Spatial Diversity Equalization for Underwater Acoustic Communications

Qian Wen
Department of Electrical Engineering
University of Washington
Seattle, WA 98195

James A. Ritcey
Department of Electrical Engineering
University of Washington
Seattle, WA 98195

Abstract

Intersymbol interference (ISI), due to long multipath durations, is a prominent characteristic of digital communication over underwater acoustic channels. Standard single channel equalizers fail to effectively reduce this ISI. We apply spatial diversity equalization (SDE) using a vertical receive array, and a joint space-time equalization criterion to minimize ISI. An ocean acoustic model is used and the parabolic wave equation is solved to find the acoustic channel impulse response. Mean square error and probability of symbol error are computed for a variety of system parameters and indicate the advantages of SDE.

1 Introduction

Underwater communication channels are difficult to compensate using single channel equalization schemes. This study concentrates on the spatial diversity equalization techniques that effectively improve the communication system performance in two ways. One is to increase the signal-to-noise ratio at the receiver by coherent combining the received signals of different receiver array elements. The other is that equalization helps to increase the bandwidth efficiency of the channel by compensating the channel distortions [7]. SDE theory has been studied recently in [1], [2] and [4]. [4] contains the solution to the optimum equalizer based on the MMSE criterion which is also used in our SDE study. Spatial diversity processing in underwater communications is also studied in [3], but equalization is not emphasized in this case.

We present a study of the SDE in a shallow water environment. The mid-range channel model is found by the Parabolic Equation solver. Performance measures such as the minimum mean squared error and probability of error are presented to demonstrate that the system performance improves rapidly along with the increase in the number of diversity channels. Adaptive equalization results are also presented to demonstrate SDE performance when the channel characteristics must be found adaptively.

2 System Models

A mid-range, shallow water ocean model is solved by the Parabolic Equation solver [8] to find the acoustic channel impulse. In this way, we incorporate the physics into a linear system model. The SDE is designed for known channels to determine a bound on expected system performance. In practice, adaptation may be necessary, so we describe a limited simulation study for an RLS joint channel equalizer.

2.1 Ocean Model

The ocean model consists of a two-dimensional ocean cross-section of 8 Nmi in length and 200 m in depth. The receiver is located at one end of this ocean cross-section and at a depth of 100 m. Since PE solutions are found at every mesh point in this ocean cross-section, including spatial diversity receiving doesn’t require extra computation.

The channel impulse response is found by first computing, using PE, complex pressure fields of a number of frequency points to form the channel frequency response, and then finding the inverse FFT of this frequency response. The channel passband has a width equal to 640 Hz and is centered at 1 kHz. By using a ray tracing model (Generic Sonar Model) [9], we found that the channel impulse response duration is 0.1 s. Hence, 64 frequency samples were computed to satisfy Nyquist criterion. The low frequency was chosen to allow a PE solution and to study wave effects.
In practice, higher frequencies and shorter ranges are likely. Simulation studies would then be carried out using ray tracing.

2.2 Communication System Model

The continuous time communication system is shown in figure 1. Because SDE is our only interest, transmit filter, modulator, demodulator and receiving filter are assumed to be perfect. Additive white Gaussian noise (AWGN) with variance $T N_0$ is added into the channel, where $T$ is the sampling period (baud).

Since both input and output of this communication system are discrete at the symbol rate, we use the equivalent discrete channel impulse response in our computer simulations (figure 2). The discrete channel $F(z)$ represent cumulative effects of transmit filter, channel and receiving filter. All operations, in latter analysis, are performed in baseband. Since the receive filter is assumed to be perfect, the filtered and sampled noise remains white with variance $N_0$ in the baseband. Figure 3 shows the block diagram of the Joint Channel Equalizer (JCE). The data sequence is transmitted, by the implicit diversity of the channel (multipath), through different channels $F(d)(z)$, where $d$ is the index to a specific diversity channel. A total of $D$ diversity channels are employed. The received signals from different channels are equalized by JCE and combined. The output of JCE is then decoded into binary samples.

