Recent Advances in Digital Signal Processing and Their Application to Antenna Pattern Synthesis

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Abstract
It is shown that the problems of antenna pattern synthesis and digital filter design are similar in nature. With this in mind, some recent techniques common to digital filter design are examined in terms of their applicability to antenna pattern synthesis. An equiripple formulation of the weighted least squares method is developed to show how it can be used for both sector and pencil beam patterns. Several design examples are presented.

1. Introduction
Linear antenna arrays are in many ways analogous to one dimensional digital filters. Restriction of the pattern synthesis problem to that of discrete arrays of finite spatial extent makes the problem similar to that of finite impulse response (FIR) digital filters. It is the purpose of this paper to examine the similarities between these two fields. It is felt that techniques in the field of digital signal processing, and specifically the area of FIR filter design can be useful to the pattern synthesis problems. With this in mind, a design procedure presented in recent digital signal processing literature will be examined, and its applicability to the antenna synthesis area will be examined.

As stated above, discrete antenna array synthesis and FIR filter design are similar problems. This is due to the fact that the relationship between the excitation currents on an antenna and the resultant pattern is that of the Fourier transform. This relationship is easy to derive from geometrical principles and is shown in figure 1. The expression for the array factor is derived by summing the components of the electric fields generated by each array element. The expression for the field is weighted by the complex current applied to each element. Under the assumption that the distance is large enough to use the far field approximation, there is an incremental phase term from each succeeding array element of $\cos(\theta) / \lambda$, where $d$ is the inter element spacing and $\theta$ is the angle measured from the array axis. With a suitable change of variables the expression for the array factor can be put in the form of the discrete Fourier transform.

$$f(\theta) = \sum_{n=-N}^{N} I_n e^{j2\pi n d/\lambda} \cos \theta$$

Figure 1 Derivation of array factor

impulse response and the frequency response of an FIR filter is that of the discrete Fourier transform.

Table 1 provides a comparison of antenna pattern synthesis and their analogs in digital filter design. The basic problem of filter design is to determine a proper set of filter coefficients given a desired frequency response. In antenna pattern synthesis, one specifies a desired pattern, and then determines the excitation currents necessary to generate it.

Antenna array patterns are usually separated into two areas, pencil beam and sector patterns. Typical techniques used to synthesize sector beam patterns include the Woodward, Lawson, and Fourier series methods. Pencil beams are typically generated using the Chebyshev or Taylor methods. The Chebyshev method provides an optimum solution in the sense that the beam width is minimized for any specified side lobe level [1]. antenna synthesis design methods are typically limited to one of these two areas. Standard digital filter design methods include the Fourier series method, frequency sampling and the equiripple (Parks-McClellan) method [2]. This paper will concentrate on the equiripple design formulation. The equiripple design method generates the same solution as the Chebyshev design method common to antenna synthesis. This solution results in patterns (or frequency response) which have the characteristic of ripple of equal magnitude in the stopband region. The Parks McClellan algorithm uses an iterative procedure which minimizes the
### Filter Design
- Impulse Response
- Frequency Response

### Antenna Synthesis
- Excitation currents

### Design Problem:
- Given desired frequency response, determine filter coefficients

### Typical Methods
- Fourier series
- Equiripple
- Parks McClellan
- FOUnifold

### Typical Methods
- Sector pattern
- Fourier Series
- Woodward
- Chebyshev
- Taylor

#### Table 1: Comparison of filter design and antenna pattern synthesis

<table>
<thead>
<tr>
<th>Filter Design</th>
<th>Antenna Synthesis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impulse Response</td>
<td>Excitation currents</td>
</tr>
<tr>
<td>Frequency Response</td>
<td>Pattern</td>
</tr>
</tbody>
</table>

#### Maximum (minimax) amplitude of the peaks of the ripples. It can be shown that this ‘minimax’ solution converges to the analytic Chebyshev solution [3]. Furthermore, the generalized minimax solution is not limited to pencil beam patterns as in the case of the standard Chebyshev synthesis procedure.

### 2. Weighted least squares design method

As stated previously, this paper will limit itself to a discussion of equiripple design techniques. The standard method for equiripple design is the Parks McClellan algorithm. The Parks McClellan algorithm is based on an iterative algorithm which minimizes the maximum amplitude of the ripples present. This algorithm is both quite complex and computationally expensive. It is also by its nature limited to one dimensional problems [4]. The Weighted Least Squares algorithm (WLS) provides an option to the Parks McClellan algorithm. The standard formulation of the WLS method provides an optimal approximation to the desired response in a least squares sense. This ensures that the power present in the stopband will be a minimum.

Figure 2a shows a typical response from a filter designed with the least squares method. Figure 2b shows the same filter designed with the Parks McClellan algorithm. It is clear that the least squares solution generates a response whose side lobes are initially larger, but gradually decay. It has been suggested that use of an appropriate frequency weighting function could be used in order to generate a response whose side lobes have constant amplitudes. Several methods for doing this have been published. We will concentrate upon the method presented by Lim et al.

