CHANNEL IDENTIFICATION USING SECOND ORDER CYCLIC STATISTICS*

Zhi Ding and Ye Li
Department of Electrical Engineering
Auburn University
Auburn, AL 36849-5201
E-mail: ding@eng.auburn.edu

Abstract

Recent work by Gardner [9] presented an interesting use of cyclostationarity for channel identification in PAM data communication systems. This paper presents new results that address the identification of linear rational channels with higher than minimum bandwidth based on the use of second order cyclic statistics. Unlike the approach taken in [9], we show that channel identification is achievable for a class of linear channels without the need for a pilot tone or training periods. Moreover, channel identification based on cyclic statistics does not preclude Gaussian or near Gaussian inputs.

1 Introduction

Most blind equalization schemes begin by sampling the channel output at the baud rate [1, 2, 3] and producing a stationary channel output sequence to be processed. Consequently, blind channel identification based on input/output statistics (without direct access to the channel input) must require the use of higher than second order statistics [1] since second order statistics is only sufficient to recover the magnitude but not the phase of the channel transfer function. Some well-known blind equalization schemes based on explicit or implicit use of higher order statistics can be found in [1, 2, 4, 5, 6, 7, 8]. Due to the long data length necessary to estimate higher order statistics, algorithms based on higher order statistics tend to be rather slow.

It is important to note, however, that the actual analog channel outputs of PAM and QAM systems are in fact cyclostationary processes instead of stationary. Thus a very important question arises as to whether or not it is possible to identify the unknown linear channel based solely on the second order (cyclic) statistics of the cyclostationary channel output. Gardner in [9] recently investigated the use of second order cyclic autocorrelation in channel identification and proposed a scheme requiring the use of a training period during which the unknown input data is transmitted at a very slow rate. This training period is effectively equivalent to a period of near-distortionless data transmission for training, since for very slow transmission the ISI becomes negligible. Assuming certain rank condition, Tong et al. [10] have also proposed a new FIR channel identification scheme based on the use of second order statistics of the channel output sampled at higher than the baud rate.

Still, the key issue needs to be addressed is whether or not second order statistics alone would be sufficient for blind channel identification without training. Motivated by the work of [9] and [10], this correspondence is aimed at establishing the channel identifiability based on second order cyclic spectra (or autocorrelations) and at deriving various conditions under which blind channel identification can be accomplished. We show that for a class of channels with rational transfer functions, the channel dynamics can be uniquely identified through the use of cyclic spectra of the cyclostationary channel output signals. The motivation is that, in contrast to $T$-spaced blind equalizers which require the use of higher order statistics, channel output signal sampled at higher than baud rate can contain important second order statistical information for channel identification. Since second order (cyclic) statistics require fewer data to estimate, faster algorithms exploiting the cyclostationarity of channel output may be developed.

The rest of this paper is divided into the following sections. In Section 2, the problem of blind channel identification/equalization is outlined and the fundamentals of cyclostationarity and cyclic statistics are described. In Section 3, the channel identifiability based on second order cyclic statistics of the channel output is shown and a channel identification procedure is presented as a constructive proof. Simulation results are given in Section 4.

2 Problem Description

A typical QAM (quadrature amplitude modulated) data communication system, simply consists of a linear unknown channel which represents all the inter-connections between the transmitter and the receiver. The transmitter generates a sequence of random input data $\{a_k\}$, each element of which comes from a finite alphabet $\mathcal{A}$ (or constellation) of the QAM symbols. The data sequence $\{a_k\}$ is sent through a linear channel whose output $z(t)$ is received by the receiver. The function of the receiver is to restore the original data $\{a_k\}$ from the observation $x(t)$ by outputting a sequence of estimates for $\{\hat{a}_k\}$.
The communication channel is assumed to be linear and causal with impulse response $h(t)$. The input/output relation of the QAM system can be written as

$$z(t) = \sum_{k=-\infty}^{\infty} a_k h(t-kT-t_0) + w(t), \quad a_k \in A,$$  \hspace{1cm} (2.1)

where $T$ is the symbol baud period. The noise $w(t)$ is stationary, white, and independent of channel input $a_k$, but not necessarily Gaussian. When the channel linear distortion is large enough so that the channel output has a closed eye-pattern, equalisation is needed to remove the intersymbol interference (ISI). Due to the presence of ISI, the recovery of the input signal sequence $a_k$ requires that the channel impulse response $h(t)$ be identified. The channel identification process is explicit in nonlinear channel equalisation schemes such as decision feedback equaliser (DFE) and maximum likelihood sequence estimator (MLSE) and implicit in linear equalisations where the channel inverse is identified.