2.3 Equalization Theory

The SDE is designed based on the minimum mean squared error (MMSE) criterion. The mean squared error (MSE) at the equalizer output is minimized [6]:

$$
\epsilon^2 = (I_k - \hat{I}_k)^2.
$$

The channel acts as a filter and adds white Gaussian noise. The receiver input, to channel, $d$, is

$$
u_k^{(d)} = \sum_{n=-L}^{L} f_k^{(d)} I_{k-n} + z_k
$$

where $(d) = 1 \ldots D$ represent the specific diversity channel, $z_k$ is the noise, and $L$ is the length of the channel filter. We let $K$ represent the number of equalizer coefficients per channel. By the orthogonality principle, we can find the MMSE equalizer coefficients by solving the following equation,

$$
\sum_{d_1=1}^{D} \sum_{m=-K}^{+K} c_m^{(d_1)} v_{k-1}^{(d_1)} v_{k-m}^{(d_2)} = (i_k v_{k-1}^{(d_1)}).
$$

where $(v_{k-1}^{(d_1)} v_{k-m}^{(d_2)})$ represent correlation of received samples of two different diversity channels, and also
the elements in $\Gamma$ matrix, the channel correlation matrix. $(f_{d_j}^{(d_j)})$ represent cross-correlation between the transmitted and received binary symbol, and also the elements in $\xi$, the cross-correlation vector of the channels.

Assuming binary symbols that are uncorrelated in time, and noise which is white in both space and in time, the above correlation terms can be expressed in term of the channel impulse response. The correlation matrix $\Gamma$ consists of $D^2$ sub-matrices $\Gamma_{d_i,d_j}$, the cross-correlation between two channels. Within a sub-matrix, the elements are,

$$
\Gamma_{ij} = \left\{ \begin{array}{ll} 
\sum_{m=1}^{2K+1-|i-j|} f_m^{(d_i)} f_m^{(d_j)} + 2N_0 \delta_{ij} \delta_{d_i,d_j} & |i-j| \leq L \\
0 & \text{otherwise}
\end{array} \right.
$$

The vector $\xi$ has $D$ subvectors of length $2K+1$. Within each subvector, the elements $\xi_i$ are,

$$
\xi_i = \left\{ \begin{array}{ll}
f_i^{(d_i)} & |i| < L \\
0 & \text{otherwise}
\end{array} \right.
$$

The matrix equation is solved for the $(2K+1)D$ unknown equalizer coefficients. $(2K+1)$ is the length of the equalizer for each diversity channels. The solution, for full rank $\Gamma$ is $\Gamma^{-1}\xi$.

Due to the presence of noise, $\Gamma$ is theoretically always full rank. Practically, the condition number grows as the number of receiving channels exceed the number of multipaths; the paths then being correlated. In this case, we have used an SVD to perform a rank reduction or pseudo-inverse solution. The matrix dimensions can be quite large, as it grows as $KD$.

### 2.4 Probability of Error Calculations

The probability of error is computed using the saddle point integration technique developed by Professor Carl Helstrom [5]. We indicate a brief derivation. A received sample is corrupted by intersymbol interference caused by the channel and additive noise,

$$
r_k = I_k a_0 + \sum_{n=-\infty}^{+\infty} I_n a_{k-n} + n_k,
$$

where $a_k$ is the generalized channel that introduces ISI and it might also include the equalizer and other filtering. $n_k$ is the additive noise at the decoder which has variance $\sigma_n^2$. In order to find the probability of error, the probability distribution of this received sample must be found, which is difficult to carry out directly.

Helstrom introduced a method that computes PBE exactly when channel impulse response is finite. Note that the ISI term is a random sum of samples of channel impulse response, therefore its characteristic function can be represented as a product. The characteristic function of a sum of two independent random variables is the product of the characteristic function of these two random variables. The characteristic functions of ISI terms $g_i(s)$, binary symbols and additive noise $g_n(s)$ can be multiplied to find the characteristic function of the received sample:

$$
g(s) \equiv \prod_{n=-N,n\neq k}^{+N} \frac{1}{2} \left[ \exp(a_k+s) + \exp(-a_k-s) \right] = \prod_{n=-N,n\neq k}^{+N} \cosh(a_k-s).
$$

