CRIMNO: Criterion with Memory Nonlinearity for Blind Equalization

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Abstract

A new criterion with memory nonlinearity (CRIMNO) is introduced for blind equalization problems. The basic idea of CRIMNO is to make use of the fact that the transmitted data are statistically independent from each other. It is shown that CRIMNO may not have local minima if its weights are chosen properly, thereby guaranteeing global convergence. An adaptive weight CRIMNO algorithm is also presented and tested with simulation examples of QAM signals. It is shown that the adaptive weight CRIMNO algorithm exhibits faster convergence speed than the Godard algorithm without any significant increase in computational complexity.

1 Introduction

In high speed data communication systems, intersymbol interference caused by channel amplitude and phase distortion may be so severe that channel equalization has to be done in order to make correct decision as to which data symbol is transmitted. Conventionally, equalization is done first through a training mode when a known data sequence is transmitted and is then followed by a decision-directed scheme. However, problems arise in such scenarios as multipoint networks, and multipath fading channels, where the receiver has to perform equalization of the channel without a training mode. When the receiver is “blind” to a training data sequence, the problem is known as blind equalization.

Several algorithms have been proposed for blind equalization. In his original paper[1], Sato proposes the first true blind equalization algorithm for PAM systems, by treating multi-level PAM data as binary ones. In [2], Godard combines blind equalization with carrier recovery and proposes a blind equalization cost function for QAM data based on statistical information about the magnitude of the transmitted data. In [3], Benveniste and Goursat combine Sato’s ideas with the decision-directed scheme and obtain a new algorithm which gradually switches to the decision-directed mode when the constellation eye begins to open. In [4], by modifying the decision-directed algorithm in a probabilistic sense, Picchi and Prati obtain the stop-and-go algorithm. In [8], Shalvi and Weinstein generalized Godard’s algorithm to a larger class.

Although these algorithms are different, they perform only memoryless nonlinear transformations on the equalizer outputs to generate the desired response. This, in turn, implies that the cost functions they try to minimize with respect to the equalizer coefficients are also memoryless. These algorithms do not employ the fact that the transmitted data are statistically independent, which is the essence of the new criterion we introduce in this paper. Since statistical independence of the transmitted data involves more than one data symbol, this results in a memory nonlinear transformation on the equalizer outputs and thus a memory nonlinear cost function.

The paper is organized as follows: In section 2, the new criterion, called CRIMNO, is introduced. In section 3, the corresponding stochastic gradient algorithm are derived. In section 4, the convergence analysis of the CRIMNO algorithms is given. In section 5, an adaptive weight CRIMNO algorithm is proposed for blind equalization. Finally, simulation results and conclusions are given respectively in section 6 and section 7.
2 Criterion with memory nonlinearity

Consider a synchronous double-sideband quadrature amplitude modulated (QAM) data communication system as shown in Fig. 1. Then the blind equalization problem can be stated as follows:

We have
\[ y_n = a_n * h_n = \sum_i a_i h_{n-i} \]
\[ z_n = y_n * c_n = \sum_i y_i c_{n-i} \tag{1} \]
and the following knowledge about the transmitted data

- The transmitted data \( \{a_n\} \) come from a finite discrete set.
- The transmitted data are statistically independent from each other.

The problem is: adjust the equalizer coefficients \( \{c_n\} \) to restore the original transmitted data except for a constant delay.

In his original paper [2], Godard proposed a cost function which is independent of the transmitted data, and yet reaches its global minimum at perfect equalization. The Godard's cost function is:

\[ D^{(p)} = E [ |z_n|^p - R_p ]^2 \tag{2} \]

where
\[ R_p = \frac{E [ |a_n |^{2p} ]}{E [ |a_n |^p ]} \tag{3} \]

Note that only the expected value of some function of the current equalizer output appears in Godard's cost function. Therefore, Godard's criterion only makes use of the knowledge about the transmitted data constellation and its probability distribution, and it does not directly use the fact that the transmitted data are statistically independent.

