Adaptive Cardioid Processing

Henry Cox  
ORINCON Corporation  
1755 Jefferson Davis Hwy., Suite 904  
Arlington, VA 22202

Robert M. Zeskind  
BBN Systems and Technologies  
1300 N 17th Street  
Arlington, VA 22209

Abstract

One of the simplest practical superdirective sensors is the cardioid formed from a matched pair of orthogonal dipoles and a co-located omni-directional sensor. The ability to achieve directivity in a very small package has made this configuration a popular choice in air deployed sonobuoys. In this paper we develop algorithms for adaptively steering the null of a cardioid to achieve interference rejection. The algorithms are basically broadband but by frequency filtering it is easy to steer independently in different frequency bands. Of particular interest is the nulling of highly directional reverberation when sonobuoys are considered as multistatic active receivers. Simple adaptive algorithms are shown to be effective in simulations.

1 Introduction

An important acoustic array configuration which is widely used in sonobuoys is the three sensor configuration consisting of a matched pair of orthogonal dipoles in the horizontal plane and a co-located omni-directional sensor. The popularity of this configuration is due to its ability to provide an estimate of the direction of arrival of a signal using a very small sensor configuration.

The dipoles are essentially gradient sensors. In an acoustic field they may directly sense velocity or they may be formed by differencing two closely spaced omni-directional pressure sensors. A dipole has a directional amplitude response proportional to $\cos \alpha$ where $\alpha$ is measured from the axis of the dipole. Dipoles are broadband in that their response pattern does not change with frequency over the band for which they are small compared to an acoustic wavelength. They are superdirective in that they provide directional information and directivity gain in a configuration which is much smaller than a wavelength.

Bearing estimation usually involves cross-correlation of both dipoles with the omni-directional sensor which also serves as a phase reference. This and other types of processing can be done broadband by direct processing of the three sensor outputs or narrowband by Fourier transforming or prefiltering the outputs and performing independent processing on different frequency bins.

When a dipole is summed with an omni-directional sensor having the same sensitivity as the maximum of the dipole's directional response, the result has a cardioid amplitude response pattern proportional to $1 + \cos \alpha$. The name cardioid derives from the heart shape of this response pattern. The cardioid is also superdirective providing directivity gain is a very small package. Of particular interest is the null in its response pattern in the direction which is diametrically opposite to the maximum response direction.

The pair of matched orthogonal dipoles can be combined to form an equivalent dipole response steered to any desired azimuthal direction. This steered dipole can be summed with the omni-directional sensor to provide a cardioid with its null steered in any azimuthal direction.

In this paper we examine adaptive techniques for steering the cardioid to achieve high gain by placing its null in the direction of strong interference. Of particular interest is the use of sonobuoys as multistatic active receivers [1]. In this situation, strong forward scatter reverberation arrives from the direction of the source while echoes of interest come from a large azimuthal sector generally opposite to the source direction. In this situation, a cardioid with its null steered in the direction of the source is a very simple and attractive receiver.

The goal of the adaptive cardioid in this application is to sense the source direction and quickly steer the null before echoes can arrive from the ranges and bearings of interest.

