A New Blind Equalization Algorithm Using Higher Order Statistics
in a Decision Feedback Structure *

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Abstract

In this paper we present a new adaptive blind equalization algorithm named POTEA the Power cepstrum and Tricoherence Equalization Algorithm using a nonlinear structure. It is based on the combined use of second- and fourth-order statistics of the received sequence. A nonlinear decision-feedback equalization (DFE) structure is proposed for equalizing nonminimum-phase channels with severe distortion, since a linear equalizer does not perform well in cases of channels with deep spectral nulls in their amplitude characteristics. The direct form structure of the DFE consists of a feed-forward T-spaced filter (FFF) with an all-pass mixed phase transfer function, and a feed-backward T-spaced filter (FBF) with a minimum phase one. In this paper the performance of POTEA for linear and decision-feedback equalizers is tested, for the cases of 16-QAM and 64-QAM.

1 Introduction

In this paper we present a new adaptive blind equalization algorithm named POTEA the Power cepstrum and Tricoherence Equalization Algorithm [4] with a nonlinear structure. It is based on the combined use of second- and fourth-order statistics of the received sequence. The only information blind equalizers require is related to statistical properties of the transmitted sequence.

A zero-forcing (ZF) constraint is imposed, for a complete cancellation of the intersymbol interference at the output of the equalizer. A nonlinear decision-feedback equalization (DFE) structure is proposed for equalizing nonminimum-phase channels with severe distortion. More specifically, in cases of channels with deep spectral nulls in their amplitude characteristics the linear equalizer does not perform well, because it places a large gain in the vicinity of the spectral null, which enhances the additive noise present in the received signal.

The direct form structure of the DFE consists of a feed-forward T-spaced filter (FFF) and a feedback T-spaced filter (FBF). The FFF is driven by the samples of the received signal \( \{y(k)\} \) while the FBF is driven by decisions at the output of the detector and its coefficients are adjusted to cancel the ISI on the current symbol that results from past detected symbols. In the DFE structure of POTEA, the FFF part has an all-pass mixed phase transfer function while the FBF has a minimum phase one.

An adaptive gradient-type scheme is employed, in order to estimate the cepstral coefficients of the transfer function of the channel \( F(z) \). The sums and the differences of the cepstral coefficients are computed, using the cepstrum of the power spectrum (second-order domain) and the cepstrum of the tricoherence index (fourth-order domain), respectively. The sums of the cepstral coefficients of the transfer function of the channel \( F(z) \) contain the amplitude information, whereas their differences contain the phase information. This property makes the POTEA algorithm capable of identifying and equalizing a nonminimum-phase channel from its output only.

We should emphasize that the algorithm performs simultaneous channel identification and equalization, because the channel and its inverse filter (equalizer) have the same cepstral coefficients with opposite signs. Finally, like all the Higher-Order Spectra (HOS) based techniques, POTEA is less sensitive to additive Gaussian noise, because HOS of Gaussian processes of order 2 are approximately Gaussian.
POTEA (with a linear structure) has been shown to have superior performance in terms of convergence than the existing TEA (the Tricestrum Equalization Algorithm) originally introduced by Hatsi- 
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nakos and Nikias [5], and accordingly, to converge faster than the Bussgang-type blind equalization algorithms (Stop-and-Go, Godard or CMA, Benveniste-Goursat, Sato etc.) at the expense of higher com- 
putation complexity [5]. Another limitation of the Bussgang-type techniques is that they are not globally 
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convergent, as POTEA is, because they are based on minimization of nonconvex cost functions.

In this paper the performance of POTEA for linear 
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equalizers and DFE is tested, especially with chan- 
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nels that have deep spectral nulls. It is demonstrated 
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that when the zeros of the channel transfer function 
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approach the unit circle, the linear equalizer fails to 
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work, while the DFE performs successful equalization 
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in the cases of 16-QAM and 64-QAM.

