A Technique for Instantaneous Tracking of Frequency Agile Signals in the Presence of Spectrally Correlated Interference*

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Abstract
A novel single-sensor technique for detecting and tracking frequency agile signals in the presence of spectrally correlated or second-order cyclostationary noise and co-channel interference is presented. The technique is developed from the maximum-likelihood estimate of the spectral frequency of a frequency agile signal received in complex Gaussian interference with unknown spectral correlation, and is implemented using a frequency-channelized LMS adaptive processing algorithm. The resulting algorithm can significantly outperform conventional frequency-channelized whitener/detector approaches when tracking agile signals corrupted by spectrally correlated interference, by using the correlation between spectrally separated interference components to reduce the interference content in each spectral bin prior to the whitening/detection operation. These results are demonstrated via computer simulation.

1 Introduction
The problem of detecting and tracking frequency agile signals is an important but difficult problem to solve. Factors that contribute to the difficulty of detecting and tracking frequency agile signals of interest (SOIs) in typical reception environments include:

1. the density of co-channel interference at each frequency occupied by the target SOI;
2. the severity and time/frequency variation of channel distortion over the SOI frequency band; and
3. the dwell time of the SOI at each frequency channel covered by the reception system.

The reception system can fail to detect or track the SOI if the severity (power level) and density of the background interference is high enough to mask the SOI energy over a critical number of channels; if the channel distortion is too severe or variable in time and frequency to allow precise measurement of SOI features difficult; or if the SOI does not dwell for a long enough time at each frequency to allow SOI features to be compiled and detected through the interference.

A new algorithm for detection and tracking of frequency agile signals, based on maximum-likelihood estimation of the instantaneous frequency of the signals, is applied here to solution of these problems in environments containing strong spectrally correlated interference [1]. The algorithm, referred to here as the Maximum-Likelihood Frequency Bin Estimator (MLFBE), adapts a single-sensor frequency-channelized processor to track a frequency-agile SOI over the SOI transmission interval, by estimating the frequency location of the SOI over each SOI dwell interval. Initial detection and despreading of the SOI is also provided as a by-product of the processor.

The MLFBE is capable of suppressing spectrally correlated background interference, allowing detection and tracking of agile signals under severe co-channel interference, and significantly reducing the time-bandwidth product required to detect the SOI frequency channel over each dwell interval. The MLFBE is also implemented using a time-dependent adaptive processor, allowing the estimator to quickly compensate for changing signal and channel conditions. Finally, the adaptive MLFBE lends itself to a parallel implementation, reducing both the computational complexity and the latency time of the detection processor.

The algorithm requires only specific knowledge of the bandwidth and location of each SOI frequency channel in its implementation. In particular, no assumptions are made about the SOI frequency sequence or any fixed characteristics of the SOI baseband such as preamble sequences in order to implement the algorithm. However, the interference is assumed to be a second-order cyclostationary or almost-cyclostationary waveform, such that it exhibits the property of spectral correlation [1]. This property is used to predict and suppress spectrally correlated interference received by the processor, by optimally combining frequency channels to remove the correlated interference components between each channel prior to the SOI frequency detection procedure. Examples of spectrally correlated interference that can
be exploited in this manner include PCM (PSK, FSK, MSK, OOK, QAM, etc.) digital communication signals with high excess bandwidth; stacked carrier, multitone, and DSB-AM analog communication signals; and FDM-FM communication signals with low modulation index and/or prominent pilot tones; direct-sequence spread spectrum communication signals; and periodic/pulsed, swept, stacked or spread radar signals. This ability provides the MLFBE with unique capability to detect and track agile signals in dense spectrally correlated co-channel interference.

A. Discussion

This section develops the basic MLFBE processor structure and algorithm, based on maximum-likelihood estimation strategies.

