Wigner-Ville Distribution: An Important Functional Block for Radar Target Detection in Clutter

Simon Haykin and Tarun Bhattacharya

Communications Research Laboratory, McMaster University
Hamilton, Ontario, Canada L8S 4K1

Abstract

In this paper we describe an application of the Wigner-Ville distribution (WVD) for the detection of a radar target signal buried in a strong clutter background. The WVD transforms the received radar signal into a time-frequency image that accounts for the nonstationary nature of the radar signal. The cross-terms, an inherent feature of the WVD, play a constructive role in this application. In particular, their presence enhances the visibility of a target in the WVD image in a unique and significant way. The WVD image provides a common input to a pair of channels, with one channel matched to clutter and the other channel matched to target plus clutter. Each channel consists of a principal components analyzer (for feature extraction) followed by a multilayer perceptron classifier. Experimental results, based on real-life, radar data, are presented to demonstrate the superior performance of the new detection strategy compared to a conventional constant false-alarm rate (CFAR) processor.

1. Introduction

In this paper we describe a novel strategy for the detection of a radar target signal buried in a strong nonstationary clutter background. The best way to visualize the task is to think of it as that of "finding a needle in a haystack". The detection strategy involves three signal processing tools:

- time-frequency analysis
- feature extraction
- pattern classification.

An important notion guiding the development the new detection strategy is the *principle of information preservation*. This principle states that the information content of a received signal (irrespective of its origin) should be preserved in its essential form until the system is ready for decision-making. With this notion in mind, we have chosen the *Wigner-Ville distribution* (WVD) to perform the time-frequency analysis. The WVD is known for its optimal information-preserving property.

Aspects of the WVD pertinent to the objectives of the paper are discussed in Section 2. In Section 3 we briefly describe the remaining two functional blocks: feature extraction using principal components analysis, and pattern classification using a multilayer perceptron. These latter two functional blocks are implemented by means of self-organized and supervised neural networks, respectively. Section 4 presents experimental results based on real-life radar data. The paper concludes with some final remarks in Section 5.

2. Wigner-Ville Distribution

In many operational radar environments, the nonstationary nature of the received radar signal mandates the use of some form of *time-frequency analysis* (TFA), which can clearly bring out the nonstationary behavior of the signal. The important virtue of TFA is that it provides an indication of the specific times during which certain spectral components of the signal are observed. However, because of the disjoint nature of the time and frequency spaces, it is impossible to carry out TFA without sacrificing some property of interest. This has lead to a plethora of TFA techniques, and which have been applied to various problems. However, since this processing step is critical to the whole detector, we devote a short sub-section to the specific reasons for choosing the Wigner-Ville distribution (WVD) in this paper.

Time-frequency analysis can be performed (maybe equally effectively) using any one of the many different distributions available in the literature. However, as it turns out, none of these distributions are effective for "every" signal class and the choice of the distribution is situation- and application-specific. In any event, TFA techniques may be classified broadly into linear and nonlinear methods. An important subclass of the latter are the so-called "bilinear distributions", of which the Wigner-Ville distribution (WVD) is an important member. Given a signal x(t), the WVD of x(t) is defined by [1]

$$W_x(t,f)=\int_{-\infty}^{\infty} x^*\left(t-\frac{\tau}{2}\right)x\left(t+\frac{\tau}{2}\right)e^{-j2\pi f\tau}d\tau \qquad (1)$$

where τ is a dummy variable, and the asterisk denotes

1058-6393/95 $4.00 © 1995 IEEE

complex conjugation.

