Fractionally Spaced Semi-Blind Equalization of Wireless Channels

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Abstract- A new procedure for fast identification and equalization of wireless channels is presented. Utilizing the finite impulse response (a realistic assumption) nature of wireless channels and the cyclostationary nature of the oversampled data, a finite matrix description of the channel is derived which is used to obtain channel estimates and equalizers. The procedure is called semi-blind because it estimates the signal subspace blindly but gets trained on phase.

1 Introduction

The demand for high bit rate digital wireless communication has increased tremendously in the recent years. Due to multipath fading (which is common in such channels) the communication environment suffers from severe intersymbol interference (ISI). In order to combat this issue several researchers have proposed using equalizers and diversity combiners (see e.g. Proakis [1], Balaban and Salz [2], and Aria et. al. [3]) while designing such systems.

Since the channel may be changing rapidly, LMS-type adaptive algorithms may not always be suitable due to the slow convergence properties of such algorithms. Although the recursive least squares (RLS) type algorithms converge quickly, they suffer from potential numerical instability. However none of the above methods work very well in case of sudden fades or channel changes.

Moreover the multipath channels are often nonminimum phase and such channels can not be modeled very well by standard LMS or RLS type algorithms.

It is well known that the incoming signal is cyclostationary. But if sampled at the baud rate, the sampled process is stationary. However sampling at the baud rate is sensitive to unknown frequency and phase jitters. Fractionally spaced equalizers [4] are known to be robust with respect to such problems. Moreover it is not unusual to expect fractional symbol delays in wireless channels. So it is natural to consider oversampling of the received data for building a good channel identifier or an equalizer. However oversampling makes the sampled signal cyclostationary and one must account for that in some way. Often the nature of the wireless channel can be described as having a few beams of radio waves arriving well separated in time (however possibly all within one symbol period 1) while each beam being made out of a number of rays from a large number of very closely spaced scatterers.
In this paper we propose a new fast method of channel identification and equalization, for short impulse response (possibly non-minimum phase) multipath channels.

This method can be used over and over again (possibly in every frame in a TDMA system) as sudden fades or other reasons make it necessary to do so. We assume throughout this paper that the channel impulse response is of finite duration (which is realistic) and hence the observation in any finite interval is influenced by a finite number of consecutive symbols.

If the channel were time invariant, the received signal \( y(t) \) could be expressed as

\[
y(t) = x(t) + n(t); \quad x(t) = \sum_{k=-\infty}^{\infty} s_k h(t - kT),
\]

where \( s_k \) is symbol from some constellation \( S \), \( h(.) \) is the impulse response of the channel that includes pulse shaping and the receiver filters as well, \( T \) is the symbol interval, and \( n(.) \) is the additive noise. In fact \( h(.) \) could also include a pre-whitening filter at the receiver to whiten the noise before equalization.

2 Vectorized Stationary Representation of the Received Cyclostationary Signal

The over-sampled received signal is

\[
y(i\Delta) = \sum_{k=-\infty}^{\infty} s_k h(i\Delta - kT) + n(i\Delta),
\]

where \( \Delta < T \) is the sampling period.

We assume that the channel impulse response (which includes pulse shaping etc.) to be of finite length (which is realistic for such channels) viz:

\[
y(t_0 + i\Delta) = \sum_{k=0}^{r-1} s_k h(t_0 + i\Delta - (k_0 + kT)) + n(t_0 + i\Delta)
\]

for some finite \( k_0, r \) within a finite observation interval \([t_0, t_0 + T_S]\), such that \( T_S = m\Delta \) for some positive integer \( m > 1 \) and \( i = 1, \ldots, m \). Thus using a so called translation series representation (TSR) of a cyclostationary process a finite size matrix description of the channel can be derived viz:

\[
Y(p) = HS(p) + N(p)
\]

where \( Y(.) \) is \((m \times 1)\), \( S(.) \) is \((r \times 1)\), \( N(.) \) is \((m \times 1)\), and \( H(.) \) is \((m \times r)\). It should be pointed out at this time that the vector observation process \( Y(.) \) is stationary while the scalar observation process \( y(.) \) is cyclostationary. Here \( N(.) \) is a vectorized noise process, obtained from the scalar noise process \( n(.) \).

