Optimal Transversal Filter Bank for 3D Real-Time Acoustical Imaging

Emad S. Ebbini
Department of Electrical Engineering and Computer Science
The University of Michigan
Ann Arbor, MI 48109-2122

Abstract
A new approach for the optimal design transversal filter bank for parallel processing of echo data in 3D real-time acoustical imaging is proposed. The approach is based on a discretized linear time-domain formulation of the imaging equation for systems that employ either coded-aperture transducers or transmitter arrays with coded-excitation. The two approaches are mathematically equivalent and result in an imaging system architecture which is based on parallel processing of sampled vector of echo data with a bank of transversal filters. The filter coefficients are obtained through a pseudoinverse operator which is designed to decouple echoes received from different image lines.

1 Introduction
Three-dimensional real-time pulse-echo acoustical imaging requires simultaneous acquisition and processing of large number of image lines. This can be achieved using transmitter arrays with coded excitation or coded-aperture transducers. Both of these approaches have been suggested in SONAR, NDE, and medical imaging applications. Ideally, these approaches produce coded wavefronts with echoes from different directions being orthogonal. Thus the echo data can be processed simultaneously by a set of matched filters where each filter is designed to recover echoes corresponding to a specific direction in the 3D space [1]. However, it is practically difficult, if not impossible, to generate coded wavefronts using finite-aperture transducers and finite-duration codes. This renders the matched filter approach impractical since correlation between echoes from different directions could produce significant image artifacts at the outputs of the matched filters that could severely degrade the image quality.

This paper proposes a new method for the optimal design of a transversal filter bank for parallel processing of echo data. The proposed approach is based on a discretized linear time-domain formulation of the imaging equation. This linearization results in a complex (due to baseband conversion) matrix equation of which can be solved using a pseudoinverse operator. Under certain simplifying assumptions given below, the pseudoinverse operator can be implemented efficiently using a set of parallel transversal filters each designed to extract echoes from a specific direction. The number of filters in the bank (the size of the imaging operator) should be large enough in order to acquire sufficient number of image lines per transmit pulse to meet the real-time constraint. On the other hand, the size of the pseudoinverse should be kept small enough to assure robust inversion. We propose a method of controlling the data flow into the receiver by using multiple-beam receive patterns with phased arrays. An attractive feature of this approach is that it requires a single beamformer to synthesize multiple beams.

2 Theoretical Formulation
2.1 Discretized Linear Model
In the following, a 2D imaging system will be assumed for simplification and for clarity of presentation. It is also assumed that a sector format imaging system as shown in Figure 1 is used. The discretized linear formulation starts by assuming that the ROI is defined by a grid in the R, ϕ domain. The grid size is Nr x Nϕ. We assume that every point on the grid is associated with a scatterer with strength sij, a complex number.

2.1.1 The Imaging Equation
The sampled complex received data vector (in-phase and quadrature after baseband conversion) is a linear sum of echoes from all the scatterers in the ROI. Hence, the sampled data vector can be expressed as follows:

\[ f = Gs + n \]  \hspace{1cm} (1)

where f = [f1, f2, ..., fN]T is the complex sampled data vector, s = [s1, s2, ..., sN]T is the complex scatter strength in the ROI (size Ns = Nr x Nϕ), n = [n1, n2, ..., nN]T is a complex noise vector, and

\[ G = (G_1 G_2 \cdots G_{N_s}) \]  \hspace{1cm} (2)

is a spatio-temporal impulse response matrix with

\[ G_i = (g_{i1} g_{i2} \cdots g_{iN}) \]  \hspace{1cm} (3)
2.1.2 Derivation of Filter Equations

In most finite-aperture imaging systems, it is reasonable to assume the impulse response vectors, \( \{r_{i1}, r_{i2}, \ldots, r_{iN_i}\} \) along the direction \( \phi_i \) are related by a linear shift. This assumption is supported by our experience based on computer simulations as well as experimental data. The adjoint, \( G^* \), can be expressed in the form,

\[
G^* = \begin{pmatrix}
G_{11}^* \\
G_{12}^* \\
\vdots \\
G_{N_i}^*
\end{pmatrix}
\]

where \( G_{i1}^* = [r_{i1}, r_{i2}, \ldots, r_{iN_i}]^T \) is the adjoint of the impulse response matrix associated with the image line, \( \phi_i \). Under the assumption that the impulse response vectors along any image line are shifted versions of each other, the matrices \( G_{i1}^*, i = 1, 2, \ldots, N_i \) become convolution matrices associated with their respective image lines. This yields the desired solution for impulse response of the transversal filter associated with direction, \( \phi_i \),

\[
h_i^T = [g_{i1}^T g_{i2}^T \ldots g_{iN_i}^T]^T
\]