At first glance, the reason for choosing this subclass is not obvious, but if we recall that one of the realizations we are seeking is a behavior akin to t-f energy density, then we must try to satisfy a local counterpart of the global energy (or power) conservation principle. That is,for any two signals, u,v and arbitrary constants α and β, we have

$$E_{\alpha u + \beta v} = \alpha^2 E_u + \beta^2 E_v + \alpha\beta^* E_{u,v} + \alpha^* \beta E_{v,u} \quad (2)$$

where E is a measure of power either in the time domain or the frequency domain. In terms of the joint time-frequency distribution $C_x(t,f)$, we want

$$C_{\alpha u + \beta v}(t,f) = \alpha^2 C_u(t,f) + \beta^2 C_v(t,f) \quad (3)$$
$$+ \alpha\beta^* C_{u,v}(t,f) + \alpha^* \beta C_{v,u}(t,f) \left(\alpha^* \beta C_{v,u}(t,f) \right)$$

which clearly demands bilinearity. In addition to this important property, other properties of interest to the signal detection problem are:

- Time-shifting property
- Frequency shifting property
- Marginal property

One other significant advantage of the WVD that is relevant to bilinear TFA, results from the availability of a unified formulation [1]. Specifically, we may state that any bilinear joint time-frequency distribution can be described in terms of a kernel Φ as follows:

$$C_x^{\Phi(t,f)} = \iint W_x(t-u, f-\theta)\, \pi(u,\theta)\, du\, d\theta \quad (4)$$

where $\pi(.,.)$ is the two-dimensional Fourier transform of a kernel Φ. Equation 4, in effect, states that all members of the bilinear class can be expressed as a smoothing over the WVD. Also, as has been shown by Janssen [2], for the important smoothing functions of Gaussian pulses, WVD requires the minimum amount of smoothing to generate a positive distribution for all signals. Moreover, the extent of smoothing required is consistent with the t-f uncertainty principle.

A criticism that is often levelled against the WVD is the generation of cross-terms or "ghost" laws due to the combined presence of two (or more) signal components. Recognizing that in a clutter-dominated environment cross-terms only arise when a target signal is present, it can be argued that the presence of cross-terms is in fact an asset. We say so because they provide another feature that can enhance the visibility of a target in the time-frequency image resulting from the application of the WVD. Indeed, cross-terms contribute to the optimal information-preserving property of the WVD. Most importantly, as demonstrated later in Section 4, the generation of cross-terms transforms a "barely visible" target into a "clearly visible" one in signal processing terms.

3. Structure of the Receiver

Unfortunately, the use of TFA leads to a significant increase in the amount of redundant information contained in the time-frequency image of a radar signal. To improve computational efficiency, it is therefore necessary to follow up the TFA with some form of data compression or feature extraction. A signal processing tool that is well suited for this task is *principal components analysis* (PCA). Basically, PCA performs an eigendecomposition on a matrix (in our case, the time-frequency image of the incoming radar signal), orders the eigenvalues in descending order, and retains the eigenvectors associated with the most significant eigenvalues. The resulting output is represented by the combination of eigenvectors retained by the PCA. Thus, the PCA is instrumental in extracting a set of *features* from the WVD image that is *optimum*, in that the original WVD image can be reconstructed from these features in a minimum mean-square error sense. In other words, information loss brought on by the extraction of features by the PCA is kept to a minimum.

The final task of the receiver is that of *pattern classification*, the purpose of which is to distinguish between two WVD images on the basis of the features extracted by the PCA. One image pertains to the presence of clutter alone. The other image pertains to the combined presence of clutter and the target signal of interest. For the application at hand, we therefore have a nonlinearly separable pattern classification problem on our hands.

Figure 1 shows a block diagram of a neural network-based implementation of the detection strategy described herein. It consists of two channels, one termed the *clutter channel* and the other termed the *target channel*. Both channels are fed from a common input, representing the WVD image. Each channel consists of a PCA network followed by a multilayer perceptron for pattern classification. The PCA networks operate in a self-organized fashion, whereas the multilayer perceptrons are designed in a supervised manner.