2 Basic Relationships

Consider the co-located trio of sensors consisting of a matched pair of orthogonal dipoles and an omni-directional sensor. The outputs of all sensors are
normalized so that their maximum response to a
directional signal is unity. The axes of the dipoles are
in the horizontal plane and the azimuthal angle \( \theta \) is
measured clockwise from the axis of the first dipole.
The outputs of the three sensors in a field containing a
directional component \( p(t) \) from direction \( \theta \) may be
expressed as
\[
x_1(t) = p(t) \cos \theta + n_1(t) \quad \text{(dipole 1)}
\]
\[
x_2(t) = p(t) \sin \theta + n_2(t) \quad \text{(dipole 2)}
\]
\[
x_3(t) = p(t) + n_3(t) \quad \text{(omni)}
\]
If the field is isotropic in the absence of the
directional component \( p(t) \), then the "noise" terms \( n_1 \)
\( n_2 \), and \( n_3 \) will be uncorrelated. In this situation, the
noise power on the omni-directional sensor will be
greater than that on the dipoles by a factor \( g^2 \), the
directivity gain of the dipoles. For spherically isotropic
noise \( g^2 \) is equal to 3, and for cylindrically isotropic
noise \( g^2 \) is equal to 2.
A pair of orthogonal dipoles with their axes rotated
by an angle \( \theta_e \) may be obtained by applying the
following linear transformation to \( x_1 \) and \( x_2 \)
\[
y_1(t; \theta_e) = x_1(t) \cos \theta_e + x_2(t) \sin \theta_e
\]
\[
y_2(t; \theta_e) = -x_1(t) \sin \theta_e + x_2(t) \cos \theta_e
\]
If the noise components \( n_1 \), and \( n_2 \) in (1) and (2) are
uncorrelated and of equal power, as in an isotropic field,
then the noise components of (4) and (5) will also be
uncorrelated with equal power.
A cardioid with its maximum response in direction
\( \theta_e + \pi \) and its null in direction \( \theta_e \) is formed from \( y_1 \)
and \( y_2 \) as follows
\[
z(t; \theta_e) = \frac{1}{2} [ x_3(t) - y_1(t; \theta_e) ]
\]
This cardioid has the following response to the
directional component \( p(t) \) from \( \theta \) in the absence of
noise
\[
z(t; \theta_e) = \frac{1}{p(t)} [ 1 - \cos ( \theta - \theta_e ) ]
\]
By adding \( y_2 \) to \( x_3 \) rather than subtracting as in (6) the
maximum of the cardioid is steered in the direction \( \theta_e \).

3 Bearing Estimation
The standard approach to bearing estimation is to
cross-correlate both dipoles with the omni-directional
sensor. This procedure is motivated by the model of a
single directional component in isotropic noise. Since
the noises are uncorrelated in this case, the following
cross-correlations result
\[
E(x_1, x_3) = E(p^2) \cos \theta
\]
\[
E(x_2, x_3) = E(p^2) \sin \theta
\]
where \( E \) denotes expectation.
Estimates of \( \cos \theta \) and \( \sin \theta \) obtained from finite
time averages are
\[
\cos \hat{\theta} = \frac{x_1 x_3}{|x_1 x_3|^2 + |x_2 x_3|^2} \frac{1}{2}
\]
\[
\sin \hat{\theta} = \frac{x_2 x_3}{|x_1 x_3|^2 + |x_2 x_3|^2} \frac{1}{2}
\]
where the overbar denotes time averaging.
The estimate of \( \theta \) is
\[
\hat{\theta} = \tan^{-1} ( \sin \hat{\theta} / \cos \hat{\theta} )
\]
the ambiguity of the arctangent can be resolved since
the signs of the cosine and sine estimates are available
from (10) and (11).
When there is more than one directional component,
the estimate of bearing provided by this procedure is a
power weighted average of the two bearings. Hence,
this procedure provides good bearing estimates only
when there is a dominant directional component. If the
two signals are in different frequency bands pre-filtering
permits independent bearing estimation.

4 Adaptive Cardioid Steering

4.1 Block Averaging
A simple direct approach to adaptively steering the
null of a cardioid in the direction of a strong interference
is to estimate \( \cos \theta \) and \( \sin \theta \) using finite time
averages as in (10) and (11). These estimates are then
used in (4) to steer a dipole so that its maximum
response is in the direction of the interference. By
subtracting the steered dipole output from that of the
omni-directional sensor the desired cardioid is obtained.
The result of these operations is
\[
z(t; \hat{\theta}) = \frac{1}{2} [ x_3(t) - (x_1(t) \cos \hat{\theta} + x_2(t) \sin \hat{\theta} ) ]\]
where \( \cos \hat{\theta} \) and \( \sin \hat{\theta} \) are given by (10) and (11).
The application to the multistatic active problem
exploits the fact that the echo of interest is much
shorter and weaker than the reverberation from the
source direction. The averaging time used in direction
estimation should be long compared to the anticipated
echo duration.

