A New Approach to Blind Identification and Equalization of Multipath Channels

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Abstract

A new blind channel identification and equalization method is proposed by exploiting the cyclostationarity of communication signals. Identification and equalization of possibly nonminimum phase multipath channels are achieved without using training signals. Unlike most of the adaptive blind equalization methods for which the convergence properties are often problematic, the eigenstructure-based channel estimation algorithm proposed here is asymptotically exact. Based on the second-order properties, the new approach identifies nonminimum phase channels and achieves equalization with less samples than most techniques based on higher-order statistics. Simulations demonstrate promising performance of the proposed algorithm in a blind equalization of a three-ray multipath channel.

1 Introduction

The promise of cellular mobile communication has propelled ever increasing research efforts in high-speed wireless communication in recent years. The implementation of digital communication receivers for universal digital portable communication poses immense engineering challenges, primarily because the environment for wireless communication is much more complicated than the regular telephone channels. In particular, the time varying multipath fading that commonly exists in a wireless communication environment leads to severe intersymbol interference (ISI). Other channel impairments that contribute to ISI include symbol clock residual jitter, carrier phase jitter, etc. In order to achieve high-speed reliable communication, channel identification and equalization are necessary to combat ISI. Traditionally, channel identification and equalization are achieved either by sending training sequences, or by designing the equalizer based on a priori knowledge of the channel. The later approach is clearly not suitable for radio communication environment since little knowledge about such channel can be assumed a priori. The standard adaptive approach, though attractive in handling time varying unknown channels, has to waste a fraction of the transmission time slot for a training sequence. Indeed even the so-called the decision feedback equalization (DFE), which does not explicitly use a training sequence, nevertheless requires the sending of known training sequences periodically to avoid catastrophic error propagation [2]. In contrast to the standard adaptive equalization, the blind equalization does not require a training sequence. Instead, the statistical properties of the transmitted signals are exploited. The equalization is performed at the receiver end without access to the symbols being transmitted. Instead of choosing the equalizer so that the equalized output sequence \( \{s_k\} \) is close to the source symbol sequence \( \{s_k\} \) as in the standard equalization formulation, in blind equalization, one chooses the equalizer so that the statistics of the equalized output sequence \( \{s_k\} \) is close to the statistics of the source symbol sequence \( \{s_k\} \).

1.1 Existing Blind Channel Identification and Equalization Methods

The innovative idea of self-recovering (blind) adaptive equalization was first proposed by Sato [8], then further developed by Godard [9], Treichler and Agee [10], Benveniste and Goursat [1], Picci and Prati [8], Foschini [4] and more recently Shalvi and Weinstein [11]. Although the blind equalization schemes proposed so far are technically different, they are all derived via some optimization criteria involving certain higher-order (cumulants) statistics of the observation; various gradient-based algorithms are then employed to achieve the optimization. The major problem with the adaptive blind equalization techniques is that their convergence properties, as shown in [7], are often problematic; as pointed out in [7], the global convergence may be jeopardized when the channel has finite impulse response; however, it should be noted that the optimization criteria proposed by Shalvi and Weinstein [11] ensure global optimization whenever there exist ideal equalizers. In addition, the effect of non-Gaussian noise will also affect the convergence and the performance of the cumulant-based approach. Currently, there is no systematic initialization method to ensure the convergence.

1.2 A New Blind Channel Identification and Equalization Method

In this paper, we propose a new blind channel identification and equalization method. The main features of our approach can be summarized as follows:
1. The algorithm provides exact identification of the possibly non-minimum phase channel impulse response if the correlation function of the received signal is known exactly; more realistically, when the estimated signal correlation is used, asymptotic exactness can be established.

2. The algorithm can be used to initialize various adaptive schemes and the equalized output can be used to facilitate decision feedback adaptation. The identified channel can be used to implement maximum likelihood sequence estimation \([3]\) to further reduce ISI.

3. The algorithm relies only on the second order statistics of the received signal. Therefore it usually requires smaller sample size than all other schemes suggested to date. In our simulation at a signal-to-noise ratio (SNR) of 30dB, 100 symbols are sufficient to give a good estimate of the channel, which is a much smaller number than for the existing techniques. This fact also implies that the algorithm can be used to estimate channels with relative faster varying fading.

4. There is no restriction imposed on the probability distribution of the source symbols. The random source may be real or complex, continuous or discrete, or even Gaussian, in contrast to the assumptions \([11]\).