Assume that perfect equalization is achievable and consider the situation where perfect equalization has indeed been achieved. That is
\[ z_n = a_{n-d} \]
where \( d \) is a positive integer, which accounts for the constant delay. Since the transmitted data \( \{a_n\} \) are statistically independent from each other, so are the equalizer outputs \( \{z_n\} \) at perfect equalization. In addition, for most transmitted data constellations, the mean of \( a_n \) is zero. Therefore, at perfect equalization, we have
\[ E(z_n z_{n-1}) = E(a_{n-d} a_{n-1-d}) = E(a_{n-d})E(a_{n-1-d}) = 0 \]

By making use of this property and combining it with Godard's criterion, we obtain a new criterion, called criterion with memory nonlinearity (CRIMNO), which is the minimization of the following cost functions:

CRIMNO Version I: (no update)
\[ M_1^{(p)} = w_0 E [ |z_n|^p - R_p ]^2 + w_1 |E(z_n z_{n-1})|^2 + \cdots + w_M |E(z_n z_{n-M})|^2 \tag{4} \]

CRIMNO Version II: (update)
\[ M_2^{(p)} = w_0 E [ |z_n|^p - R_p ]^2 + w_1 |E(z_n z_{n-1}^{(n)})|^2 + \cdots + w_M |E(z_n z_{n-M}^{(n)})|^2 \tag{5} \]

where \( z_{n-i}^{(n)} \) is the calculated equalizer output at \( t = n - i \) using equalizer coefficients at \( t = n \).

The rationale behind the CRIMNO is that since each term reaches its global minimum at perfect equalization, by appropriately combining them, we can increase the convergence speed of the corresponding stochastic gradient algorithms and eliminate the stable local minima inherent in Godard's criterion. The CRIMNO version II is proposed because we hope to increase the convergence speed further by using the most recent equalizer coefficients.

Remarks:
1. The CRIMNO cost functions I and II depend not only on the current equalizer output, but also on the previous equalizer outputs. As such, they result to the criterion with memory nonlinearity.
2. When \( w_0 = 1 \), \( w_i = 0 \) for \( i \neq 0 \), CRIMNO cost functions reduce to Godard's cost function. Therefore, the CRIMNO algorithms may be seen as generalizations of Godard's algorithm.

3 CRIMNO algorithms

Two versions of CRIMNO were proposed in section 2, whose global minimum corresponds to perfect equalization. Therefore, the blind equalization problem reduces to adjusting equalizer coefficients to seek for the global minimum of the CRIMNO cost functions.
Equalizer coefficients are adjusted using steepest descent method. As is usually done, by substituting the expected value with the current value, and setting $p$ equal to 2, we obtain the following equalizer coefficient adaptation formulas:

**CRIMNO version I:**

$$C_{n+1} = C_n - \alpha Y_n^* z_n [4w_0 |z_n|^2 + 2w_1 |z_{n-1}|^2 + \cdots + 2w_M |z_{n-M}|^2 - 4w_0 R_2]$$  \(6\)

**CRIMNO version II:**

$$C_{n+1} = C_n - \alpha [4w_0 Y_n^* z_n (|z_n|^2 - R_2) + 2w_1 (Y_n^* z_n)_{n-1}^2 + Y_{n-1}^* (n-1)|z_n|^2 + \cdots + 2w_M (Y_n^* z_n)|z_{n-M}|^2 + Y_{n-M}^* (n-M)|z_n|^2)](7)$$

where $Y_n = [y_n, \ldots, y_{n+N-1}]^T; C_n = [c_{n-N+1}, \ldots, c_{n-1}]^T; N = N_1 + N_2 - 1$ is the length of the equalizer; $\alpha$ is the step-size of the algorithm.

**Remarks:**

1. The first $M + 1$ terms in the square bracket of (6) is a weighted sum of $|z_n|^2$, $|z_{n-1}|^2$, $\cdots$, $|z_{n-M}|^2$. As such, it can be regarded as a way of calculating the expected value $E(|z_n|^2)$. Consequently, the CRIMNO algorithm I amounts to substituting the original single point estimate of $E(|z_n|^2)$, with a more accurate estimate involving a weighted sum of $M + 1$ terms. This perhaps explains why the CRIMNO algorithm exhibits faster convergence speed than the original memoryless Godard algorithm.

2. At perfect equalization, $E(|z_n|^2) = E(|z_{n-1}|^2) = \cdots = E(|z_{n-M}|^2).$ Thus the first $M + 1$ terms in the square bracket of (6) is an estimate of $\gamma E(|z_n|^2)$, where $\gamma = 4w_0 + 2w_1 + \cdots + 2w_M$. Comparing (6) with the Godard algorithm and noting that the coefficient before $R_2$ is still $4w_0$, we conclude that the CRIMNO algorithm I will exhibit the problem of constellation eye shrinkage. A solution to this problem is to add an Automatic Gain Control (AGC) unit after the equalizer. The AGC dynamically scales the equalizer output such that it attains the same variance as the transmitted data.