time reversed and shifted by \( T_c = 2R_n^c \), where \( g_{iN_i} \) is the impulse response at a grid point midrange along the \( \phi_i \) direction, \( R_n^c \) the depth of that grid point, \( N_i \) its order on the grid in the \( R \) direction, \( c \) is the speed of sound in the medium, and \( T_c \) is the two-way travel time from the center of the transducer to the grid point. Time reversal is used based on the same grounds as the derivation of the matched filter. In fact, without the operator, \( (GG^*)^T \), (6) yields the matched filter. The superscript, \( ^T \), is used to signify that the derived filter is an approximation of the inverse filter. In subsequent sections of this proposal, this filter will be referred to as the pseudoinverse filter.

3 Multiple Beam Phased Array Pattern Synthesis

The use of phased array receivers could prove very significant in improving the SNR of the imaging system. This would be achieved by the use of multiple beam phased array patterns which can be synthesized based on a technique described in [3, 4, 5, 6, 7]. The use of multiple beam patterns allows only echoes from a finite set of image lines to be processed simultaneously. The number of beams should be large enough to allow the imaging system to meet real-time constraints. By suppressing echoes from directions other than those being processed at a given point in time, the SNR of the system can be improved significantly (compared to the case where all echoes from all directions form the sampled data vector irrespective of the number of image lines being processed).

The main advantage of the proposed multiple beam phased-array pattern synthesis technique is that it utilizes a single conventional delay-and-sum beamformer. Furthermore, receive-only or transmit-only beamformer, depending on the coding algorithm, is needed. This results in further simplification of the system requirements. Hence, a hybrid system of coded-excitation transmitter array and a phased-array receiver could be used. Alternatively, a phased-array
transmitter and a coded-aperture receiver can be used. Both of these approaches are, at least theoretically, equivalent and result in an imaging system which is much less complex than a 3D imaging system designed based on a fully sampled 2D array with half-wavelength spacing between elements. A complete description of the multiple-beam synthesis approach can be found in [3, 4, 5, 6, 7].

4 Simulation Results

Computer simulations were performed to demonstrate the validity of the proposed discretized mathematical model. A two-dimensional imaging system was simulated which consists of 24-element linear transmitter array. The array elements were driven simultaneously using 64-chip segments of 512-chip M-sequences obtained using 9-stage shift registers with different feedback arrangements [8, 11, 15]. A total of 24 512-point sequences were available with “good” crosscorrelation properties. By good cross-correlation properties we mean that the peak cross-correlation between any two sequences of the set was less than -20 dB (with respect to the maximum autocorrelation of 512). The segmentation of the 512-point sequences into 64-point sequences produced 192 sequences to choose from with peak crosscorrelation of -13 dB. The autocorrelation of the code segments used in our simulations was impulse-like with range sidelobes below -15 dB.

A 24-element transmitter array with element-to-element spacing of 1 mm was used to transmit 24 64-chip codes from the m-sequences described above into a medium where the nominal speed of sound is 1500 m/s. The clock frequency for the code was 10 MHz. An approximately omnidirectional wavefront was obtained.

First, we use simulation results to illustrate the difference between using the matched filter approach and the pseudoinverse filter approach in the reconstruction of the scatterer distribution. For this purpose, we assume a grid of size $1 \times 31$ at 50 mm from the center of the array with equispaced scatterers from -15° to 15° in steps of 1°. A single scatterer with amplitude of 1 was positioned at 0° to examine the azimuthal point spread function (PSF) of the imaging system when implemented with both matched filters and pseudoinverse filters. Figure 2 shows the reconstructed scatterer distribution with the matched filters (solid line) and pseudoinverse filters (dashed line). In the figure, the reconstructed values using the matched filters and pseudoinverse filters are marked with ‘o’ and ‘+’, respectively, while the actual values are marked with ‘x’. In this case, the pseudoinverse approach achieves exact reconstruction at the grid points. To examine the effect of measurement noise on the quality of reconstruction, a white Gaussian noise was added to the received signal such that the SNR of the received signal defined as

$$SNR_t = 20 \log \left( \frac{||r||}{||n||} \right),$$

was 16 dB. In (7), $r$ and $n$ are the signal and noise components, respectively, of the received signal, $f$, and $||.||$ is the standard Euclidian norm. The result of this simulation is shown in Figure 3 where an error in reconstruction when using the pseudoinverse approach can be seen. Even though $SNR_t$ was only 16 dB, the peak sidelobe was -26 dB. The performance of the matched filters does not vary significantly in this case. However, it is unacceptable under both ideal and low $SNR_t$ situations where the peak sidelobe level is -14 dB.