It may seem impossible at first glance that a nonminimum phase system can be identified using only the second order statistics of the system output. However, it may be less surprising if one assumes that the system is driven by a nonstationary process. This is indeed the case for most communication channels, where the input signals are cyclostationary (or periodically correlated) rather than stationary.

### 1.3 Problem Statement

When the channel is time invariant, the received complex signal \(x(.)\) can be expressed as

\[
r(t) = \sum_{k=-\infty}^{\infty} s_k h(t - kT),
\]

\[
x(t) = r(t) + n(t),
\]

where

- \(s_k\): an information symbol in a signal constellation \(S\)
- \(h(.): the "composite" channel impulse response that includes pulse shaping filter, multipath channel, and receiver filters.
- \(T\): the symbol interval, \(n(.)\): the additive noise.

The objective of blind channel identification is to estimate \(h(.)\) given only the received signal \(x(.)\). Once such identification is achieved, the estimation of the information symbols \(s_k\)'s becomes more or less a classical problem, and various equalization and sequence estimation techniques can be applied.

We assume the following throughout the development in the sequel:

1. The impulse response \(h(t)\) has finite support.
2. \(\{s_k\}\) is an i.i.d. sequence drawn from \(S\) with certain distribution not necessarily known.
3. \(n(.)\) is a white noise process.

### 2 A Multichannel Representation

An important observation is that, under the first assumption that the channel impulse response has finite support, the space of signal restricted to any finite time interval \(I\) (referred to as an observation interval) is a linear space spanned by a basis comprised of time-shifted copies of \(h(.)\). Specifically, the signal space \(\mathcal{R}_s(s, h(I))\), or simply \(\mathcal{R}(I)\), is defined by

\[
\mathcal{R}(I) = \{ r(t), r(t) = \sum_{k=-\infty}^{\infty} s_k h(t - kT), s_k \in S, t \in I \}
\]

Moreover, when the observation interval \(I\) is finite and the channel impulse response has only finite support, \(\mathcal{R}(I)\) is finite dimensional and its dimension can be computed directly with knowledge of \(I\) and the duration of \(h(.)\). Consider, for example, the received (noise free) signal \(r(.)\) in an observation interval \((t_0, t_0 + L)\) for some finite \(L\). The signal space \(\mathcal{R}(r(t_0, t_0 + L))\) is spanned by \(h(t - K_0T), \ldots, h(t - (K_0 + (d-1))T)\) with \(h(t - iT)'s\) being restricted to \((t_0, t_0 + L)\). The integer \(K_0\) and the dimension \(d\) are functions of \(t_0\), \(L\) and the duration \(L_a\) of the impulse response \(h(.)\) as follows

\[
K_0 = \left\lceil \frac{t_0 - L_a}{T} \right\rceil, \tag{4}
\]

\[
d = \left\lceil \frac{t_0 - (K_0 - 1)T + L}{T} \right\rceil, \tag{5}
\]

where \(\lceil x \rceil (\lfloor x \rfloor)\) stands for the smallest (greatest) integer that is greater than (less than) \(x\). If \(x(.)\) is sampled in \((t_0, t_0 + L)\) with a sampling period \(\Delta\), we then have

\[
x(t_0 + i\Delta) = \sum_{k=0}^{d-1} s_{K_0+k} h(t_0 + i\Delta - (K_0 + k)T) + n(t_0 + i\Delta), i = 1, \ldots, m.
\]

A matrix formulation of the above will be convenient. We can write

\[
x(t_0) = H(t_0)s(t_0) + n(t_0), \tag{7}
\]

where

\[
x(t_0) = (x(t_0 + \Delta), \ldots, x(t_0 + m\Delta))^T, \tag{8}
\]

\[
H(t_0) = [h(t_0 + i\Delta - (K_0 + j - 1)T)], \tag{9}
\]

\[
s(t_0) = [s_{K_0}, \ldots, s_{K_0+d-1}]^T \tag{10}
\]

\[
n(t_0) = [n(t_0 + \Delta), \ldots, n(t_0 + m\Delta)]^T. \tag{11}
\]
Similarly, if the sampling is performed in any interval of the form \((t_0 + nT, t_0 + nT + L)\), replacing \(t_0\) and \(K_0\) by \(t_0 + nT\) and \(K_0 + nT\) in (7) yields,

\[
x(t_0 + nT) = H(t_0 + nT)s(t_0 + nT) + n(t_0 + nT).
\] (12)

