Information Transmission Using Phase Coupling

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Abstract
The ability to transmit information via the presence or absence of phase coupling is investigated. A ternary hypothesis problem, with one signal zero, one signal composed of harmonically related cosines each with a random phase, and one signal composed of harmonically related and phase coupled cosines is examined. The optimal detector with perfect knowledge, the Bayes optimal detector under the assumption of unknown phase, and a heuristic detector, based on higher order moments are designed and analyzed. It is shown that the moment-based detector can distinguish between the three hypotheses, without the exact phases being known.

1 Introduction
In some communication systems it is desirable to encode a message into three symbols instead of the usual two. For example, with Morse code, one could transmit no signal for the symbol "space", the signal $S_1$ for the symbol "dot", and the signal $S_2$ for the symbol "dash". If each symbol is allotted $T$ seconds, and if the channel is distortionless, then the received signal will be described by the ternary hypothesis test:

$$
H_0 : X(t) = N(t) \quad 0 \leq t \leq T
$$

$$
H_1 : X(t) = N(t) + S_1(t)
$$

$$
H_2 : X(t) = N(t) + S_2(t)
$$

where $N$ is noise. Throughout this work, the channel noise, $N$, is constrained to be a zero mean, white Gaussian random process with spectral level $\sigma^2$.

The goal of this work is to determine the utility of encoding information into the phase structure of a signal. Hence, $S_1$ is designed to consist of harmonically related cosines of known frequencies while the $S_2$ consists of harmonically related and phase coupled cosines. Specifically, define

$$
S_1(t) = \cos(\omega_1 t + \phi_1) + \cos(\omega_2 t + \phi_2) + \cos(\omega_3 t + \phi_3)
$$

$$
S_2(t) = \cos(\omega_1 t + \phi_1) + \cos(\omega_2 t + \phi_2) + \cos(\omega_3 t + \phi_3)
$$

where the frequencies, $\omega_1$, $\omega_2$, and $\omega_3 = \omega_1 + \omega_2$ are known, the phases, $\phi_1$, $\phi_2$, $\phi_3$ are independent random variables uniform on $[0, 2\pi)$, and the phase $\phi_4 = \phi_3 + \phi_2$. Further, for analytical convenience, the observation time $T$ is selected to be an integer multiple of the period of the signals $S_1$ and $S_2$. Thus, the presence of the cosines indicates that either a "dot" or a "dash" has been transmitted, and the presence or absence of phase coupling distinguishes between these two symbols.

2 Signal Detection
By appropriate sampling, an equivalent hypothesis test to that of Eq. (1) is:

$$
H_0 : X = N
$$

$$
H_1 : X = N + S_1
$$

$$
H_2 : X = N + S_2
$$

Here $X$, $S_1$, and $S_2$ are $M \times 1$ vectors of samples from $X(t)$, $S_1(t)$, and $S_2(t)$ respectively. The vector length $M = f_s T$ where $f_s$ is the sampling frequency.

Bayes theory can be used to derive a statistical test which will minimize the average cost, $C$. To specify the test, some additional notation is required. Let $C_{ij}$, $i,j = 0,1,2$, be the cost associated with making decision $H_i$ given that hypothesis $H_j$ is true. Further, constrain these costs such that $C_{ij} > C_{jj} \geq 0$ for $i, j = 0,1,2$. Define the likelihood ratios $L_1$ and $L_2$ as:

$$
L_1(x) = \frac{p(x|H_1)}{p(x|H_0)} \quad L_2(x) = \frac{p(x|H_2)}{p(x|H_0)}
$$

where $p(x|H_j)$ is the probability density function of the observation $x$ conditioned on hypothesis $H_j$ being true. Define $P(H_j)$ to be the prior probability of hypothesis $H_j$. Now the decision regions, $\Omega_j$ for $j = 0,1,2$, for the three hypotheses can be defined. In this study, the cost of an incorrect decision, $C_{ij}$ for $i \neq j$, is unity, the cost of a correct decision, $C_{jj}$, is zero, and the hypotheses are equally likely. Hence,