The probability of error is found as the inverse Laplace transform of the characteristic function integrated in the erroneous decision region. The integral, evaluated numerically using the trapezoidal rule, is integrated along a suitable contour passing through a saddle-point. This integral is given by

$$
\begin{align*}
\tau_e &= \int_{-\infty}^{0} p_k(\nu_k) d\nu_k \\
&= \int_{-\infty}^{0} \int_{-\infty}^{+j\infty} g_r(s) \exp(\nu_k) \frac{ds}{2\pi j} d\nu_k \\
&= \int_{-\infty}^{+j\infty} \int_{-\infty}^{0} s^{-1} g_r(s) ds / 2\pi j.
\end{align*}
$$

Since the integral $\int_{-\infty}^{0} \exp(\nu_k) d\nu_k$ must be finite, $s$ is constrained to the right half plane. The path of integration is selected to be a vertical contour passing through the right half plane saddlepoint, for reasons discussed by Helstrom. The integral converges quite rapidly, in tens of steps.

### 3 Numerical Results

Here we present some typical results of our study. Some issues we want to consider include: the effect of element placement along the vertical array, advantages in increasing the number of elements, and the feasibility of adaptive spatial equalization using RLS. Other suboptimal criteria were also developed in our study, but are not presented here.
First, consider a SDE whose receiving elements are arbitrarily placed in the 200 m water column, and have spacings greater than 20 m. Figure 4 shows a family of MMSE curves as functions of the SDE size, and are labeled by the number of diversity channels used. The single channel MMSE decreases very slowly, whereas SDE has much smaller MMSE values. The 4-channel SDE (div 4) of size 20 has MMSE that is 46 dB lower than the 1-channel equalizer MMSE. The plot shows that significant reduction in MMSE can be achieved by using a number of spatial diversity channels. For this channel, 4 multipath rays adequately describes the propagation. Although we don't show the figure, our study indicates that the single channel equalizer has a performance which is very sensitive to element location. This sensitivity is mitigated with spatial diversity.

Next we examine the convergence characteristics of an adaptive spatial diversity equalizers. Figure 5 shows the MSE of a 2-channel adaptive RLS SDE with 31 taps in each channel. During an iteration step, 62 taps are updated simultaneously. This long computation time, especially with the RLS algorithm, is undesirable in practice. The initial convergence rate of 150 iterations is quite fast.

The diversity gain of SDE is illustrated in the plot of PBE as a function of SNR with 31 equalizer coefficients (figure 6). At a constant PBE value, the 1-channel equalizer requires a SNR 6.5 dB higher than that of the 4-channel SDE. This agrees with the predicted diversity gain of approximately 6 DB for 4-channel diversity over no diversity. Thus, we see that SDE realizes the gain available due to implicit chan-

nel diversity. Simply, summing the element outputs, or using a beamformer directed at a single ray would not provide this gain. ISI is simply not considered in the usual formulation of optimal beamformers. However, it is the limited factor for a digital communication system over these channels. In practice, probing, rather than adaption may be required to learn the channel impulse response. An experiment using this procedure has been proposed.

When only considering the presence of Gaussian noise and ignoring the ISI, a lower bound of the probability of error can be easily calculated. This lower bound is plotted against the PBE with ISI in figure 7. Two closely spaced curves in the middle show the lower bound and the calculated PBE. The difference of which shows the effects of residue ISI in the signal. Clearly, the single channel equalizer has the biggest difference and less difference is detected in the case of more diversity channels. Certainly, we notice that after equalization, the residue ISI become less significant in affecting the receiver performance. Why? The ISI power has been greatly reduced by the equalizer.

The calculated PBE without equalization (the slowly decreasing curve on the top) and the matched filter bound PBE using SNR as the only parameter (the lowest curve) are both shown on the same plots. The loss respect to the Matched Filter Bound is caused by the channel, and can't be improved by linear equalization.
4 Conclusions

Success in removing ISI is demonstrated by Spatial Diversity Equalization. The results show the advantages of jointly optimizing the space-time system. However, SDE performance is very much dependent on channel characteristics, and channel characteristics vary with environmental conditions. We have also studied decision feedback equalizers and suboptimal combining techniques, but have insufficient space to present the results here.

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References


Figure 6: Probability of error of the SDE as function of SNR with 31 equalizer taps per channel. The PBE of different number of diversity channels are shown.

Figure 7: The lower and upper bounds of PBE as function of SNR. The equalizer size is 31 taps per channel. The top most curve is the PBE without equalization. The bottom most curve is the matched filter output PBE. The two curves in the middle is the PBE at the output of SDE with and without the effects of ISI.