Making a closer examination of the \((i,j)^{th}\) entry of \(H(t_0 + nT)\), we see that

\[
h(t_0 + nT + i\Delta - (K_0 + n + j - 1)T) = h(t_0 + i\Delta - (K_0 + j - 1)T).
\] (13)

Hence for all integer \(n\),

\[
H(t_0) = H(t_0 + nT).
\] (14)

We then have

\[
x(t_0 + nT) = H(t_0)s(t_0 + nT) + n(t_0 + nT), \quad n = 0, 1, \ldots
\] (15)

When no confusion arises, we drop \(t_0\) (or let \(t_0 = 0\)) for the sake of simplicity, with the understanding that a certain \(t_0\) is fixed throughout; then the multichannel representation of the received signal is

\[
x(iT) = Hs(iT) + n(iT), \quad i = 0, 1, \ldots
\] (16)

where \(x(iT)\) and \(n(iT)\) are \(m\)-dimensional vectors formed from the \(m\) samples of \(x(t)\) and \(n(t)\) inside the interval \((t_0 + iT, t_0 + iT + L)\) respectively. \(H = H(t_0)\), independent of \(i\), is given by (9), and \(s(iT)\) is a \(d\)-dimensional vector consisting of symbols that have "contribution" to the received signal inside the observation interval \((t_0 + iT, t_0 + iT + L)\) given in (10).

In an actual implementation, the vector \(x(iT)\) can be obtained via time-division demultiplexing [5].

### 3 A New Channel Identification and Equalization Method

#### 3.1 Channel Identifiability

In presenting the ideas of our approach, we ignore the noise for simplicity. The case with noisy data can be treated in a straightforward way and is elaborated in the presentation of the algorithm.

Following the development in the previous section, the "blind" channel identification problem can be phrased as follows:

Consider a vector process \(x(i)\), \(i = 1, \ldots\), obtained from a linear model

\[
x(i) = Hs(i), i = 0, 1, \ldots
\] (17)

along with the following constraints:

1. \(H\) is an \(m \times d\) complex matrix of full column rank.
2. \(s(i)\) is a zero mean stationary process with autocorrelation function

\[
R_s(k) = E(s(i)s^H(i - k))
\] (18)

of the following form:

\[
R_s(k) = J^k, k \geq 0,
\] (19)

\[
R_s(k) = (J^T)^{|k|}, k < 0,
\] (20)

where

\[
J = \begin{pmatrix}
0 & \cdots & 0 \\
1 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 1 & 0
\end{pmatrix}.
\] (21)

The objective of blind channel identification and equalization is to identify \(H\) (channel identification) and estimate \(s(i)\) from \(x(i)\) (channel equalization).

The following theorem settles the issue of channel identifiability.

**Theorem 1** Suppose \(H\) and \(s(i)\) satisfy the linear model (17) and its constraints. Then \(H\) is uniquely determined up to a constant by \(R_x(0)\) and \(R_x(1)\).

The following lemma provides a constructive proof of the above theorem. Its significance is that it offers a computational method to identify \(H\) from \(R_x(0)\) and \(R_x(1)\).

**Lemma 1** Let \(R_x(0)\) have the following singular value decomposition,

\[
U^H R_x(0)V = \text{diag}(\sigma_1^2, \ldots, \sigma_d^2, 0, \ldots, 0)
\] (22)

Let \(u_i\) denote the \(i^{th}\) column of \(U\), and let

\[
U_s = [u_1, \ldots, u_d],
\] (23)

\[
\Sigma = \text{diag}(\sigma_1, \ldots, \sigma_d),
\] (24)

\[
F = \Sigma^{-1} U_s^H
\] (25)

Suppose

\[
R = F R_x(1) F^H
\] (26)

has the singular value decomposition of the form

\[
[y_1, \ldots, y_d]^H R [z_1, \ldots, z_d] = \text{diag}(\gamma_1^2, \ldots, \gamma_d^2)
\] (27)

Then there exists a real phase \(\phi\) such that

\[
H = U_s \Sigma Q e^{j\phi}
\] (28)

where

\[
Q = [y_d, R y_d, \ldots, R^{(d-1)} y_d]
\] (29)

or equivalently,

\[
Q = [ (R^H)^{(d-1)} z_d, (R^H)^{(d-2)} z_d, \ldots, z_d ]
\] (30)
3.2 Algorithm Implementation

Lemma 1 provides the essential parts of the algorithm. We remark here that the constant phase ambiguity is unavoidable and is usually harmless in practice. Nevertheless, such ambiguity can be removed if, for instance, the channel is real. We first outline the algorithm and then address certain technical points of the algorithm.