Each of the multilayer perceptrons for pattern classification in the clutter and target channels has three output nodes. The rationale for the choice of three output nodes is explained elsewhere [3]. The outputs of the two multilayer perceptron classifiers are *linearly combined* into a single output node, where the final decision is made. If a preset threshold is exceeded by the overall output of the receiver, a decision is made that a target is present;

otherwise, a decision is made that the received radar signal consists of clutter alone. The important point to note however is that the synaptic weights of both perceptrons are *trained simultaneously* by presenting to the whole network a set of WVD images known to contain clutter only, or target signal plus clutter.

4. Experimental Results

To test the performance of the new radar receiver, we performed a case study involving the detection of a *growler* floating in an ocean environment. A growler is a small piece of ice that is broken off an iceberg. Typically, it is about the size of a grand piano; but recognizing that about ninety percent of the volume of ice lies below the water surface, a growler represents an object large enough to be hazardous to navigation in ice-infested waters, such as those found on the East Coast of Canada during the Spring and early Summer. The radar task at hand is that of detecting a growler in the presence of sea clutter. For the collection of radar data representative of this environment, an instrument-quality radar system called the IPIX radar was used. The IPIX radar is a fully coherent, polarimetric, X-band radar system, equipped with computer control and a digital data acquisition capability. The present study is confined to the use of coherent data collected under the polarimetric condition of horizontal transmit and horizontal receive.

To appreciate the importance of the WVD for the radar detection problem at hand, we present three sample WVD images of real radar returns representing three different situations:

- strong radar returns from a large growler, shown in Fig. 2a
- relatively weak radar returns from a small growler, shown in Fig. 2b
- sea clutter alone, shown in Fig. 2c.

Each figure also includes plots of the actual time series and its power spectrum, shown along the horizontal and vertical axes, respectively. From these figures, we see that the WVD image presents a much clearer picture about the presence or absence of a target than either the time series or the power spectrum acting alone. In particular, the WVD images in Figs. 2a and 2b exhibit a *zebra-like pattern alternating between black and white narrow stripes*, which occupy an area located between the instantaneous frequency plot of the target (growler) centered around zero Hertz and that of the clutter. This unique and constructive pattern is indeed a manifestation of the cross-terms. The important point to note is that the presence of the zebra-like pattern is almost unaffected by how strong or weak the target signal is. This appears to confirm the assertion that we made earlier, namely, that the generation of cross-terms due to the presence of multicomponents, which is an inherent feature of the WVD, is indeed helpful to the radar detection process for the task at hand.

Figure 3 shows the actual power of the radar returns versus the output of the two detectors for a long dwell (around 35s) along a range swath of 200 m. All the figures have been normalized separately to remove any bias introduced by changes in the dynamic ranges. The most significant observation is the ability of the neural network to separate the two classes significantly, as could be expected from a nonlinear processor such as a neural network.

The detection results for three different approaches, namely Doppler, CFAR, two-node neural network-based receiver and three-node neural network-based receiver are shown in Fig. 4. It appears, from Figure 4 that at high false-alarm rates the Doppler CFAR performs better. This is a cost we have to pay for using a nonlinear classifier such as a neural network; the neural network is trying to classify any pattern as belonging to either class, whereas CFAR is able to smooth out the two classes over an overlapping region. When, however, the false-alarm rate is low, which is how it should be in practice, we see from Fig. 4 that the neural network-based receivers outperform the CFAR processor, and the neural network-based receiver with three output nodes per channel outperforms the receiver with two output nodes per channel.

5. Conclusions

In this paper we have described an important application of the Wigner-Ville distribution for the enhanced detection of a target signal buried in a clutter background. The WVD forms the front end of an adaptive radar receiver. Basically, the receiver consists of two channels that are fed from a common WVD image; each channel consists of a feature extractor (implemented in the form of a principal components analyzer) followed by a multilayer perceptron classifier. The superior performance of the new detector over a conventional Doppler CFAR processor has been demonstrated using real-life radar data.

References

[1] Cohen, L., "Generalized phase-space distribution functions", J. Math. Physics, vol. 7, pp. 781-786, 1966.

[2] Janssen, A.J.E.M., "Bilinear phase-space distribution functions and positivity", J. Math, Phys., vol. 26, pp. 1986-1994, 1985.