#### 2.1 Analysis

In order to facilitate the derivation of the WLS method, we will limit ourselves to the case of an odd number of excitation elements with symmetric excitation currents. Under this assumption we get the following expression for the discrete Fourier transform of the excitation currents \(a(\omega)\):

\[
H(\omega) = a_0 + \sum_{m=1}^{M} 2a_m \cos m\omega \quad (1)
\]

This expression can be put into matrix form \(H = XA\). Where:

\[
H = \begin{bmatrix} H(\omega_1) & H(\omega_2) & \cdots & H(\omega_p) \end{bmatrix}^T
\]

\[
X = \begin{bmatrix} 1 & 2\cos \omega_1 & 2\cos 2\omega_1 & \cdots & 2\cos p\omega_1 \\ 1 & 2\cos \omega_2 & 2\cos 2\omega_2 & \cdots & 2\cos p\omega_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2\cos \omega_p & 2\cos 2\omega_p & \cdots & 2\cos p\omega_p \end{bmatrix}
\]

\[
A = \begin{bmatrix} a_1 & a_2 & \cdots & a_M \end{bmatrix}^T
\]
In the above expressions [ ]^T represents the transpose operation. If we let the vector D be the desired response then we have the relation D = XA. The problem is now one of parameter estimation. We need to determine the value of A to best approximate the desired response given by D. Note that we have specified D on a grid of 'P' points, and that the size of the array is 2M+1. In order to achieve a solution we require that P ≥ M. If P = M, then the solution is given precisely by A=X^(-1)D. If P > M then we must use a least squares approximation. Let E be the error term given by E = D - XA. This represents the error present in our approximation to the desired response. The least squares approach finds the solution which minimizes the Euclidean distance given by J = E^T E. Using this we see that:

\[ J = (D - XA)^T (D - XA) \]
\[ = D^T D - A^T X^T D - D^T X A + A^T X^T X A. \]  

To minimize this, set the first derivative with respect to A equal to zero, and solve. Doing this yields:

\[ A = (X^T X)^{-1} X^T D. \]  

Expanding the above derivation to include a weighting vector leads us to our desired result:

\[ A = (X^T W X)^{-1} X^T W D. \]  

As stated earlier, proper choice for the weighting term in equation 4 will yield an equiripple solution which can be shown to be analytically equivalent to the Chebyshev solution. Lim et al published an iterative method for deriving the appropriate weight function. Their method is based on increasing the weight at those points for which the error is high while decreasing the weights at those points that exhibit low error. In their formulation a multiplicative update is taken i.e.

\[ W_{k+1} = W_k \cdot \beta_k \]  

\[ W_{k+1} - \text{Weight vector for iteration } k+1 \]
\[ W_k - \text{Weight vector for iteration } k \]
\[ \beta_k - \text{Weight update vector} \]

\( \beta_k \) is chosen such that it will tend to equal out the errors at different values of \( \omega \). More specifically, \( \beta_k \) is chosen in such a way that if:

\[ |E_k(\omega_n)| > |E_k(\omega_m)| \]
then:

\[ \beta_k(\omega_n) > \beta_k(\omega_m) \]

This has the desired effect of increasing the weighting function at those points where the error is high. This will ensure that in the subsequent iteration the error will be decreased at the expense of those regions for which the error is low. This results in the error tending to equalize over successive iterations.

There are several advantages gained in using this algorithm over the Parks McClellan algorithm. The WLS method is both simple to program and computationally efficient. The most computationally intensive part of the algorithm is the necessary matrix inversion. The matrix in question is Toeplitz in nature and thus can take advantage of the many numerical methods that exist to do this efficiently. Another benefit of interest to the antenna system designer is that this method is not limited to the one dimensional synthesis problem. Algazi et al reported that application of this algorithm to a two dimensional synthesis problem provides a near optimal solution, achieving results within 1dB of optimality [4]. This fact is particularly relevant since the Parks McClellan algorithm is by its nature limited to one dimension. This is due to the fact that it is dependent upon the alternation property of errors which does not hold in two dimensions.

![Figure 3: Comparison of Chebyshev and weighted least squares methods](image)

<table>
<thead>
<tr>
<th>Method</th>
<th>Directivity</th>
<th>Half Power Beamwidth</th>
<th>Transition Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>WLS</td>
<td>20.0423</td>
<td>6.5713 deg.</td>
<td>.0682</td>
</tr>
<tr>
<td>Chebyshev</td>
<td>20.0304</td>
<td>6.3889</td>
<td>.0681</td>
</tr>
<tr>
<td>Taylor</td>
<td>20.2998</td>
<td>6.4297</td>
<td>.0712</td>
</tr>
</tbody>
</table>

Table 2: Comparison of Chebyshev, Taylor and weighted least squares methods
3. Design examples

In order to demonstrate the broad range of problems for which this technique is applicable, examples are presented for both pencil beam and sector patterns. The pencil beam synthesis problem was specified to have a maximum side lobe level of 20 dB. This particular problem was approached using the standard Chebyshev method, the discrete Taylor array and the WLS method. A comparison between the resultant patterns are given in figure 3. Table 2 compares the directivity, half power beamwidth (HPBW), and the transition width for the three procedures. Figure 4 shows a comparison with a discrete Taylor array. Note the decrease in side lobe level away from the main lobe inherent to the Taylor design. This design example is presented to suggest the idea that with a suitable choice for the weighting function, it might be possible to achieve a result which would approach the Taylor design.

The next design example is that of a sector pattern. The results are given in figure 5. The design was specified for a cutoff of $w = .4$. This relates to a value of $\theta = 66.4$ degrees.

4. Conclusion

We have examined the WLS method as an alternative to the standard Parks McClellan algorithm for equiripple filter design. It was shown that a suitable choice of weighting function could result in the WLS method converging to the Chebyshev solution. An iterative method provided by Lim et al was examined for accomplishing this. Using this method several design examples common to the field of antenna synthesis were presented, indicating the usefulness of this technique. It was also mentioned that this method can be applied to two dimensional design scenarios, thus establishing its place in an area where the standard Parks McClellan algorithm is limited.

References