2.1 Problem Statement

Consider an environment where a signal-of-interest with a frequency agile modulation format is received in the presence of uncorrelated co-channel interference and broadband noise, digitized and converted to complex baseband format, and passed through an N-channel nonoverlapped (but possibly windowed) FFT. For analysis purposes, assume that the collection system is ideally configured so that each SOI frequency dwell starts at the beginning of an FFT block and each SOI frequency location falls on an FFT bin center, and assume that each dwell interval extends over an integer number of FFT blocks. Then the channelizer input signal \( x(n) \) can be modeled by

\[
x(n) = s(n) + i(n) = \delta_k(n)
\]

where \( \{s(n)\}_{n=0}^{N-1} \) is the FFT window function, and \( \{i(n)\}_{n=0}^{N-1} \) is the channelized interference signal and \( \{\delta_k(n)\}_{k=0}^{N-1} \) is the downconverted channelized SOI. If the SOI bandwidth is small (less than \( 1/N \)) and the frequency response of \( h(m) \) is sufficiently narrowband, then \( \delta_k(n) \) can be approximated by \( \delta_k(n) \), where \( s(n) \) is the \( 1/2 \) decimated SOI and \( \delta_k \) is the Kronecker delta function, and the channelizer output signal can be approximated by

\[
x_k(n) = \begin{cases} s(n) + i(n), & k = 0 \\ i_k(n), & \text{otherwise} \end{cases}
\]

\[
\Rightarrow x(n) \approx e_{(n)}s(n) + i(n)
\]

where \( x(n) = \{x_k(n)\}_{k=0}^{N-1} \) and \( i(n) = \{i_k(n)\}_{k=0}^{N-1} \) are the vector channelized data and interference signals, respectively, and where \( e_{(n)} \) is the SOI frequency steering vector.

The problem of interest here is to estimate the SOI frequency bin \( \ell(n) \) at each block \( n \), so that the SOI can be tracked from dwell-to-dwell over the reception interval. It is assumed that this estimation must be performed on an instantaneous basis, i.e., without using past estimates of the SOI frequency bin to aid the estimation procedure. It is further assumed that the received interference \( i(m) \) has no first-order periodicity (line spectra) and is random with a circularly symmetric complex Gaussian (CSCG) probability distribution, such that \( i(n) \) has a zero-mean vector CSCG probability density,

\[
f_{i(n)}(z) = \pi^{-N} |R_{ii}|^{-1} \exp \left( -z^H R_{ii}^{-1} z \right),
\]

where \( (\cdot)^H \) and \( | \cdot | \) denote the conjugate-transpose and determinant operations, respectively, and \( R_{ii} \) is the autocorrelation of \( i(n) \), and where \( \gamma_{ii} \) denotes infinite time averaging. However, it is also assumed that \( i(m) \) can have second-order periodicity or spectral correlation (second-order cyclostationary or almost-cyclostationary signal components [1]) at some multiples of the FFT bin resolution \( 1/N \), such that \( R_{ii} \) has a nondiagonal form.

These assumptions are reasonable in many practical environments. The interference vector \( i(n) \) can usually be approximated as a vector CSCG sequence if the components of \( i(m) \) are random and have bandwidths much greater than \( 1/N \). The interference can also possess spectral correlation at many values of frequency separation if \( i(n) \) contains communication or radar signals with AM, PCM, stacked-carrier or PAM/PPM/PWM modulation formats.

If the SOI has already been detected through some previous processing stage, then it is also reasonable to assume that \( R_{ii} \) is known, e.g., if it is measured before the SOI appears at the receiver. This immediately

\[
\text{To simplify the notation used here, any amplitude gating or dwell-to-dwell phase modulation induced by the SOI transmitter is subsumed into the baseband SOI \( i(n) \).}
\]
suggests that \( f(n) \) can be estimated using a maximum-likelihood (ML) approach. This estimator is developed in the next section.

### 2.2 Estimator Development

Development of the ML estimate of \( f(n) \) is straightforward given assumptions (3)-(4). Assuming that \( s(n) \) and \( f(n) \) are unknown and \( i(n) \) is white and modelled by (3), then \( x(n) \) can be modelled as a circularly-symmetric complex Gaussian random sequence with mean \( \mu_x(n) \) and autocovariance \( R_{ii} \), such that \( x(n) \) has conditional probability density

\[
 f(x(n) | s(n), f) = \pi^{-N} |R_{ii}|^{-1} e^{-[x(n)-\mu_x(n)]^H R_{ii}^{-1} (x(n)-\mu_x(n))},
\]

\[
 = \pi^{-N} |R_{ii}|^{-1} \exp \left(-x^H R_{ii}^{-1} x \right)
\]

\[
 \times \exp \left[ 2R_{ii}^{-1} s^*(n) - e_i^H R_{ii}^{-1} e_i |s(n)|^2 \right].
\]

The ML estimate of \( f \) is then derived by maximizing (5) with respect to \( s(n) \) for every \( f \), and then finding the value of \( f \) that maximizes the resultant partially optimized objective function.