2 Problem Definition - Preliminaries

Let us assume that the received sequence, \{y(n)\}, is a filtered and noise corrupted version of a transmitted sequence \{x(n)\):

\[ y(n) = z(n) + w(n) = \sum_{k=-L_s}^{L_s} f(k) z(n-k) + w(n) \]  

where \{f(n)\} is the impulse response of the channel. The input complex sequence \{x(n)\} is non- 
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Gaussian white, i.i.d. with mean \( E\{x(n)\} = 0 \), second-order moment \( \beta = E\{|x(n)|^2\} \neq 0 \), skewness \( \gamma = E\{x(n)^3\} - 2E\{x(n)^2\} \neq 0 \). Finally, \{w(n)\} is zero-mean AGN (additive Gaussian noise) and statistically independent from \{x(n)\}.

The transfer function of the system \( F(z) \) may be expressed by its poles and zeros as follows:

\[ F(z) = G z^{-r} I(z^{-1}) O(z), \] 

where \( G \) is a constant, \( r \) is an integer,

\[ I(z^{-1}) = \prod_{i=1}^{L_1} (1-a_i z^{-1}) \quad \text{(minimum phase component)} \] 

\[ O(z) = \prod_{i=1}^{L_2} (1-b_i z) \quad \text{(maximum phase component)} \] 

because \( |a_i| < 1, |b_i| < 1 \) for all \( i \).

Let \( Y(z) \) be the Z-transform of \( \{y(n)\}\). The cepstrum of the Power spectrum is defined as:

\[ c_{P_y}(m) \triangleq Z^{-1}[\ln P_y(z)], \] 

where \( Z^{-1}[.\.\.\] denotes inverse Z-transform and \( P_y(z) \) is the power spectrum of \( \{y(n)\}\).

For symmetric QAM signals the second order moments are defined as follows:

\[ r(m) = E\{y(n) y^*(n+m)\} \neq 0, \] 

or equivalently \( Z[r(m)] = Q F(z^{-1}) F^*(z^{-1}) \), where \( Q = E\{|x(n) x^*(n)\} \).

The cepstrum of the power spectrum of \( \{y(n)\}\) is given by the sum of \( A(k) \) and \( B(k) \) (the minimum and maximum phase cepstral coefficients of \( F(z) \)) [8], [4]. These are:

\[ A(k) = \sum_{i=1}^{L_1} a_i^k, \quad B(k) = \sum_{i=1}^{L_2} b_i^k, \] 

The tricoherence function is given by:

\[ t_y(z_1,z_2,z_3) = \left[ \frac{T_y(z_1,z_2,z_3)}{T_y(z_1^{-1},z_2^{-1},z_3^{-1})} \right]^{1/2} \] 

where \( T_y(z_1,z_2,z_3) \) is the trispectrum; i.e., the Z- 
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transform of the fourth-order cumulants defined by:

\[ L(m,n,l) \triangleq E\{y(i) y^*(i+m) y(i+n) y^*(i+l)\} - E\{y(i) y^*(i+l)\} E\{y(i+m) y(i+n)\} - E\{y(i) y^*(i+m)\} E\{y(i+n) y^*(i+l)\} \] 

The cepstrum of the tricoherence is defined as the in- 
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verse Z-transform of the complex logarithm of the tri- 
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coherence function (7). That is

\[ c_{t_y}(m,n,l) \triangleq Z^{-1}[\ln[t_y(z_1,z_2,z_3)]] \] 

It can be shown [4] that:

\[ c_{t_y}(m,n,l) = j Z^{-1} [\phi(z_1)+\phi(z_3)-\phi(z_1^{-1}) z_2^{-1} z_3^{-1}] \] 

From the above equation, it is obvious that the cepstrum of the tricoherence contains the phase 
information of the channel only.

The complex cepstrum of the tricoherence function can be shown [4] to be equal with the difference of the cepstral coefficients \( A^k, B^k \), along the 3 axes and the main diagonal in a 3-D plot.