4.2 Recursive Method
In the recursive method a pair of orthogonal dipoles
rotated by a steering angle \( \theta_e \) is formed using (4) and
(5). A cardioid with its null steered in direction \( \theta_e \)
is formed using (6). The recursive algorithm attempts to
minimize the average output power \( E_z^2(t; \hat{\theta}) \) by taking
a step in the direction opposite to an estimated gradient.
From (6) the derivative of \( z^2 \) with respect to \( \theta_a \) is

\[
d \frac{z^2}{d \theta_a} = -z(t; \theta_a) \frac{d y_1(t; \theta_a)}{d \theta_a}
\]

But from (4) and (5)

\[
d \frac{y_1(t; \theta_a)}{d \theta_a} = y_2(t; \theta_a)
\]

The resulting recursive algorithm is

\[
\theta_a(t+1) = \theta_a(t) + \mu \frac{d z^2}{d \theta_a}
\]

where \( \mu \) as usual is the small positive step size parameter which determines speed of response and mean squared error. A variation on the algorithm which limits the maximum step size to say \( \pi / 6 \) can be used to avoid modulo \( 2\pi \) problems for very strong interferences.

The new estimate \( \theta_a(t+1) \) obtained from (17) is used in (4), (5) and (6) to adjust the steering. This approach is illustrated in the block diagram of Fig. 1.

In this algorithm, we estimate a parameter \( \theta_a \) which is nonlinearly related to the weights \( \cos \theta_a \) and \( \sin \theta_a \) applied to the sensor outputs. This is unlike the usual LMS algorithm in which the weights are directly estimated.

5 Optimum Beamforming

Of course one can apply well known optimum beamforming techniques to the outputs of the three sensors. This can be done using the estimated cross-correlation matrix \( R \) of the three sensor outputs or by using an LMS algorithm to adjust the weights. We will not pursue this approach here. Suffice it to say that in general, this will result in a different beam pattern than the adaptive cardioid.

The cardioid is generally a suboptimum processor. It has the advantages of a controlled beam pattern and ease of implementation.

6 Example

In order to gain insight into the performance of the adaptive cardioid algorithms, a simulation of a bistatic active situation has been conducted. The sensor outputs consist of three components:

1. continuous uncorrelated noise of unit power (0 dB) on each sensor
2. strong extended reverberation (+20 dB) from azimuth of 30 degrees during the time step interval: 100 to 900
3. a short duration moderate strength (+10 dB) echo from 170 degrees during the interval: 600 to 610

The power output of the omni-directional sensor is shown in Fig. 2. Here the reverberation totally masks the echo.

The block processing algorithm using non-overlapping blocks of 50 samples was used to adaptively steer a cardioid. The resulting cardioid power output is shown in Fig. 3. The cardioid has successfully rejected the strong reverberation and achieved several dB of gain against the noise, making the echo clearly visible. The maximum response direction of the cardioid is at 210 degrees, 40 degrees from the echo. This causes only about 1 dB loss in output echo level.

The recursive algorithm was applied to the identical sensor time series using a value of 0.01 for the step size parameter \( \mu \). The cardioid output power is shown in Fig. 4. Except for the strong transient at the on-set of the reverberation at \( t = 100 \), the output is nearly identical to that for the block algorithm. The steering of the cardioid is quickly adapted to suppress the reverberation. The slow response sometimes associated with LMS algorithms is not observed. Of course one would reject the possibility of the transient being a short echo by examining the output of the omni-directional sensor.

7 Conclusion

Adaptive processing to steer the null of a cardioid can have high payoff in situations with strong directional interference such as the multistatic active environment. Simple adaptive algorithms of both the block averaging and the recursive types have been shown to be effective in rapidly nulling strong directional interference. The choice between approaches may be based on other consideration in the overall receiver system design.

8 References

Figure 1. Recursive Algorithm

Figure 2. Omni Output Power

Figure 3. Cardioid Output Power

Figure 4. Cardioid Output Power