$$
\Omega_0 = \{ x : L_1(x) < 1, L_2(x) < 1 \} \quad \Omega_1 = \{ x : L_1(x) > 1, -L_1(x) + L_2(x) < 0 \} \quad \Omega_2 = \{ x : L_2(x) > 1, -L_1(x) + L_2(x) > 0 \}.
$$
The average cost is

$$\bar{C} = \sum_{j=0}^{2} P(H_j) \sum_{i \neq j} C_{ij} P(H_i | H_j), \quad (6)$$

where $P(H_j | H_j)$ is the probability of deciding hypothesis $H_j$ given hypothesis $H_j$ is true. Note that $P(H_j | H_j)$ is the probability of correctly deciding hypothesis $H_j$ while the probability of an incorrect decision is $\sum_{i \neq j} P(H_i | H_j)$. The effectiveness of the detectors studied will be evaluated using the average cost criterion.

### 2.1 Known Signal Detection

In the simplest case, the signals $S_1$ and $S_2$ are known in every detail. Such a case could arise when the sequence of phases used would not be truly random, but would be drawn from a code book. The known signal detector will achieve the lowest average cost, hence its evaluation will allow for the performance of sub-optimal detectors to be interpreted relative to the best possible performance.

Let

$$g(x; \mu, \sigma^2) \equiv \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad . \quad (7)$$

Since the noise samples are independent, the three conditional densities are:

$$p(x|H_j) = \prod_{i=1}^{M} g(x_i; S_j(i), \sigma^2) \quad j = 1, 2, \quad (8)$$

and $p(x|H_0)$ is as above with $S_j(i) = 0$ for all $j$ and $i$. The likelihood ratios $L_j$, $j = 1, 2$ can be written as

$$L_j(x) = \exp \left\{ \frac{3M\theta^2}{4\sigma^2} \sum_{i=1}^{M} x_i S_j(i) \right\} \quad (9)$$

with the sample-by-sample signal-to-noise ratio (snr) defined to be $3/2\sigma^2$.

#### 2.1.1 Performance Results

Initially, the dependence of the phase difference $|\phi_3 - \phi_4|$ on the performance level was evaluated by fixing $\phi_1$ and $\phi_2$ and allowing the phase difference to vary from zero to $\pi$. The simulation results indicate that when no signal is present and except for the low snr levels, the probability of a correct decision, $P(H_0|H_0)$, is only weakly dependent on the value of the phase difference. In contrast, $P(H_1|H_1)$ and $P(H_2|H_2)$ are strongly dependent on the phase difference. For small phase differences, the signals $S_1$ and $S_2$ are nearly indistinguishable. Hence, for large snr, $P(H_1|H_1)$ varies from nearly 0.5 to 0.9 as the phase difference ranges from zero to $\pi$. Figure 1 shows typical curves for the probability of a correct decision for each of the three hypotheses versus phase difference.

Next, the performance of the optimal detector with perfect knowledge was simulated with the phases $\phi_j$, $j = 1, 2, 3$ being independent random variables and $\phi_4 = \phi_1 + \phi_2$. The results of the simulation are contained in Table 1. The average cost $\bar{C}$ decreases quickly as the noise variance decreases, indicating that at the higher snr one can transmit a digital signal using the coupling between cosines. Even at lower snr's, the optimal detector's average cost is much lower than the cost of 0.67, what one could achieve through guessing.

![Figure 1: Probability of correct detection for different phase differences. Noise variance $\sigma^2 = 3.036$. Note that $P(H_1|H_0) \approx P(H_2|H_0)$ for all snr levels tested. Thus, when $H_0$ is true, the detector is essentially testing the energy of the received signal. If the energy is too large, then, the decision is split nearly equally between the two hypotheses $H_1$ and $H_2$. In contrast, $P(H_0|H_1) \approx P(H_2|H_1)$ only when the snr is small. For larger snr, $P(H_0|H_1) < P(H_2|H_1)$, indicating that the detector is better at differentiating between hypotheses $H_0$ and $H_1$ and it is in differentiating between hypotheses $H_2$ and $H_1$. The simulation results indicate that $P(H_0|H_0) > P(H_1|H_1)$ for $j = 1, 2$. Such a result suggests that phase evaluation is more difficult than measurement of the received signal energy.