Maximization of (6) with respect to \( s(n) \) is straightforward, yielding

\[
 \hat{s}_{ML}(n) | f = \frac{e_i^H R_{ii}^{-1} x(n)}{e_i^H R_{ii}^{-1} e_i}
\]

\[
 \Rightarrow L[f, \hat{s}_{ML}(n) | s(n)] = \left| \frac{e_i^H R_{ii}^{-1} x(n)}{e_i^H R_{ii}^{-1} e_i} \right|^2
\]

for every \( f \). The ML estimate of \( f \) is then equal to

\[
 \hat{f}_{ML}(n) = \arg \max_{0 \leq k < N} d_k(n)
\]

where \( \arg \max_{\theta \in \mathcal{D}} f(\theta) \) is the value of \( \theta \) that maximizes \( f(\theta) \) for any \( f : \mathcal{D} \rightarrow \mathcal{R} \), and where \( \{d_k(n)\}_{k=0}^{N-1} \) is the frequency bin discriminator spectrum given by (8).

\[
 d_k(n) \triangleq \left| \frac{e_i^H R_{ii}^{-1} x(n)}{e_i^H R_{ii}^{-1} e_k} \right|^2
\]

If the background interference \( i(n) \) is stationary, then the channelized interference components \( \{i_k(n)\}_{k=0}^{N-1} \) satisfy \( \{i_k(n)x(n)\}_{k=0}^{N-1} = 0 \) for every \( k \neq k' \), and \( R_{ii} \) can be approximated as a diagonal matrix. The ML frequency bin estimator then reduces to the conventional time invariant or time independent (TI) frequency bin estimator shown in Figure 1. This estimator can be mathematically expressed as

\[
 \hat{f}_{TI}(n) = \arg \max_{0 \leq k < N} d_k(n)
\]

\[
 \hat{d}_k(n) \triangleq \left| x(n) \right|^2
\]

where \( R_{kk} = \langle |i_k(n)|^2 \rangle \) is the received interference power on the \( k \)th channelizer bin. The TI frequency bin estimator essentially adjusts the level of each frequency channel to yield unity response when the frequency agile SOI is absent, and then searches for the maximum deviation from that response when the signal appears in a given channel. Note that the TI estimator cannot suppress or cancel the interference prior to the estimation procedure. This processor can only exploit the inherent processing gain provided by the (time invariant) channelization operation, which reduces the energy of the overall interference signal by dividing that signal into separate frequency components.

[Diagram of a frequency bin estimator]

However, in practice \( i(n) \) is many times not stationary, and the general ML frequency bin estimator given in (9)-(10) can provide a much more powerful estimation capability. In particular, if \( i(n) \) exhibits second-order cyclostationarity or second-order spectral correlation [1] at multiples of the channelizer resolution \( 1/N \), then the ML frequency bin estimator reduces to a time-dependent (TD) processor that can provide significant interference cancellation over the TI frequency bin estimator given in (11)-(12).

The general ML frequency bin estimator is shown in Figure 2. This estimator exploits the spectral correlation of \( i(n) \) in the latter manner, by using correlation between the interference \( \{i_k(n)\}_{k=0}^{N-1} \) present in each frequency channel to maximally suppress the interference before estimating the SOI frequency bin. Comparison of Figures 1 and 2 reveals that the general ML processor differs from the TI processor only in an overwhitening or covariance-inversion operation that is applied to the channelized data \( x(n) \) prior to
the normalization operation. If \( i(m) \) is spectrally correlated at multiples of the channelizer resolution \( 1/N \), however, then the cross-correlation \( (i_k(n)G_k(n))_\infty \) is nonzero for some values of \( k \neq k' \), and the autocorrelation matrix \( R_{ii} \) is therefore nondiagonal. This overwhitening operation is equivalent to time-dependent filtering of the input interference signal \( i(m) \), resulting in a significant reduction of interference power over the conventional TI processor if \( i(m) \) contains significant spectral correlation. The ML processor then searches for the maximum deviation from the suppressed interference signal in order to estimate the SOI frequency bin. This estimator can be mathematically expressed as

