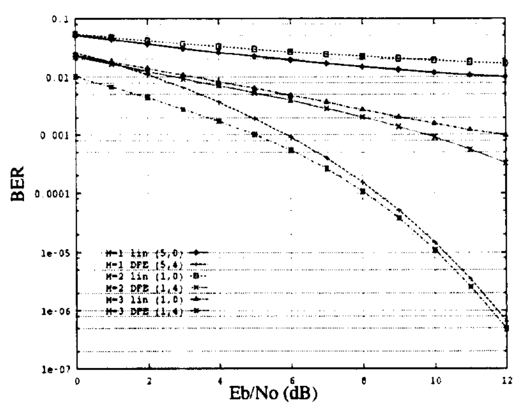

Figure 4 - BER for Spatial filters and 1 sensor equalizer

The TU model shows large delays between the different paths, thus the 2 sensor spatial filter, having two degrees of freedom (adaptive antenna theory) is unable to compensate for all the paths. But with three sensors satisfactory BER are obtained, and if a decision loop is added, the performance is improved.

Figure 5 represents the BER obtained for a M-LE (5 coefficients on each sensor) and M-DFE (5,4) scheme. The performances obtained are significantly improved.

This is due to the combined effects of the gain in signal to noise ratio at the output of the M-LE or M-DFE and of their capability to eliminate ISI. The decision loop contributes furthermore in the elimination of the ISI.

The BER obtained with these multisensor equalizers clearly indicates the significant performance improvements as compared to a single sensor equalizer.

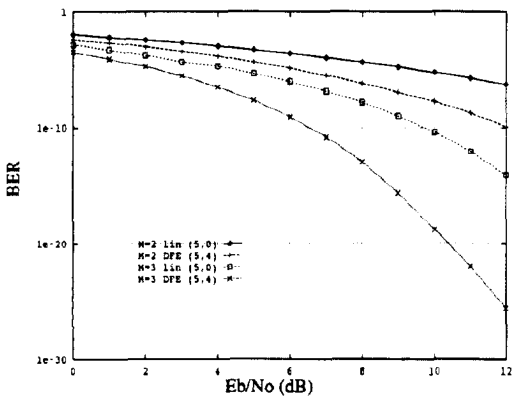

Figure 5 -BER of M-LE and M-DFE

4.4. RA model

Consider the next channel model given in Table 1, the Rural Area model. Firstly the performance of a purely spatial filter and a FIR filter, linear and with a decision loop, (fig.6) is compared.

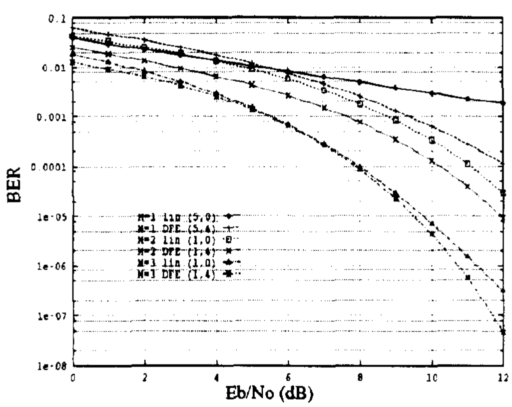

Figure 6 -BER for Spatial filters and 1 sensor equalizer

For this model the purely spatial filter offers better performance than the 1 sensor equalizer, even in the case of a spatial filter with 2 sensors only, contrary to the results obtained for the TU model. Furthermore the BER decreases with the number of sensors.

It can be observed that the delays betweeen successive paths for the RA model, in Table 1, are all equal to 0.1 μs, where as the symbol duration is 1.686 μs. Even the delay between the direct path and the 6th path is 0.6 μs, so they can still be considered to be correlated paths. Therefore the 2 sensor filter (2 degrees of freedom) offers better performance than in the case of the TU channel, and also better results than the 1 sensor equalizer. The improvement in performance due the decision loop is also verified.

For the same channel (RA) model, the BER obtained with a M-LE (5 coefficients on each sensor) and a M-DFE (5,4) is shown on Figure 7. The results confirm once more the ability of the multisensor equalizer to eliminate ISI and to introduce a gain in the signal to noise ratio at its output.

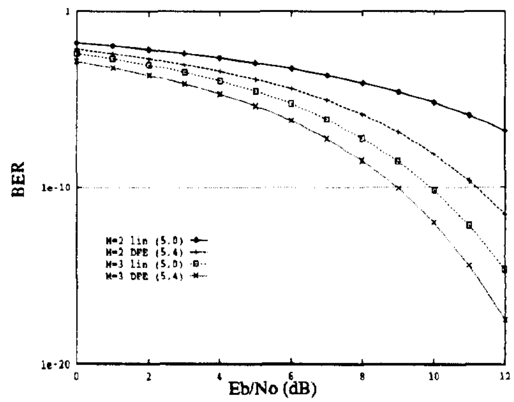

Figure 7 -BER of M-LE and M-DFE

4.5. HT model

The remarks which can be done from the results obtained for the Hilly Terrain channel model are the same as those observed for the RA channel model.

5. CONCLUSIONS

A theoretical study of the performance (BER) of a M-LE or M-DFE with the MMSE criterion has been done for three different channel models given by the GSM recommendations. After a comparative study of the results, the following conclusions can be drawn:

Firstly for the AWGN, the signal to noise ratio is increased by a factor M, the number of sensors in the equalizer. Otherwise for the other four channel models (Equalization test, TU, RA and HT) a purely spatial filter with three sensors offer better BER than a single sensor equalizer. however a definite superiority of the multisensor equalizer is observed from the results, the best BER are obtained. Moreover if a decision loop is added to the M-LE the performance are improved.

Finally an M-DFE (3-sensors (5,4)) presents a very interesting BER.

6. REFERENCES

[1] PROAKIS J.G, " *Digital Communications*", Mc GrawHill.

[2] R.E BLAHUT, " *Digital Transmission of Information*", Addisson-Wesley.

[3] K.E. SCOTT, "*Adaptive Equalization and Antenna Diversity in Digital Radio*", Phd Thesis, Department of Electrical and Computer Engineering, CALGARY, ALBERTA, Aug 1991.

[4] S. BULJORE, J.F. DIOURIS and J. SAILLARD, "*Comparison of Structures of Single and Multisensor for Blind Equalization of the Radiomabile Channel*", COST 229, BORDEAUX, 1992

[5] J.F. DIOURIS, "*Comparison of Different Structures for Blind Adaptive Beamforming and Equalization*", 27th ASILOMAR Conference on Signals, Systems and Computers, PACIFIC GROVE, CALIFORNIA, Nov 1993.

[6] GSM 05.05-DCS (Version 3.0.0) pp 19-22.