#### 2.2 Unknown Signal Phases

To generalize the previous results, the signal phases are allowed to be random variables. Since the phases of the signals $S_1$ and $S_2$ are now unknown, these parameters cannot be incorporated into the detector structure. The likelihood ratio $L_1$, as defined in Eq. (4), can be written as

$$L_1(x) = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(\phi_1, \phi_2, \phi_3) \, d\phi_1 d\phi_2 d\phi_3$$

$$= e^{-\frac{3M\theta^2}{2\sigma^2} \sum_{i=1}^{M} x_i S_i(i)}$$

$$x e^{\frac{3M\theta^2}{2\sigma^2} \sum_{i=1}^{M} x_i S_i(i)}$$

$$\times \prod_{i=1}^{M} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(\phi_1, \phi_2, \phi_3) \, d\phi_1 d\phi_2 d\phi_3$$

where $f$ is the joint probability density function of the unknown phases, then $[1, 2, 3, 4]$. In effect, the likelihood ratio averages over all possible phase values. The likelihood ratio $L_1$ can be simplified as follows. Define $c_2(i) = \cos(\omega_1(i-1)\Delta t + \phi_2)$, i.e. the $i$th
sample of the cosine with the kth phase and frequency, then

\[ L_1(x) = e^{-\frac{3M^2}{4}x} \int_{-\pi}^{\pi} \frac{1}{2\pi} e^{2\pi i M \sum_{i=1}^{M} r_i c_i(i)} d\phi_1 \times \int_{-\pi}^{\pi} \frac{1}{2\pi} e^{2\pi i M \sum_{i=1}^{M} r_i c_i(i)} d\phi_2 \int_{-\pi}^{\pi} \frac{1}{2\pi} e^{2\pi i M \sum_{i=1}^{M} r_i c_i(i)} d\phi_3 \]

At first glance, \( L_1 \) appears to be extremely complex. But on defining \( M \)

\[ r_k \cos \eta_k = \sum_{i=1}^{M} x_i \cos(\omega_k (i-1) \Delta t) \]
\[ r_k \sin \eta_k = -\sum_{i=1}^{M} x_i \sin(\omega_k (i-1) \Delta t) \]

the integrals can be written in terms of modified Bessel functions.

\[ \int_{-\pi}^{\pi} \frac{1}{2\pi} e^{2\pi i M \sum_{i=1}^{M} r_i c_i(i)} d\phi_1 = I_0 \left( \frac{\theta_1}{\sigma^2} \right) \]

Therefore, the likelihood ratio \( L_1 \) is

\[ L_1(x) = e^{-\frac{3M^2}{4}x} I_0 \left( \frac{\theta_1}{\sigma^2} \right) I_0 \left( \frac{\theta_2}{\sigma^2} \right) I_0 \left( \frac{\theta_3}{\sigma^2} \right) \]

The likelihood ratio \( L_2 \) can be written as

\[ L_2(x) = e^{-\frac{3M^2}{4}x} \int_{-\pi}^{\pi} \frac{1}{2\pi} e^{2\pi i \sum_{j=1}^{M} r_j c_j(i)} d\phi_2 \int_{-\pi}^{\pi} \frac{1}{2\pi} e^{2\pi i \sum_{j=1}^{M} r_j c_j(i)} d\phi_3 \]

where \( \alpha_1 = \phi_1 - \eta_1 \), \( \alpha_2 = \phi_2 - \eta_2 \), \( \alpha_3 = \sqrt{(r_1 + r_3 \cos(\alpha_2 - \alpha_3))^2 + (-r_3 \sin(\alpha_2 - \alpha_3))^2} \), and \( r_3 = \eta_3 - \eta_2 - \eta_1 \).

2.2.1 Performance Results

The performance of the detector defined by the likelihood ratios in Eq. (12) and (13), and the decision regions in Eq. (5) was simulated for a variety of snr's.