### 2.1 Channel Identification Based On Stationary Statistics

In traditional blind equalisation systems, the channel output is sampled at the baud rate $1/T$ as the timing clock is known. The sampled channel output

$$z(nT) = a_k \odot h(nT-t_0) + w(nT)$$  \hspace{1cm} (2.2)

is a stationary process. Since both $a_k$ and $h(nT-t_0)$ are unknown to the receiver, it is well known [1] that channel identification of $h(nT-t_0)$ based on the second order statistics of the sampled channel output $z(nT)$ is insufficient to identify channels $h(nT-t_0)$ of mixed phase. As a result, traditional blind equalisation schemes require the use of higher order statistics (or equivalently polyspectra) to find the transfer function phase in channel identification. Once the sampled (non-zero) channel impulse response $h(nT-t_0)$ is identified, the actual input sequence $\{a_k\}$ can be recovered from the sampled channel output sequence $\{z(nT)\}$ through the use of existing schemes such as MLSE.

Unfortunately, time-average approximation of higher order statistics demands a large number of data samples compared to second order statistics. Furthermore, symmetric channel input constellations typical in QAM systems result in no new information in third order statistics. Thus channel identification requires at least fourth order statistics, which worsens the time-average approximation problem. This problem can be further exacerbated when certain coding schemes utilised by the channel input may cause the channel output to have zero fourth order cumulants [11], resulting in the need for even higher order statistics whose approximations are even harder to obtain. These limitations are some of the obstacles to the wider application of existing blind equalisation schemes.

### 2.2 Cyclostationarity of the Channel Output Signal

While the use of higher order statistics is necessary for the identification of non-minimum phase channels based on channel outputs sampled at the baud rate, the actual channel output $x(t)$ as in (2.1) is cyclostationary instead of stationary. It can be verified that for stationary channel input $a_k$ and noise $w(t)$ with autocorrelation functions

$$R_a[k-l] = E(a_k a_{k+l}^*), \quad R_w(t_2-t_1) = E(w(t_1) w(t_2)),$$

we have

$$R_a(t_1,t_2) = E(a(t_1) a^*(t_2)) = R_a(t_1 + T, t_2 + T),$$  \hspace{1cm} (2.3)

where the symbol duration $T$ is the fundamental period of the cyclostationary process.

Hence, failure to identify the channel using second order statistics of the stationary channel output samples at the baud rate does not necessarily prevent the use of second order statistics of the analog cyclostationary channel output signal or the channel output sampled at a higher than baud rate. It is important to learn whether or not a channel can be identified based on the second order statistics of the cyclostationary channel output or the cyclostationary channel output sampled at a rate higher than $1/T$. Finding an answer to this crucial question is the focal point of this paper.

### 2.3 Second Order Statistics of Cyclostationary Channel Output

Our notations of second order statistics for cyclostationary processes generally follow those in the pioneer works of Gardner [12]. Specifically, for cyclostationary process $z(t)$ with fundamental period $T$, its cyclic autocorrelation function (CAF) is defined to be

$$R^c_a(r) = \frac{1}{T} \int_{-T/2}^{T/2} R_a(t + \frac{r}{T}, t) e^{-j2\pi\alpha t} dt, \quad \alpha = k/T.$$  \hspace{1cm} (2.4)

Correspondingly, the spectral-correlation density (SCD) is defined as

$$S^c_a(j\omega) = \int_{-\infty}^{\infty} R^c_a(r)e^{-j\omega r} dr.$$  \hspace{1cm} (2.5)