[3] Haykin, S., and T. Bhattacharya, "Modular Learning Strategy for Target Detection in Clutter", submitted to IEEE Transactions on Signal Processing.

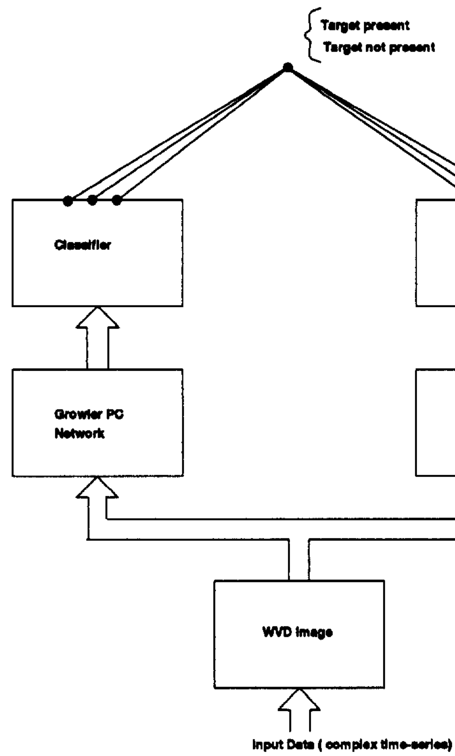

Figure 1: Block Diagram of the two channel receiver

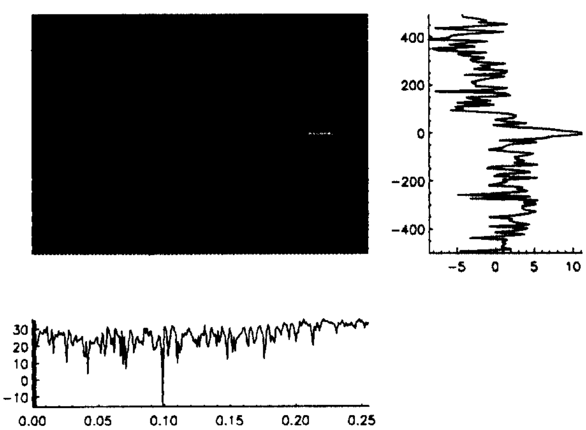

Figure 2(a): WVD for a clearly visible growler