The performance results are listed in Table 2. In examining the table entries, note that the detector is able to identify when no signal is present with a reasonably high accuracy for all snr levels tested. This suggests that the optimal detector is evaluating the energy of the received signal, and if this energy is small enough, the detector decides that hypothesis \( H_0 \) is true. Unlike the case when the signal is known exactly, note that the error probabilities, \( P(H_1|H_0) \) and \( P(H_2|H_0) \) are now substantially different for all snr levels.

The relative sizes of \( P(H_0|H_1) \) and \( P(H_2|H_1) \) is very dependent on the snr. For high snr, the detector successfully determines that a signal is present, but it has difficulty determining whether the signal present is \( S_1 \) or \( S_2 \). For low snr, the detector has a very high error probability. In contrast to the known signal case, \( P(H_0|H_2) \) and \( P(H_1|H_2) \) do not follow the same pattern as \( P(H_0|H_1) \) and \( P(H_2|H_1) \). The results indicate that the detector is better able to recognize when the highly structured signal \( S_2 \) is present than when \( S_1 \) is present.

We conclude that the detector makes the correct decision when either \( H_0 \) or \( H_2 \) is true. Even at low snr’s, \( P(H_0|H_0) \) and \( P(H_2|H_2) \) are both greater than one-third. In contrast, the detector is poor at determining when the harmonic signal \( S_1 \) is present. However, in terms of average cost, the detector is better than what would be achieved with guessing. The average costs listed in Table 1 and 2 indicate that the lack of phase information results in a substantial penalty.

3 Higher Order Moments

It is theoretically possible to distinguish between the three hypotheses if the power spectrum and the bispectrum of the signal are utilized. Therefore, we propose to use the following test:

\[ H_0 : \text{power spectrum } = \begin{array}{c} 0; \text{bispectrum } = 0 \\ H_1 : \text{power spectrum } \neq 0; \text{bispectrum } = 0 \\ H_2 : \text{power spectrum } \neq 0; \text{bispectrum } \neq 0 \end{array} \]

Theoretically, under \( H_1 \) and \( H_2 \), the power spectrum will be non-zero only at the frequencies of the tones in the two signals \( S_1 \) and \( S_2 \). Likewise, the bispectrum (which is the two-dimensional Fourier transform of the third-order moment sequence)
$$E\{X(t_1)X(t_2)X(t_3)|H_j\}$$ for \(j = 0, 1, 2\) will be identically zero everywhere when \(H_0\) or \(H_1\) is true, but will be non-zero for the frequency pair \((\omega_1, \omega_2)\) (in the principal domain). Thus the proposed test will use the following information to make a decision: 

### 3.1 Signal Structure

Since the bispectrum is the transform of a moment, an ensemble average is used to get the result that the bispectrum of the noise is zero. To achieve the same result in practice, one must use a time-average. However, in this case, this translates into requiring the signals \(S_1\) and \(S_2\) to contain several different phase triples. Therefore, the structure of the signal set used in the remainder of this article is a slightly modified form of the given by Eq. (2).

Let the signal duration be \(LMT_4, sec\) instead of \(K_M, sec\), where the phases are chosen randomly every \(M\) samples. Thus, the received signal will contain \(L\) observations of the phases. For \(L = 1\), the third order spectra for signals \(S_1\) and \(S_2\) are identical, but as \(L\) increases, the third order spectrum for signal \(S_1\) will approach zero, for all frequencies while \(S_2\) will remain unchanged.

To determine the number of segments necessary, the amplitude of the bispectra at the frequency pair \((\omega_1 = 1, \omega_2 = 2)\) was evaluated for different \(L\). A value of \(L = 10\) resulted in \(|b_{signal}(\omega_1, \omega_2)| \approx 0.28\). Increasing the number of segments to \(L = 20\) resulted in the ratio decreasing to only slightly. Hence, a \(L = 10\) for all further work.