### 4 Convergence analysis

There are two kinds of equilibria in blind equalization problem. One is with respect to the equalizer coefficients while the other is with respect to the combined impulse response of channel and equalizer. The former includes the latter as a subset. It has been shown in [7] that Godard’s criterion has no stable undesired equilibria with respect to the combined impulse response when an infinite length equalizer is used. However, what is required in practice is no stable local minima with respect to equalizer coefficients. In this section, we will show that CRIMNO will not have any local minima if the weights are appropriately chosen.

Assume that the equalizer coefficients consist of $c_0, c_1$ and that the initial states of the received signals $y$ are $0$. After some algebra, we obtain the following results:

For the CRIMNO version II (memory size $M = 1$):

$$M_2^{(2)} = w_0 J_0 + w_1 J_1$$  \(8\)

where

$$J_0 = (c_0^2 - 1)^2 + c_1 (c_1 - 0.6c_0)^2 + c_2 (6c_0^2 - 2)(c_0 - 0.6c_0)^2$$

$$J_1 = c_3 (c_1 - 0.6c_0)^2 (c_0 - 0.6c_1)^2$$  \(9\)

$\kappa_1, \kappa_2, \kappa_3$ are constants.

### 5 Adaptive weight CRIMNO algorithm

In this section, an ad hoc way of adjusting the weights on-line is presented.
The basic idea is: we estimate the value of each term in the CRIMNO cost functions and set the weights proportional to the deviations of the corresponding terms from their ideal value at perfect equalization.

Rewrite the CRIMNO cost function as

$$M^{(p)}_1 = w_0 J_0 + w_1 J_1 + \cdots + w_M J_M$$

(10)

and define the deviation of the $i$th term $D(J_i)$ by

$$D(J_i) \equiv |J_i - J_{i0}^{(p)}|$$

(11)

where $J_{i0}^{(p)}$ is the value of $J_i$ at perfect equalization. Then the weights are adjusted using the following formulas:

$$w_0 = \begin{cases} \gamma_0 D(J_0) & \gamma_0 D(J_0) < \lambda \\ \lambda & \gamma_0 D(J_0) \geq \lambda \end{cases}$$

$$w_i = \begin{cases} \gamma D(J_i) & \gamma D(J_i) < \lambda \\ \lambda & \gamma D(J_i) \geq \lambda \end{cases}$$

(12)

where $\gamma_0, \gamma$ are positive scaling constants; $\lambda$ is a constraint on the maximum value of the weights for the sake of stability of the algorithm.

The CRIMNO algorithm with weights so adjusted online is called adaptive weight CRIMNO algorithm.

Remark:

1. When the deviations of all terms vary proportionally, the adaptive weight scheme becomes an adaptive step-size algorithm. Moreover, the adaptation is done automatically. So when the algorithm converges, the weights decrease toward zero. Hence, the adaptive weight CRIMNO algorithm acquires as a byproduct the decreasing step-size, which has been proven to be an optimal strategy for equalization [6].

6 Simulation results

Fig. 3 shows the amplitude and phase characteristics of the channel in our simulation. Fig. 4 shows the performance of the CRIMNO, and the adaptive weight CRIMNO algorithms, compared with that of Godard algorithm. The reason that there is little difference between the performance of the two versions of CRIMNO is the small step-size $\alpha$ used for the stability of the algorithm. Fig. 5 shows the performance of the adaptive weight CRIMNO algorithm for different memory size $M$. Table 1 summarizes the computational complexity of Godard’s, the CRIMNO, and the adaptive weight CRIMNO algorithms.

7 Conclusions

A new criterion with memory nonlinearity and its corresponding blind equalization algorithms have been introduced for blind equalization. The new criterion has been shown to have no local minima if the weights are chosen properly, therefore guaranteeing its global convergence. In addition, the adaptive weight CRIMNO algorithm has been demonstrated to exhibit much faster convergence speed than Godard’s algorithm at the expense of little increase in computational complexity.

References


Fig. 1. Diagram of communication systems.

Fig. 2. (a) Plot of CRIMNO cost function; (b) Contour of CRIMNO cost function.

Table 1. Comparison of Computational Complexity

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Real Multiplication</th>
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<tbody>
<tr>
<td>Godard</td>
<td>4N+5</td>
</tr>
<tr>
<td>CRIMNO (Version I)</td>
<td>4N+3M+5</td>
</tr>
<tr>
<td>CRIMNO (Version II)</td>
<td>MN+8M+1N+5</td>
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<tr>
<td>AW CRIMNO (Version I)</td>
<td>4N+8M+5</td>
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Fig. 3. Channel characteristics.

Fig. 4. Comparison of convergence speed.

Fig. 5. Convergence speed with memory order.