\[
y(n) = W^H x(n), \quad W = R_{ii}^{-1/2} \quad (13)
\]

\[
y_k(n) = \frac{y_k(n)}{\sqrt{\epsilon_k}}, \quad \epsilon(n) = W^H i(n) \quad (14)
\]

\[
R_{kk} = \frac{y_k(n)}{\sqrt{\epsilon_k}} \cdot W = [u_k]_{k=0}^{N-1} \quad (15)
\]

\[
d_k(n) = \|y_k(n)\|^2 \quad (16)
\]

\[
d_k(n) = \frac{\epsilon_k}{\epsilon_k} \cdot \frac{R_{ii}^{-1} x(n)}{\epsilon_k} \quad (17)
\]

where \( R_{kk} = \langle |y_k(n)|^2 \rangle_\infty \) is the signal power on the \( k \)th overwhitener output channel, \( \epsilon(n) = [\epsilon_k]_{k=0}^{N-1} \) is the overwhitened interference signal, and (17) is used in place of (16) in order to save a square-root and power normalization operation. Note that both the interference suppression weights \( W \) and the normalization constants \( \{u_k\}_{k=0}^{N-1} \) can in principle be obtained directly from the inverse interference autocorrelation matrix \( R_{ii}^{-1} \), without the need for additional matrix operations.

![Figure 2: General (Time-Dependent) ML Frequency Bin Estimator](image)

The general time-dependent ML frequency bin estimator can also be interpreted from a whitener/template-matcher viewpoint, as shown in Figure 3. Passing \( x(n) \) through an \( N \times N \) matrix combiner \( W = R_{ii}^{-1/2} \) simultaneously whitens the background interference \( i(n) \) and distorts the frequency bin steering vector \( \epsilon_k \). The estimated frequency bin is then computed by finding the closest match between the whitened data \( \hat{x}(n) = R_{ii}^{-1/2} x(n) \) and the predistorted steering vectors \( \{\epsilon_k\}_{k=0}^{N-1} = \{R_{ii}^{-1/2} \epsilon_k\}_{k=0}^{N-1} \). The resultant template-matching statistic is then given by

\[
d_k(n) = \frac{|\epsilon_k^H \hat{x}(n)|^2}{\epsilon_k^H \epsilon_k} \quad (18)
\]

\[
d_k(n) = \frac{|\epsilon_k^H R_{ii}^{-1} x(n)|^2}{\epsilon_k^H R_{ii}^{-1} \epsilon_k} \quad (19)
\]

\[
d_k(n) = \delta_k(n),
\]

i.e., the template matching statistic is equivalent to the ML detection statistic. This result agrees with intuition, since the template-matching statistic should be the optimal estimator if the signal is received in the presence of white interference. However, the overwhitening approach shown in Figure 2 should be the superior implementation approach, since it uses \( R_{ii}^{-1} \) (which is easily computed via steepest-descent) and \( \{\epsilon_k\}_{k=0}^{N-1} \) rather than \( R_{ii}^{-1/2} \) and \( \{\epsilon_k\}_{k=0}^{N-1} \) to compute \( d_k(n)_{k=0}^{N-1} \).

![Figure 3: Whitener/Template-Matcher Interpretation of the General ML Frequency Bin Estimator](image)

2.3 Estimator Implementation

Given the general estimator structure developed in Section 2.2, it remains to devise an algorithm to actually compute the combiner weights \( W \) shown in Figure 2. Methods must be developed for adaptively computing \( R_{ii}^{-1} \) both prior to reception of the SOI, and during reception of the SOI. If \( N \) is large, it is also very important to devise methods for thinning \( W \) to reduce the complexity of the overall processor, e.g., by incorporating a priori knowledge of the interference background into the estimation procedure.