For QAM signal $z(t)$ of (2.1), its SCD can be shown to relate to the input power spectrum density (PSD) and the frequency response of the channel through [12]

$$S^c_a(j\omega) = \frac{1}{T} H(j\omega + j\alpha) H^*(j\omega - j\alpha) \int_{-\infty}^{\infty} H(t) e^{-j\omega t} dt, \quad \alpha = k/T,$$  \hspace{1cm} (2.6)

in which $H(j\omega) = \int_{-\infty}^{\infty} H(t) e^{-j\omega t} dt$ is the channel frequency response and $S_a(\omega) = \sum_{k=-\infty}^{\infty} R_a[k] e^{-j\omega kT}$ is the PSD of the stationary channel input sequence $a_k$. It is important to note that for white stationary noise $w(t)$, we have

$$S^c_a(j\omega) = \int_{-\infty}^{\infty} R^c_a(r) e^{-j\omega r} dr = N_0 \delta[k].$$  \hspace{1cm} (2.7)

The PSD of the stationary input must be known. If the channel input $\{a_k\}$ is i.i.d. with zero mean, its autocorrelation function is simply

$$R_a[k] = E(\{a_k \oplus a_k\}) = E(|a_k|^2) \delta[k],$$  \hspace{1cm} (2.8)

and its PSD is constant $S_a(\omega) = R_a[0] = E(|a_k|^2)$.  

---

336
3 Channel Identification based on Channel Output SCD

From the preliminary discussions of Section 2, it is hence clear that the desired objective is to identify the channel frequency response $H(j\omega)$ from the SCD of the channel output $x(t)$. Without loss of generality, our discussion will be limited to the case of zero mean, i.i.d. input. Thus, our objective is to identify $H(\omega)$ from

$$S_{x}^k(\omega) = \frac{R_{x}[0]}{T} H[j(\omega + \frac{k\pi}{T})]H^*[j(\omega - \frac{k\pi}{T})]e^{-j\omega k} N_0 \delta[k], \quad k = 0, \pm 1, \pm 2, \ldots$$

(3.1)

It is apparent that our result directly applies also to correlated channel inputs if the correlation is known.

3.1 Basic Assumptions

Assume the channel transfer function is given or can be closely approximated by the following rational transfer function

$$H(j\omega) = A e^{-j\omega t_d} \prod_{i=1}^{M} \frac{[j(\omega + \pi \alpha - \pi z_i)]}{\prod_{i=1}^{N} [j(\omega - \pi \alpha - \pi p_i)]}$$

(3.2)

The orders of the channel model $(M, N)$ should be no less than those of the actual rational channel. We require the channel to be strictly stable such that all of its poles are inside the stability region $\{Re(p_i) < 0\}$. We also require the channel to have no allpass section. The channel bandwidth $B_\nu$ is required to be above the minimum bandwidth, $B_\nu > 0.5/T$.

3.2 Uniqueness of SCD

Our identification of the channel transfer function relies on

$$S_{x}^k(\omega) = A^2 \frac{R_{x}[0]}{T} e^{-j\omega t_d} \prod_{i=1}^{M} [j(\omega + \pi \alpha - \pi z_i)] \prod_{i=1}^{N} [j(\omega - \pi \alpha - \pi p_i)]$$

(3.3)

$$\times \prod_{i=1}^{M} [-j(\omega - \pi \alpha - \pi z_i)] \prod_{i=1}^{N} [-j(\omega + \pi \alpha - \pi p_i)]^*$$

The cyclic spectrum is a rational function of the complex variable $j\omega$. It has $2N$ poles that are not on the imaginary axis in $C$ because of the stability condition $Re(p_i) < 0$. Thus the complex, rational function $S_{x}^k(\omega)$ is analytic on a complex open set $\Omega$, which includes the entire real axis. We now invoke the following well known lemma from complex analysis:

Lemma 3.1 If $S_1(z)$ and $S_2(z)$ are two analytic functions in a region $\Omega$ and $S_1(z) = S_2(z)$ for all $z$ in some set which has a limit point in $\Omega$, then $S_1(z) = S_2(z), \forall z \in \Omega$.

The cyclic spectrum $S_{x}^k(\omega)$ is therefore uniquely determined by its values given over any frequency (open or closed) interval. In other words, the complex function $S_{x}^k(\omega)$ is uniquely specified by

$$S_{x}^k(\omega), \quad \omega \in [\omega_1, \omega_2] \subset \mathbb{R} \quad (3.4)$$

3.3 Identifiability

It is well known that the channel poles can be identified from the PSD

$$S_{x}^k(\omega) = A^2 \frac{R_{x}[0]}{T} \prod_{i=1}^{M} [j(\omega - \pi z_i)]^2 \prod_{i=1}^{N} [j(\omega - \pi p_i)]^2 + N_0$$

(3.5)

except for those allpass fractions that have $p_i = -z_i$. Since we require the channel to have no such pole/zero relationship, the key to channel identification is to identify all the zeros based on the cyclostationary information contained in $S_{x}^k(\omega)$.