### 4 Moment Detector Implementation

There are many different methods which could be used to implement the detector defined in Eq. (14). Here, the power spectrum, \(P_z\), and the bispectrum, \(B_z\), are implemented as follows. Let \(X(f)\) be the DFT of \(x\). Then, the power spectrum is estimated to be

$$P_z(f) = |X(f)|^2$$

and the bispectrum as

$$B_z(f_1,f_2) = X(f_1)X(f_2)X^*(f_1 + f_2)/\Delta f^3,$$

where \(\Delta f\) is the spacing of the samples in the frequency domain [5]. Since the three power spectral components are identical, the proposed detector first fuses these three components and then compares the fusion result and the bispectrum estimate to two thresholds. The block diagram of this moment-based detector is illustrated in Figure 2.

#### 4.1 Performance

For this work, four different fusion rules were implemented. Each rule has been used in prior research on distributed detection [6]. The four rules are:

- **AND:**
  $$\tilde{P}_z(f_1,f_2) = \max\{P_z(f_1), P_z(f_2), P_z(f_1 + f_2)\};$$

- **OR:**
  $$\tilde{P}_z(f_1,f_2) = \min\{P_z(f_1), P_z(f_2), P_z(f_1 + f_2)\};$$

- **AVG:**
  $$\tilde{P}_z(f_1,f_2) = (P_z(f_1) + P_z(f_2) + P_z(f_1 + f_2))/3;$$

- **MED:**
  $$\tilde{P}_z(f_1,f_2) = \text{median}\{P_z(f_1), P_z(f_2), P_z(f_1 + f_2)\}.$$

For a given set of thresholds, \(t_1\) and \(t_2\), the performance rule can be evaluated for the four different fusion rules. Unfortunately, the selection of the "best" pair of thresholds is difficult. One approach which can be used is to determine the sample mean of \(\tilde{P}_z\) and \(B_z\) for the three different hypotheses when the noise is present. Once these sample means are known, then the threshold \(t_1\) can be selected as the midpoint between the sample means of \(\tilde{P}_z\) given \(H_1\) is true and \(B_z\) under \(H_1\) and \(H_2\) should be identical. The second threshold, \(t_2\), can be chosen as the midpoint between the sample mean of \(B_z\) under \(H_1\) and under \(H_2\).

![Figure 2: Block diagram of the moment-based detector defined in Eq. (14).](image)

**Table 2:** Performance of Optimal Detector when Phases are Unknown

<table>
<thead>
<tr>
<th>snr (dB)</th>
<th>Noise Power</th>
<th>Average Cost</th>
<th>True Hypoth.</th>
<th>Probability Decide (H_0)</th>
<th>Probability Decide (H_1)</th>
<th>Probability Decide (H_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-18.06</td>
<td>96.00</td>
<td>0.64</td>
<td>(H_0)</td>
<td>0.59</td>
<td>0.13</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(H_1)</td>
<td>0.49</td>
<td>0.15</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(H_2)</td>
<td>0.49</td>
<td>0.15</td>
<td>0.36</td>
</tr>
<tr>
<td>-13.06</td>
<td>30.36</td>
<td>0.57</td>
<td>(H_0)</td>
<td>0.65</td>
<td>0.11</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(H_1)</td>
<td>0.37</td>
<td>0.22</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(H_2)</td>
<td>0.39</td>
<td>0.19</td>
<td>0.43</td>
</tr>
<tr>
<td>-8.06</td>
<td>5.60</td>
<td>0.43</td>
<td>(H_0)</td>
<td>0.82</td>
<td>0.06</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(H_1)</td>
<td>0.16</td>
<td>0.35</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(H_2)</td>
<td>0.16</td>
<td>0.29</td>
<td>0.54</td>
</tr>
<tr>
<td>-3.06</td>
<td>3.036</td>
<td>0.27</td>
<td>(H_0)</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(H_1)</td>
<td>0.01</td>
<td>0.55</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(H_2)</td>
<td>0.02</td>
<td>0.31</td>
<td>0.67</td>
</tr>
</tbody>
</table>
t_{1} = \sigma_{2}^{2}/M \Delta f + 1/8 \Delta f. A similar relationship for the sample mean of the bispectrum is unknown to the author. Hence, the threshold \( t_{2} \) was determined by calculating the sample mean of \( B_{2} \) under the three different hypothesis. To determine the sample mean, 10,000 independent trials were run. Table 3 contains the average cost as a function of the snr and the two best fusion rules; the AVG and MED rules. The two thresholds used are listed for convenience. If the average cost is compared to that of Table 1, we see that the optimal detector with a known signal of duration \( MT_{0} \) seconds, has about the same performance as this moment-based detector when the signal is of duration \( LMT_{0} \) seconds, and \( L = 10 \).