A Blind Channel Identification and Equalization Algorithm

1. Select a \( t_0, \Delta \) such that \( T = T_j \Delta, L = m \Delta \), and form the vectorized observation \( x(i) = \left[ x(t_0 + \Delta + iT_1), \ldots, x(t_0 + m \Delta + iT_l) \right]^T, i = 0, 1, \ldots \).

2. Estimate \( \hat{R}_x(0) \) and \( \hat{R}_x(1) \) from \( x(i) \) via, for example, time average.

\[
\hat{R}_x(0) = \frac{1}{N} \sum_{i=1}^{N} x(i)x^H(i),
\]

\[
\hat{R}_x(1) = \frac{1}{N} \sum_{i=1}^{N} x(i)x^H(i - 1).
\]

3. From \( \hat{R}_x(0) \), estimate the noise covariance \( \sigma_n^2 \) and the dimension \( d \) of the signal space.

4. Compute the SVD of \( R_0 \),

\[
R_0 = \hat{R}_x(0) - \sigma_n^2 I
\]

and form \( U_0 \) which consists of the singular vectors associated with the \( d \) largest singular values, \( \Sigma \) which consists of the square-root of the \( d \) largest singular values, and then \( F = \Sigma^{-1} U_0^H \).

5. Compute the SVD of \( R \),

\[
R = F(Rx(1) - Rn(1))F^H,
\]

where

\[
Rn(1) = \sigma_n^2 J^T,
\]

\[
J = \begin{pmatrix}
0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots \\
0 & 0 & \cdots & 1
\end{pmatrix}
\]

Denote \( y_d \) and \( z_d \) as the left and right singular vectors corresponding to the smallest singular value.

6. Form the estimate of \( H \) (and consequently \( h(\cdot) \) if necessary) by

\[
H = U_0 \Sigma Q
\]

where

\[
Q = [y_d, Ry_d, \ldots, R^{(d-1)}y_d]
\]

or

\[
Q = [(R^i)_{d-1}z_d, (R^i)_{d-2}z_d, \ldots, z_d]
\]

or certain combination of the above.

7. Extract the information symbols

\[
s(i) = H^Tx(i)
\]

or

\[
= Q^Hfx(i),
\]

or by implementing various types of equalizer or maximum-likelihood estimation scheme based on the estimated channel.

4 A Simulation Example

In this simulation, a 3-ray multipath channel is constructed from a raised cosine pulse \( c(t, \alpha) \) (\( \alpha \) is the roll-off factor) and its delays,

\[
h(t) = (c(t, 0.11) + 0.8c(t-1, 0.11) - 0.4c(t-3, 0.11))W_6(t).
\]

where \( W_6(t) \) is a square window of duration 6 symbol intervals, i.e. \( L_h = 6T \). Among 23 zeros of the system, there are 20 nonminimum phase zeros. The source symbols are drawn from the 16-QAM signal constellation. Figure 1 is the plot of 1000 output symbols of the unequalized channel (obtained by sampling the received signal at \( kT, k = 1, 2, \ldots \)). The signal to noise ratio is 30dB. Clearly, the intersymbol interference is severe and a high error rate is expected. 100 symbols are used to estimate \( Rx(0) \) and \( Rx(1) \). 1000 symbols are then transmitted and the equalized channel output is shown in Figure 2, which indicates that the channel is equalized rather nicely.

5 Conclusions

A new approach to blind identification and equalization of multipath channels is developed in this paper. By exploiting the nonstationarity of the received signal, we are able to identify the nonminimum-phase channel with only the second-order statistics. This leads to a more accurate estimation with a smaller sample size. The over-sampling of the received signal is similar to that used in fractionally spaced equalization, where it is known to be insensitive to various timing errors. The algorithm achieves channel identification and equalization without interrupt the communication of the information bearing symbols. In addition, it can be easily incorporated into various existing equalization methods.

Acknowledgements

This work was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by the Air Force Office of Scientific Research under contract F49620-90-C-0014 (the United States Government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation thereon).
References

Figure 1: The output sampled at Nyquist instant of unequalized channel. SNR=25dB, 1000 symbols plotted.

Figure 2: The output of the equalized channel. 1000 symbols are plotted. 100 symbols are used in the estimation. SNR=25dB.