Estimation of Cyclic Polyspectra

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Abstract

The term polyspectra refers to the spectral cumulants of a stationary random process or time-series, which are equivalent to the multidimensional Fourier transforms of the temporal cumulants. The Fourier coefficients of periodic (and multiply-periodic) cumulants of cyclostationary random process or time-series are called cyclic cumulants. The multidimensional Fourier transforms of cyclic cumulants are called cyclic polyspectra, and are themselves spectral cumulants. In this paper the general formulas for spectral and temporal cumulants of cyclostationary time-series are presented, some applications are briefly discussed, and methods for measuring cumulants from finite-length data records are described and shown to produce consistent estimators.

1 Introduction

The power spectral density function is of great significance in the mathematical analysis of signal processing problems that involve wide-sense stationary time-series. Since the power spectral density is the Fourier transform of the autocorrelation function, it provides a complete frequency-domain description of the second-order statistical characteristics of such time-series. Similarly, the collection of cyclic spectral density functions, indexed by the cycle frequency \( \alpha \), completely describes the second-order statistical behavior of cyclostationary time-series in the frequency-domain [1]. The set of polyspectra through order \( n \) are sufficient to describe the \( n \)-th order statistical characteristics, in the frequency domain, of \( n \)-th order stationary time-series [2]. In the case of an \( n \)-th order cyclostationary time-series, we require knowledge of the collections of \( n \)-th order cyclic polyspectra for all cycle frequencies for orders \( m = 1 \) to \( m = n \) [3] [4].

The implementation of signal processing algorithms that are based on such idealized functions must naturally entail measurements of the functions that use only finite segments of data. Thus we are led to consider measurement techniques for the power spectrum [1][5], the cyclic spectrum [1][6], the polyspectrum [8][9], and the cyclic polyspectrum. Each of these frequency-domain functions can be characterized as the Fourier transform of a time-domain function. For the PSD it is the autocorrelation, for the cyclic spectrum it is the cyclic autocorrelation, for the polyspectrum it is the temporal cumulant function, and for the cyclic polyspectrum it is the cyclic temporal cumulant function. Thus, measurements of the time-domain parameters can be used to indirectly obtain measurements of the corresponding frequency-domain parameters. It is also sometimes feasible to measure the frequency-domain parameters directly in the frequency domain. Both of these approaches are considered herein.

In this paper we review the definitions of and relations between cyclic cumulants, cyclic moments, and cyclic polyspectra, and we consider nonparametric estimation of both cyclic cumulants and cyclic polyspectra. It is shown that cyclic polyspectra can be estimated consistently by first measuring the cyclic cumulant, multiplying it by a tapering window, and then Fourier transforming it. Measurement of cyclic polyspectra directly in the frequency domain is shown to be relatively difficult due to the fact that infinite-strength spectral functions (containing Dirac delta functions) must be estimated and combined to obtain estimates of finite-strength spectral functions (in which all Dirac deltas cancel each other). Examples are provided to illustrate the theory.

As explained in [10], there is a duality between the statistical theory of parameter (moment or cumulant) estimation formulated within the stochastic-process framework, where moments and cumulants are defined by ensemble averages, and the alternative statistical theory formulated within the nonstochastic framework...
of time-series, where moments and cumulants are defined in terms of time averages. Consequently, the consistency of the estimates presented here within the nonstochastic framework of time-series applies equally well when these estimators are interpreted as estimators for parameters of stochastic processes, provided that the mathematical models of the processes exhibit appropriate cycloergodic properties. Otherwise, there is no reason to expect that estimators that rely on only a single time-series should be consistent. Moreover, the absence of cycloergodic properties in a mathematical model is often indicative of an inappropriate model.

Cyclic cumulants and cyclic polyspectra can be used to perform signal processing tasks that cannot be accomplished by using techniques that involve only second-order statistics. This is so because the higher-order cyclic cumulants can reflect underlying periodicities that the cyclic spectrum cannot. Some applications include signal-selective time-delay estimation, synchronization, and weak-signal detection. Cyclic cumulants also play a fundamental role in the design of low-probability-of-intercept signals.