Given that all $N$ poles have been identified, define

$$D_{x}^k(\omega) = S_{x}^k(\omega) \prod_{i=1}^{N} [j(\omega + \pi \alpha - \pi z_i)]^2 [j(\omega - \pi \alpha - \pi p_i)]^2$$

(3.6)

Then from (3.4), the zeros of $D_{x}^k(\omega)$ are at

$$\pm Re(z_i) + jIm(z_i) = j\alpha, \quad i = 1, 2, \ldots, M.$$ Without loss of generality, choose $\alpha > 0$. Find all the zeros of $D_{x}^k(\omega)$ to form a set $U_M$.

(1) Find the element $a_M$ in $U_M$ that has the maximum imaginary part. If it is not unique, arbitrarily choose one. Then we have

$$Im(z_M) = Im(a_M) = \pi \alpha, \quad Re(z_M) = Re(a_M).$$

(2) Form a new set $U_{M-1}$ by removing two zeros:

$$U_{M-1} = U_M - \{ \pm Re(z_M) + jIm(z_M) \mp j\pi \alpha \}$$

(3) Repeat the same process for $U_i, \quad i = M - 1, M - 2, \ldots, 1$ to identify all $M$ zeros of the channel transfer function $H(\omega)$:

$$a_i = Arg \max_{e^{j\omega t_i}} Re(z_i); \quad Im(z_i) = Im(a_i) \pm \pi \alpha, \quad Re(z_i) = Re(a_i); \quad U_{i-1} = U_i - \{ \pm Re(z_i) + jIm(z_i) \mp j\pi \alpha \}.$$ The above procedure identifies all the zeros of the channel transfer function.

Once the poles/zeros of $S_{x}^k(\omega)$ are identified, the constant gain $|A|$ and the combined delay $t_0 + t_d$ can also be identified modulo $T$. Hence, from the cyclic spectrum of the received channel output signal $S_{x}^k(\omega)$, the zero of the channel transfer function can be uniquely identified. While $\alpha$ can be any positive multiple of $1/T$, it is simpler to rely on $S_{x}^k(\omega)$. The ambiguity of a possible linear phase $\omega t_d$ is not crucial to the objective of recovering the input sequence $\{x_n\}$ since we have identified the delayed channel impulse response $h(t - t_0)$. On the other hand, the sign ambiguity in $|A|$ is inherent in the blind identification/equalization problem [1].
3.4 Existence of SCD

The major cause of ISI is the limited bandwidth of the channel. From the channel output SCD given in (2.1), the bandwidth limitation of \( H(j\omega) \) may cause the SCD \( S_{2}^{1/T}(j\omega) \) to be negligible for most of the frequency range. However, since it is assumed that the unknown channel has bandwidth higher than the minimum bandwidth, i.e., \( B_{w} > 0.5/T \), we have

\[
S_{2}^{1/T}(j\omega) \neq 0, \quad |\omega| < 2\pi(B_{w} - \frac{1}{2T}). \tag{3.7}
\]

As a result, SCD of the channel output over the frequency interval \((0.5/T - B_{w}, B_{w} - 0.5/T)\) can be used to determine the function \( S_{1}^{1/T}(j\omega) \) which can then be used to identify the linear channel.

3.5 Remarks and Comments

The proof of channel identifiability based on second order cyclic statistics outlines a procedure for channel identifiability based on SCD. Essentially, \( S_{2}^{1/T}(j\omega) \) and \( S_{2}^{1/T}(\omega) \) should first be parametrically determined from data via spectral estimation. The channel transfer function \( H(j\omega) \) can then be identified from the gain, poles, and zeros of the SCDs.

Our result differs from that of [9] in that we do not require the use of a training period during which slowly transmitted data help to identify the channel frequency response at multiples of \( 0.5/T \). Our identifiability scheme operates without the aid of a training period and is based on the channel output signal \( z(t) \) obtained during normal transmission.