Further, detailed examination of the performance indicates that when the noise variance is small, the proposed test is essentially a sequential test. That is, the detector decides between the pair of hypotheses, \( H_{0} \) and \( (H_{1}, H_{2}) \) by examining the magnitude of \( \hat{P}_{2} \). If \( \hat{P}_{2} \) is large enough, then the magnitude of \( B_{2} \) is used to distinguish between \( H_{1} \) and \( H_{2} \). For larger noise powers, this decoupling does not occur.

### 4.2 Comparative Performance

The performance of the following two detectors was evaluated and compared to the performance of the detector defined in Eq. (14).

**OPT-KP:** The optimal detector, Eq. (9), assuming known phases and for \( L = 10 \) was implemented.

**UNK-A:** Due to the phase changing every \( M \)-samples, the optimal detector with unknown phase is extremely difficult to derive. This detector calculates the likelihood ratios defined in Eq.’s (12) and (13) for each \( M \)-sample segment. An average likelihood ratio value is calculated for both \( L_{1} \) and \( L_{2} \). These average likelihood ratios are used along with Eq. (5) to make the final decision.

Simulation results are presented in Table 3. When interpreting these numbers, it important to emphasize that due to extremely long computational time, only 100 trials were run of detector UNK-A where as 10,000 trials were used for the moment-based detector and the OPT-KP detector. In effect, the increase in the signal length allows for perfect detection if the phases are known when the noise variance is small enough. Since the known phase constraint is unreasonable, it is more instructive to compare the moment-based detector to the extremely complex and sub-optimal detector UNK-A. Except for the large noise variance case, the moment-based detector has a lower average cost.

### Table 3: Average cost versus snr for moment-based detector.

<table>
<thead>
<tr>
<th>( \text{snr} )</th>
<th>( \text{AVG} )</th>
<th>( \text{MED} )</th>
<th>( \text{OPT-KP} )</th>
<th>( \text{UNK-A} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.63</td>
<td>3.53</td>
<td>0.074</td>
<td>0.075</td>
</tr>
<tr>
<td>6</td>
<td>3.18</td>
<td>3.22</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>9.6</td>
<td>5.91</td>
<td>6.49</td>
<td>0.17</td>
<td>0.27</td>
</tr>
<tr>
<td>30.36</td>
<td>15.06</td>
<td>16.2</td>
<td>0.46</td>
<td>0.53</td>
</tr>
<tr>
<td>96.9</td>
<td>40.44</td>
<td>30.16</td>
<td>0.61</td>
<td>0.63</td>
</tr>
</tbody>
</table>

5 Conclusions

The Bayes optimal detector, assuming perfect signal knowledge has been derived and its performance evaluated. This detector has been shown to be computationally simple and to be able to successfully distinguish between the three hypotheses with high accuracy. Next, the phases of the cosines were assumed to be unknown and likelihood ratios were determined by averaging over all possible phases. The two likelihood ratios were shown to be either a product of Bessel functions or the integral of a Bessel function. This optimal detector performed well in the high snr cases, but had difficulty identifying the presence of signal \( S_{1} \) in low snr cases.

Finally, a new detector which takes advantage of the received signals power spectrum and bispectrum has been proposed and evaluated. This new detector uses only the easily computed power spectrum and the bispectrum of the observation. Further, the proposed detector is non-parametric since it requires only sample moments of the observation.

It was shown that the choice of fusion rule affected the average cost, and that the AVG fusion rule was the best choice. Due to complexity, when the signal is allowed to have \( L \) segments, the optimal detector with unknown phase information was not derived. However, a sub-optimal form was proposed and analyzed. It was shown that the moment-based detector is simpler and performs better than this sub-optimal detector.

### References


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