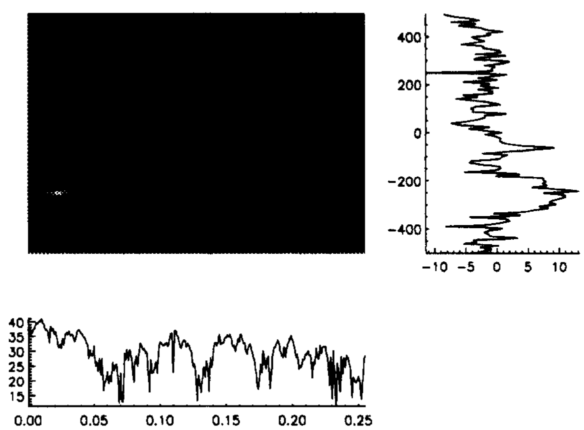

Figure 2(b): Wigner-Ville Distribution for a barely visible growler

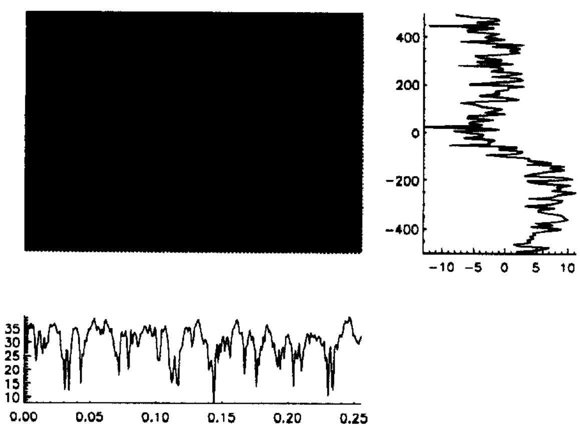

Figure 2(c): WVD for sea clutter

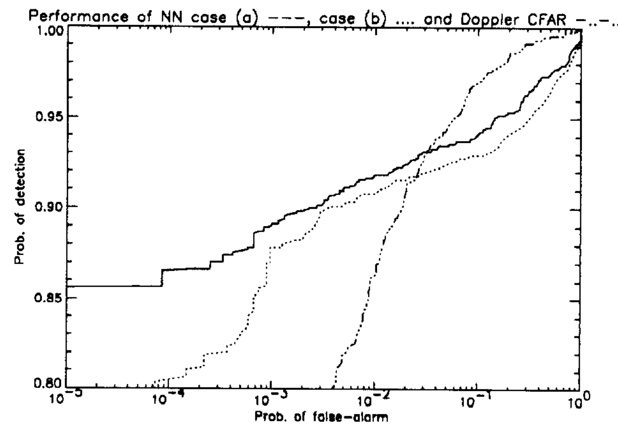

Figure 4: The composite ROC

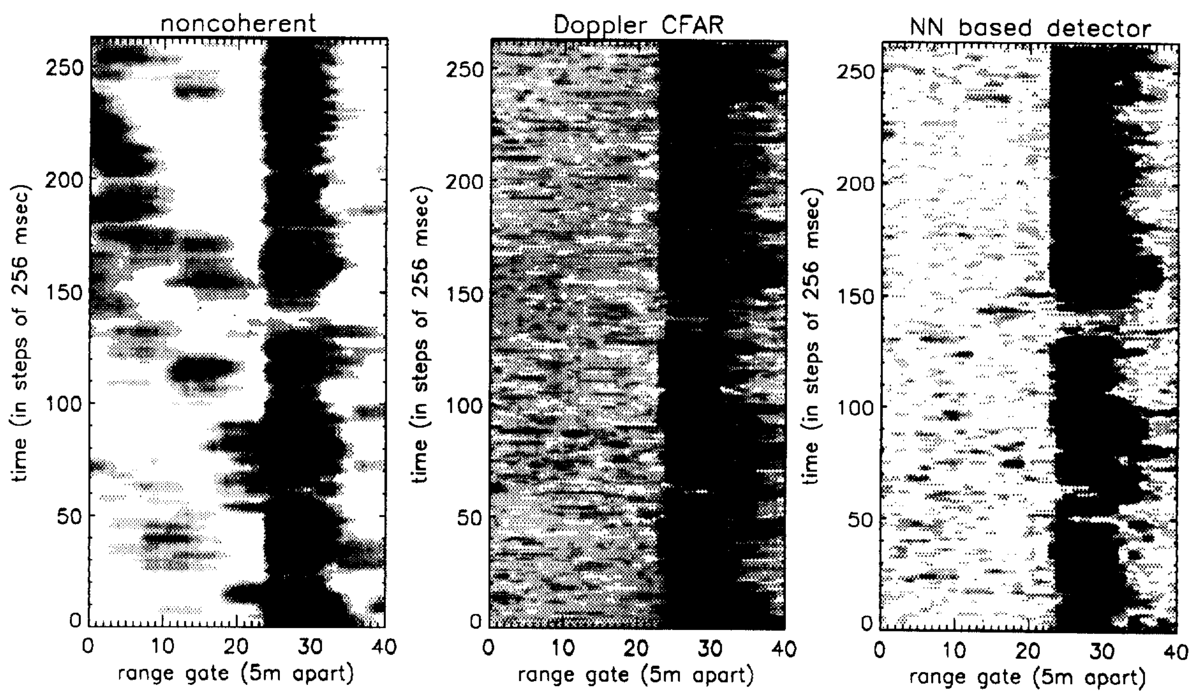

Figure 3: The detection statistics for three different schemes