While we require the channel to have no allpass section, this condition can be relaxed if the allpass poles do not result in any pole-zero cancellation in \( H(j\omega - j\pi a)H^{*}(j\omega - j\pi a) \). Under this relaxed condition, the allpass poles can also be identified using the same procedure in identifying zeros of \( D_{2}^{1/T}(j\omega) \).

It is also important to note that for stationary signals and systems, if the input is Gaussian or nearly Gaussian, it then becomes impossible to identify the unknown channel and to deconvolve the unknown input. However, if a cyclostationary output is available, our result shows that even when the input is Gaussian, channel identification and blind deconvolution may still be accomplished based on second order cyclic statistics. This feature has practical importance since recent work [11] has shown that certain constellation shaping tends to make the channel input nearly Gaussian.

4 Spectral Estimation and Simulation

The computation of the SCD must be accomplished from discrete samples of the channel output \( z(t) \). Denote the sampling period as \( T \), we have

\[
z(nT) = \sum_{k=\infty}^{\infty} a_{k} h(nT - kT + t_{c}), \quad a_{k} \in A, \tag{4.1}
\]

which is a discrete cyclostationary process if \( T < T \). Its CAF and SCD can be obtained from

\[
\hat{R}_{n}^{\omega}(\Delta T) = \frac{1}{2N+1} \sum_{n=-N}^{N} z(nT + \Delta T)z^{*}(nT) e^{-j2\pi(n+1/2)T_{m}}, \tag{4.2}
\]

\[
\hat{S}_{n}^{\omega}(\omega) = \sum_{\lambda=-\infty}^{\infty} \hat{R}_{n}^{\omega}(\Delta T) e^{-j\omega T_{m}}. \tag{4.3}
\]

The sampling interval \( T_{m} \) must be sufficiently small to avoid spectral aliasing (see [12]). Once \( \hat{S}_{n}^{\omega}(\omega) \) is obtained, we solve for a least square parametric model \( S_{n}^{\omega}(\omega/\theta) \). The resulting parameter vector \( \theta \) can then be used for pole/zero channel identification.

We simulate a BPSK \((a_{k} = \pm 1)\) system in which the data rate is 1 kbits/s and the linear channel has two conjugate poles and a nonminimum phase zero. The channel impulse response is given by

\[
h(t) = e^{-0.001t} \sin(0.829t - 1.200t)u(t). \tag{4.4}
\]

The channel output is sampled at 16kbits. Channel identifiability results are obtained for 1024, 256, and 64 data symbols, respectively. In Figure 1, the actual SCD \( S_{1}^{1/T}(j\omega) \) and impulse response \( h(t) \) (solid) are shown along with estimated results from 1024 symbols (dotted), 256 symbols (dashed), and 64 symbols (dashdot), respectively. The corresponding channel frequency responses are shown in Figure 2. Apparently, good identification results are obtained from cyclostationary statistics even with only 256 input symbols. Such a short data set would be insufficient for most identification methods based higher order statistics.

5 Concluding Remarks

The cyclostationarity of the QAM channel output can be used for the blind identification of the unknown channel. By merely using the second order cyclic statistics or cyclic spectra of the channel output, the parameters of a class of rational channels can be uniquely identified. For bandlimited channels, the channel identification can be achieved from second order cyclic statistics of the sampled channel output provided that sampling rate is higher than Nyquist frequency to avoid aliasing. The potential advantage of using second order statistics for blind equalisation may be the significant improvement in convergence rate compared to algorithms based on higher order statistics.

Our identifiability results only demonstrate the potential of exploiting the cyclostationarity of the QAM channel output for channel identification. While the identification scheme presented here need not be the most efficient, the identifiability result may lead to the future development of highly effective and fast algorithms based on second order statistics only.

References


---

Figure 1: Ideal and estimated SCD of the channel output and the impulse responses of the original and identified channels. Solid lines represent the actual responses. Dotted lines, dashed lines, and dashdot lines represent estimated results from 1024, 256, and 64 data symbols, respectively.

Figure 2: Magnitude and phase of the actual and the identified channel transfer functions. Solid lines represent the actual responses. Dotted lines, dashed lines, and dashdot lines represent estimated results from 1024, 256, and 64 data symbols, respectively.