Here the objective of channel identification is to estimate \( H \) given the received signal \( y(.) \), while the problem of channel equalization is to estimate \( S(.) \).

3 Pre-Whitening, Order Selection, and Channel Model Estimation

Let the symbol process \( s(.) \) be zero mean and white with unity power spectral density. It will be assumed that the noise process \( n(.) \) is zero mean and 'in-band white' with the 'in-band' power spectral density equal to \( \sigma_n^2 \). Since \( n(.) \) is only 'in-band

\[^{1}\text{Present North American Digital Cellular IS-54 standard requires equalization of all multipaths spaced within one symbol period only.}\]
white',

\[ R_N = E\{N(i)N^*(j)\} \neq \sigma^2 I_m. \]

However, there exists a non-singular matrix \( T \) such that

\[ TR_NT^* = E\{(TN(i))(TN(j))^*\} = \sigma^2 I_m. \]

This matrix \( T \) can be precomputed and for obvious reasons will be called a *whitening transformation*.

Let us define

\[ Y = TY. \]

Then the autocovariance of \( Y \) must be given by,

\[ R_Y = E\{Y(i)Y^*(j)\} = GG^* + \sigma^2 I_m, \]

where \( * \) stands for a Hermitian transpose, \( G = TH \), and \( I_m \) denotes a \( m \times m \) identity matrix. Then the SVD of \( R_Y \) would be given by

\[ R_Y = U_{r} \Sigma_{r} V^*, \]

where \( U_{r} \) is the first \( r \) columns of \( U \), and

\[ \Sigma_{r} = \text{diag}\{\sigma_{1}^{2}, \sigma_{2}^{2}, \ldots, \sigma_{r}^{2}\}. \]

It can be shown that \( G \) can be represented as

\[ G = U_r Q \]

where \( U_r \) is made up of the first \( r \) columns of \( U \), and

\[ Q = \Sigma_{r}^{1/2} \Phi; \Phi^* \Phi = I_r = \Phi \Phi^*; \Sigma_{r} = \text{diag}\{\sigma_{1}^{2}, \ldots, \sigma_{r}^{2}\}. \]

4 Estimation of the Channel Model Parameters and a Semi-Blind Equalization Algorithm

It is clear that both \( r \) and \( U_r \) can be estimated blindly (i.e. without a training sequence) observing the most significant singular values and the corresponding singular vectors of \( R_Y \). At this point the best linear least square estimate of \( Q \) is computed using a very short training sequence. This is why this approach has been called *semi-blind*.

\( Q \) is computed as follows. Let \( \{s_i(k)\}_{k=1}^{q} \) such that \( q \geq 2r - 1 \), be the training sequence. If \( \{Y(p)\}_{p=1}^{q-r+1} \) are the corresponding vectorized observations, then we can write,

\[ Y_t = GS_t, \]

where \( G^P \) is a pseudoinverse of \( G \) such that \( G^P G = L_r \), \( Y_t = [Y(1), Y(2), \ldots, Y(q-r+1)] \) and \( S_t \) is a \( r \times (q-r+1) \) Hankel matrix whose first row is \( [s_t(1), \ldots, s_t(q-r+1)] \) while the last row is \( [s_t(q-r+1), \ldots, s_t(q)] \). This immediately leads to the following formula for the best least squares estimate \( \hat{Q} \) of \( Q \), viz:

\[ \hat{Q} = U_r^* Y_t S_t^r [S_t S_r^r]^{-1}. \]

The symbol estimates \( \hat{S} \) are then obtained from

\[ \hat{S} = L\{G^P Y\}, \]

where \( Y \) is any \( m \times 1 \) vectorized observation and \( S \) is the corresponding \( r \times 1 \) vector of symbols affecting the observation \( Y \), \( L \) is the desired decision operation and \( G^P \) is the pseudoinverse of \( G \).

5 Practical Considerations and Simulations

It turns out that at the present (IS-54) digital cellular rate of transmission, one symbol (which is about
41\mu sec) delay translates into a distance of about 8 miles, traveled by radio wave. In all possibilities the delays will possibly be smaller. A semi-blind equalizer for a three-way non-minimum phase multipath channel has been simulated. Simulation results indicate rapid convergence even in somewhat low SNR.

References