The channel identification procedure in POTEA is based on the computation of the cepstral parameters \( A^l \) and \( B^l \), respectively, of the received sequence.
\{y(n)\} and it can be shown [6] that the cepstral parameters of the output sequence are identical to those of the channel impulse response \{f(n)\} because the channel input is white:
\[
c_p(m, n, l) = c_f(m, n, l) + \delta(m, n, l)
\]
which implies that \(c_p(m, n, l) = c_f(m, n, l)\) for \((m, n, l) \neq (0, 0, 0)\), where \(c_p(m, n, l)\) is the tricepstrum of the received sequence \{y(n)\} and \(c_f(m, n, l)\) is the tricepstrum of the channel \{f(n)\}.

3 POTEA with a DFE Structure.

In this section the POTEA algorithm is derived for a DFE structure with a FFF part consisting of \(N_1\) taps and having transfer function \(U_1(z)\) and a FBF part with \(N_2\) taps and transfer function \(U_2(z)\), see Figure 1. Using a zero forcing (ZF) equalization constraint [10] we obtain:
\[
U(z) = \frac{1}{F(z)} = U_1(z) \frac{1}{1 + U_2(z)}
\]
Combining equations (2), (12) we obtain the following:
\[
\begin{align*}
U_1(z) &= \frac{O^*(z^{-1})}{G.O(z)} \\
U_2(z) &= I(z^{-1})O^*(z^{-1}) - 1
\end{align*}
\]
The above infinite impulse response (IIR) transfer functions are approximated by equivalent finite impulse response (FIR) filters.

The steps of the POTEA-DFE algorithm are as follows:

1. Let \(N_1, N_2\): lengths of FFF and FBF respectively.
2. Choose \(p\): Length of minimum and maximum phase cepstral parameters.
3. Estimate adaptively the \(L^0(m, n, l) - M_4 \leq m, n, l \leq M_4\), and \(r^0(m) - M_2 \leq m \leq M_2\) from a finite length window of \(\{y(n)\}\) and then generate the following functions:
\[
\begin{align*}
\hat{r}^0(m, n, l) &= L^0_y(-m-n-l) \ast L^0_y(m, n, l) \\
\hat{r}^0_2(m, n, l) &= L^0_y(-m-n-l) \ast mL^0_y(m, n, l)
\end{align*}
\]
4. Form the following equations:
\[
\sum_{k=1}^{p} S_t^k[-r_s(m-k)] + \sum_{k=1}^{p} S_t^k[r_s(m+k)] = m_S(m)
\]
where \(S_t^k = A^{(k)} + B^{(k)}, k = 1, \ldots, p, m = 1, \ldots, 2p\).
5. Solve adaptively the above systems employing gradient-type adaptation [11].
6. Calculate \(A^{(k)}\) and \(B^{(k)}\) as follows:
\[
\hat{A}(i) = \frac{S(i) + D(i)}{2}
\]
7. Calculate the normalized \((G=1)\) estimate onorm(i, z) at iteration \(i\) is given by equations (11), (12).
8. The reconstructed transmitted sequence at iteration \(i\) is the inverse Z-transform of \(\hat{X}(z)\):
\[
\hat{X}(i, z) = \hat{U}_{\text{norm}}(i, z) Y(z)
\]

4 Simulation Results

In this section the effectiveness of POTEA under a nonlinear decision feedback structure in terms of rate of convergence and robustness in signal conditioning is demonstrated with simulation examples. For the simulations a real-life channel was used with a transfer function having only a single zero, of the form \(1 + ax^{-1}\). The values for the parameters in step 2 are: \(p=0, w=0, z=1, h=0\). The SNR shown in the figures is the signal-to-noise ratio at the input of
the equalizer. The performance of linear and nonlinear POTEA was studied comparatively by approaching the zero closer to the unit circle. In Figure 2 the case of $1 + 0.7z^{-1}$ is shown for 16-QAM where both equalizers obtain equalization. In Figure 3 the case of $1 + 0.8z^{-1}$ is shown for 16-QAM and 64-QAM and the linear equalizer does not perform equalization while the DFE does. Finally in Figure 4 the successful performance of DFE is shown even for the difficult case of $1 + 0.95z^{-1}$.

References


FIGURE 3. Performance of POTEA/DFE

FIGURE 4. Performance of POTEA/DFE