2 The Parameters of Higher-Order Cyclostationarity

For the time-series \( z(t) \) for \(-\infty < t < \infty \), we define the \( n \)th-order lag-product time-series by

\[
L_\mathbb{Z}(t, \tau)_n \triangleq \prod_{j=1}^{n} z(t + \tau_j),
\]

where \( \mathbb{Z} \triangleq [z(t+\tau_1) \cdots z(t+\tau_n)]^t \) and \( \tau \triangleq [\tau_1 \cdots \tau_n]^t \), and \([\cdot]^t\) denotes matrix transposition. The cyclic temporal moment function (CTMF) of order \( n \) is defined by the limiting time average

\[
R_\mathbb{Z}^n(\tau)_n \triangleq \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} L_\mathbb{Z}(t, \tau)_n e^{-i2\pi \alpha t} dt
\]

\[
\triangleq < L_\mathbb{Z}(t, \tau)_n e^{-i2\pi \alpha t} >_T,
\]

and is simply the Fourier coefficient associated with the component \( e^{i2\pi \alpha t} \) in the time-series \( L_\mathbb{Z}(t, \tau)_n \). It can be seen that the CTMF is a Fourier coefficient of a moment function because the \( n \)-order fraction-of-time probabilistic moment (the temporal moment function [TMF]) associated with the lag product (1) can be expressed as [1][12]

\[
R_\mathbb{X}(t, \tau)_n = \sum_{\alpha} R_\mathbb{Z}^n(\tau)_n e^{i2\pi \alpha t},
\]

where the sum is over all real numbers \( \alpha \), called cycle frequencies, for which \( R_\mathbb{Z}^n(\tau)_n \neq 0 \). The functions (2) and (3) exist and are well-behaved for appropriate models of many time-series including amplitude modulated (AM), pulse-amplitude-modulated (PAM), and phase-shift-keyed modulated (PSK) signals. It is assumed in this paper that the limit (2) exists for all \( \alpha \) pointwise in \( \nu \) and in temporal mean-square over \( \nu \), and is continuous in \( \tau \), and that the series (3) converges.

The temporal cumulant function (TCF) of order \( n \) for the set of time-series translates \( \{x(t + \tau_j)\}_{j=1}^{n} \) is defined by

\[
C_\mathbb{X}(t, \tau)_n = \sum_{P=\{\nu_1\}_{j=1}^{p}} k(p) \prod_{j=1}^{p} R_\mathbb{Z}(t, \tau_{\nu_j})_{n_j},
\]

which is completely analogous to its stochastic-process counterpart in [7]. The sum in (4) is over all distinct partitions \( P \) of the set of indices \( \{1, 2, \ldots, n\} \), where each partition \( \nu_j \) has \( p \) elements, \( 1 \leq p \leq n \), and \( k(p) = (-1)^{p-1}(p-1)! \). The vector \( \tau_{\nu_j} \) is the vector of \( n_j \), lags with indices in the set \( \nu_j \). The cyclic temporal cumulant function (CTCF) is the Fourier coefficient of the TCF:

\[
C_\mathbb{Z}(t, \tau)_n \triangleq < C_\mathbb{X}(t, \tau)_n e^{-i2\pi \alpha t} >_1.
\]

Combining (3)–(5) reveals that the CTCF is given by the following explicit function of lower-order CTMFs:

\[
C_\mathbb{Z}(t, \tau)_n = \sum_{\alpha} \sum_{1 \leq p \leq n} \left( \prod_{j=1}^{p} R_\mathbb{Z}^n(\tau_{\nu_j})_{n_j} \right),
\]

where \( 1 = [1 \cdots 1]^t \), and \( \alpha = [\alpha_1 \cdots \alpha_p]^t \). The CTCF was derived in [4] as the solution to the problem of removing from the Fourier coefficient \( R_\mathbb{Z}^n(\tau)_n \) all contributions from Fourier coefficients \( R_\mathbb{Z}^n(\tau_{\nu})_{n_j} \) of lower order. This is equivalent to removing from the finite-strength additive sine-wave component of frequency \( \alpha \) in the lag product time-series \( L_\mathbb{X}(t, \tau)_n \), all contributions from products of sine-wave components in lag products \( L_\mathbb{X}(t, \tau_{\nu_j})_{n_j} \) of lower order that can be obtained by factoring \( L_\mathbb{X}(t, \tau)_n \).