There are a number of ways to implement an adaptive ML frequency bin estimator [2]. A steepest-descent approach is presented here that approximates \( W = R_{ii}^{-1/2} \) using a modified Griffiths algorithm [4], given generally by

\[
W \leftarrow W - \mu \left( \langle x(n)y^H(n) \rangle_M - I_N \right) \quad (18)
\]

where \( \langle \cdot \rangle_M \) is an \( M \)-sample smoothing operation (beginning at block \( n \)) and \( \mu \) is the adaptive stepsize of the algorithm,

\[
\mu = \frac{\sigma}{\|R_{xx}\|} \quad (19)
\]
and where $0 < \sigma < 1$ and $||R_{xx}||$ is the maximum eigenvalue (strong norm) of $R_{xx}$.

If the SOI dwell time is low enough, the MLFBE can be operated without modification during the SOI reception interval. However, if necessary, the algorithm can be adjusted in several ways to minimize the effect of the SOI on $w$. In particular, if the SOI frequency bin $k$ can be reliably detected over the SOI reception interval, then the weights $\{w_k\}_{k=0}^{N-1}$ and $\{w_k\}_{k=0}^{N-1}$ affected by the SOI can be frozen until the SOI moves to another frequency bin.

3 Estimator Evaluation

This section tests the ML Frequency Bin Estimator developed in Section 2 via computer simulation, and demonstrates the advantages and operating principles of the ML Frequency bin estimator.

3.1 Implementation and Simulation Parameters

The ML frequency bin estimator readily lends itself to an adaptive implementation. One possible form of a ML frequency bin estimator is shown in Figure 4. This implementation Fourier transforms the data and filters the data block by block. The adaptive implementation is updated using the LMS algorithm. The weights $w_k = [w_k(n)]_{k=0}^{N-1}$ are used to form the estimate of the $k$th bin over the $n$th FFT block. Thus, in Figure 4, each output bin would require its own set of weights. This set of weights performs the overwhitening operation included in (10). The weight $g_k(n)$ performs the whitening operation on the $k$th FFT bin. The peak detector assigns the SOI frequency location to the bin with the largest value. The ML frequency bin estimator has a structure similar to the Optimal Time-Dependent Receiver used for demodulation of signals corrupted by interference [6] the spectral correlation discriminator [5] used for sorting signals.

$$d_k(n) = \frac{|y_k(n)|^2}{g_k(n)}$$

$$\ell(n) = \text{arg max}_{0 \leq k < N} d_k(n)$$

Here, the overwhitening operation $e_k^H R_{xx}^{-1} x(n)$ in equation (10) is accomplished by filtering $x(n)$ with $w(n)$ to produce $y_k(n)$, and the whitening operation $e_k^H R_{xx}^{-1} e_k$ is accomplished by dividing by $g_k(n)$. The vector $e_k$ is an $N \times 1$ column vector containing all zeros except for 1 on the $k$th element, i.e., $e_k = [0, \ldots, 1, \ldots, 0]^T$.

In practice, the dimensions of the $w_k(n)$ vector are thinned, having a dimension as small as 1 or as large as $N$. When the dimension of $w_k(n)$ is one, only the time-independent term, $w_k(n)$, is used, and this configuration corresponds to the time-independent ML frequency bin estimator. When a subset of possible bins are used, the ML frequency bin estimator can be viewed as a periodic filtering process, having filter periodicities determined by the frequency separation of these bins from the $k$th bin. In practice, the dimension of $w_k(n+1)$ is much less than $N$ since the interference is band limited, and spectrally correlated information is limited to a few adjacent bins about the $k$th bin.

3.2 Simulation Results

To demonstrate the operation of the adaptive ML frequency bin estimator, a frequency agile signal is generated having parameters listed below in Table 1. Two interfering signals having parameters listed below in Table 2 are then added to the frequency agile signal. The ML frequency bin estimator FFT block size is 256 points, and each FFT block covers a single frequency dwell for display purposes. There are a total of 8192 points contained in the file.