Another test of performance for imaging operators commonly used in medical imaging is a test where a zero scattering region (a cyst) is surrounded by scattering region. This is a test of contrast resolution of the imaging system. Such a test is sometimes more meaningful in systems requiring high dynamic range as is the case in medical imaging. Figure 4 shows the result of a simulation assuming the scatterers on the grid of Figure 1 have a random distribution except for 5 scatterers (from -2° to 2°) which have been set to zero. Once again, the solid line is the reconstruction
due to the matched filter approach and the dashed line is the reconstruction with the pseudoinverse approach. Under this ideal $SNR_t$ condition, perfect reconstruction is achieved with the pseudoinverse filters while poor reconstruction is achieved with the matched filter. The simulation was repeated with an $SNR_t$ of 16 dB and the result is shown in Figure 5. One can clearly see that significant error in reconstruction now results (even though the pseudoinverse approach still provides some contrast between the cyst and the surrounding scatterers). This represents an example where the pseudoinverse approach might fail when the data is received with an omnidirectional receiver and all image lines are processed simultaneously. In an actual imaging system, it is neither necessary nor practical to process all echoes from all directions simultaneously. Instead, a finite number of image lines representing a subset of the underlying region of interest can be reconstructed at a time while echoes from other directions are blocked from entering the receiver. The complete image can be reconstructed by multiple acquisitions where on each acquisition, a different set of image lines is processed. To give an idea about the usefulness of this approach in the enhancement of the reconstruction quality, the simulation shown in Figure 5 was repeated with only 16 echoes out of 31 are processed with 16 filters designed with a reduced pseudoinverse. The reduced pseudoinverse is designed only for echoes from directions from $-15^\circ$ to $15^\circ$ with a step of $2^\circ$. Figure 6 shows the reconstructed scatterer distribution in these directions assuming echoes from remaining scatterers have been completely blocked by a multiple receive beam pattern which passes the desired echoes and blocks all other echoes. In Figure 6, the solid line and dashed line are reconstruction of scatterer distribution using matched filters and pseudoinverse filters with marks as before on 16 of 31 scatterer positions. The 'x' marks on the horizontal indicate complete blockage of echoes from undesired directions. The complete image can be formed using two acquisitions in this case. One can see that the reduced pseudoinverse significantly reduced reconstruction error when compared to the full pseudoinverse reconstruction in Figure 5. It should be noted that the results in both Figure 6 and Figure 6 are obtained with the same $SNR_t$ value of nearly 16 dB.

5 Summary and Conclusions
At this point, an important point should be stressed regarding multiple acquisitions with reduced pseudoinverse filters. The reduced pseudoinverse is designed based on a finite set of directions which is a subset of the underlying infinite dimensional space. In fact, any finite grid, no matter how large, is a finite dimensional approximation of the infinite dimensional space. The idea of using a reduced pseudoinverse is strongly coupled with the blocking of echoes from scatterers that are not aligned to grid points as defined by the reduced pseudoinverse. As demonstrated by the simulation results, this blocking does not have to be complete. This is an important practi-
tical issue since complete masking of undesired echoes is not feasible with finite aperture receivers. The "design" of the reduced pseudoinverses for multiple acquisitions is a result of a tradeoff involving the condition number of the impulse response matrix (which calls for a smaller operator), the degree of blocking of undesired echoes (determined by the beamformer and the receive array), and the real-time constraint on the system (which calls for a large enough pseudoinverse to process the necessary image lines simultaneously upon each acquisition). The "optimal design" of the set of reduced pseudoinverses should result in a PSF which suites the intended application.

We have several issues relating to the design of the "optimal operator" under various noise conditions. The objective of this paper was to provide a brief exposition of the approach and demonstrate the effect of multiple-beam receive patterns on the quality of reconstruction with the pseudoinverse operator. Some of the related issues that were investigated (results not shown):

1. An optimal rank reduction technique based on a bias-variance trade-off in the presence of white gaussian measurement noise [12, 1].

2. In cases when errors exist in both $f$ and $G$, we have shown that the solution to the total least squares (TLS) problem is a weighted minimum norm solution. The weighting matrix is derived from the SVD of the augmented matrix $[-f G]$. This means that the imaging operator can still be realized as a set of parallel transversal filters even if $G$ is corrupted [2, 9, 12].

3. We have examined 2D and 3D PSFs of different transmit array geometries and different operator designs. The basic conclusions of this paper carry to 2D and 3D. The results of this study will be reported in a later report.

Acknowledgements

The author wishes to acknowledge Professors Matthew O'Donnell at UM and A. Moeness Amin at Villanova University for helpful comments and insight on this work. The author also acknowledges graduate student Jian Shen at UM for helping with computer simulations related to this work.

References