Table 1: Frequency Agile Signal Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation Format</td>
<td>QPSK</td>
</tr>
<tr>
<td>Carrier Frequency</td>
<td>0-200 kHz</td>
</tr>
<tr>
<td>Dwell Time</td>
<td>1 msec</td>
</tr>
<tr>
<td>Baud Rate</td>
<td>4 kHz</td>
</tr>
<tr>
<td>SNR</td>
<td>4 dB</td>
</tr>
<tr>
<td>Sample Rate</td>
<td>256 kHz</td>
</tr>
</tbody>
</table>

Table 2: Interference Signal Parameters

<table>
<thead>
<tr>
<th>Interference 1</th>
<th>Interference 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mod. Format</td>
<td></td>
</tr>
<tr>
<td>Carrier Freq.</td>
<td>163 kHz</td>
</tr>
<tr>
<td>Baud Rate</td>
<td>156 kHz</td>
</tr>
<tr>
<td>SNR</td>
<td>98 kHz</td>
</tr>
<tr>
<td>Sample Rate</td>
<td>256 kHz</td>
</tr>
</tbody>
</table>

In theory, the autocorrelation matrix of the background interference should be estimated when the agile SOI is absent. In order to allow this, the first 4096
points in the data file are generated without the frequency agile SOI, and are used initially to train the adaptive ML frequency bin estimator. However, in practice experiments show that $R_{ii}$ does not need to be estimated prior to the arrival of the agile SOI, because the dwell-time of the SOI in each frequency bin is much shorter than the convergence time of the processor adaptation algorithm used here. Consequently, the adaptation algorithm is not significantly affected by the presence of the frequency agile signal. This allows the interference statistics to be continuously updated, even when an agile signal is present.

The time-frequency activity of the transmitted SOI is shown in Figure 5, plotting the SOI energy over sequential nonoverlapping FFT blocks taken at 256 point intervals, each block containing a single SOI frequency dwell. The time-frequency activity of the corrupted signal is shown in Figure 6 using the same graphical format. Notice that there are several intervals in which the SOI cannot be located.

For this simulation, the filter periodicities are chosen to be equal to the baud rate of the interfering QPSK signal, ±30 kHz.

The first simulation examines the performance of the time-independent ML frequency bin estimator. The output of the time-independent processor is shown in Figure 7. Even after processing, the agile SOI is not evident in several FFT intervals. When processed by the time-dependent frequency bin estimator, some of these frequencies, previously undetected, are now evident, as seen in Figure 8. For this simulation, the filter periodicities are chosen to be equal to ± the baud rate of the interfering QPSK signal, ±30 kHz.

The next example shows the ability of the time-dependent ML frequency bin estimator to adaptively identify the background spectral correlation properties with minimal prior knowledge, and to use this information to provide a frequency bin estimate that is superior to that provided by the time-independent frequency bin estimator. For this example, we compare the signal processed by the time-dependent ML frequency bin estimators, using the periodicity sets 0, ±30 kHz and 0, -30, -31, -33, -35 kHz, to the time-independent ML frequency bin estimator at the 17th FFT block. The results are shown in Figure 9, Figure 10 and Figure 11. Notice the agile SOI is not detectable using the time-independent ML frequency bin estimator but is easily detected using the time-dependent ML frequency bin estimators. This set of filter periodicities contains periodicities at -31 and -33 kHz that do not contribute spectral spectrally correlated information, and thus, these periodicities do not aid in providing a performance advantage of the time-dependent ML frequency bin estimator. The adaptive processor suppresses the contribu...
tion attributed to these periodicities and uses only the periodicities that contribute spectrally correlated information.

Figure 9: Time-Independent Processor Output Statistic, 17th FFT Bin

Figure 10: Time-Dependent Processor Output Statistic, 17th FFT Bin, Periodicities 0 and ±30 kHz

4 Summary and Conclusion
A new technique for detecting frequency agile spread spectrum signals, the Maximum-Likelihood Frequency Bin Estimator (MLFBE), is proposed, analyzed, and simulated. The MLFBE is similar to the conventional whitening filter but includes an interference cancellation stage that can significantly reduce cochannel interference having the property of spectral correlation. The improved performance of the MLFBE is the result of exploiting the cyclostationary nature of the interference. In addition, the MLFBE can be implemented using computationally efficient adaptive structures that quickly adjust to dynamic background interference. These results are demonstrated by computer simulations verifying the advantages of the ML processor over the conventional whitening filter when detecting a frequency agile SOI corrupted by phase modulated interference.

References