The CTMF and the CTCF are not in general integrable due to the presence of sinusoidal components. These components formally result in Dirac deltas in the \( n \)-dimensional Fourier transform of the CTCF. However, a reduced-dimension version of the CTCF is absolutely integrable for many time-series of interest.\(^1\)

\(^1\) A condition for integrability of the reduced-dimension ver-
and, therefore, it is strictly Fourier transformable [4]. The reduced-dimension CTCF is simply the CTCF associated with the \( n \) variables \( \{x(t + \tau_j)\}^n_{\tau_j=0} \) with \( \tau_n = 0 \). We use the notation
\[
\hat{C}_2(u)_n \triangleq C_2(\tau)_n |_{\tau = u, \tau_n = 0},
\]
where \( u \) is an \((n-1)\)-dimensional vector for the reduced-dimension CTCF. The \((n-1)\)-dimensional Fourier transform of (7) is denoted by \( \hat{P}_2(f')_n \):
\[
\hat{P}_2(f')_n = \int_{-\infty}^{\infty} \hat{C}_2(u)_n e^{-i2\pi u f'} du,
\]
where \( f' = [f_1 \cdots f_{n-1}]' \).

The cyclic polyspectrum is defined as follows. Consider the \( n \) complex-demodulate time-series \( X_T(t, f_j) \) for \( j = 1, \ldots, n \), associated with narrow bandpass filtered versions of \( c(t) \), where
\[
X_T(t, f) = \int_{t-T/2}^{t+T/2} z(v)e^{-i2\pi f v} dv.
\]
The limit as \( T \to \infty \) of the limiting time-average of the product of these spectral components is called the spectral moment function (SMF)
\[
S_2(f) \triangleq \lim_{T \to \infty} \prod_{j=1}^{n} X_T(t, f_j),
\]
and it can be shown that Dirac deltas can be factored out as follows:
\[
S_2(f) = \sum_\alpha S_2(f')_n \delta(f^1 - \alpha),
\]
where \( \delta(\cdot) \) is the Dirac delta function. However, the factor \( S_2(f')_n \) typically contains additional Dirac deltas. The spectral cumulant function (SCF) is given by
\[
P_2(f) = \sum_{P=(\nu)_{j=1}^n} \left[ k(p) \prod_{j=1}^{n} S_2(f_{\nu_j}) \right],
\]
where \( f_{\nu_j} \) is the vector of frequencies with subscripts in the set \( \nu_j \), and it follows from (11) that Dirac deltas can again be factored out:
\[
P_2(f) = \sum_\alpha \hat{P}_2(f')_n \delta(f^1 - \alpha).
\]

Analogous to the definition of the cyclic spectrum (and power spectrum) in [1], the factor \( \hat{P}_2(f')_n \) is defined to be the cyclic polyspectrum (CP) and is given explicitly by
\[
\hat{P}_2(f')_n = \sum_{P=(\nu)_{j=1}^n} \left[ k(p) \prod_{j=1}^{n} (\delta(f_{\nu_j} - \alpha_j)) \right]
\]
where \( \delta(\cdot) \) is the Kronecker delta function. As first shown in [4], the CP is the \((n-1)\)-dimensional Fourier transform of the CTCF \( C_2(\tau)_n \) (cf. (8)). This is a generalization of the Wiener relation between the power spectrum and autocorrelation from second-order stationary time-series (cf. [1]) to nth-order cyclostationary time-series. Within the stochastic-process framework of generally nonstationary processes, it should be called the cyclic Shiryaev-Kolmogorov relation [11], which is the generalization of the Wiener-Khinchin relation (cf. [1]). Consequently, we can call the relation (8) for nonstochastic time-series the SKG (Shiryaev-Kolmogorov-Gardner) relation. The SKG relation suggests a time-domain-based method for estimating the CP in addition to the frequency-domain-based method suggested by the definitions (10), (11), and (14). Because the CP can be absolutely integrable, the CP—unlike the SMF—contains no Dirac deltas. That is, all Dirac deltas present in the individual terms in (14) cancel.

3 Time-Domain Estimation Methods

To estimate the CP, we first consider estimation of the CTCF \( C_2(u)_n \), which in turn means that we must consider estimates of the CTMFs \( R_2(\nu)_n \), which we can then combine according to (6). Given the finite segment of data \( z(v) \) for \( t \leq T/2 \leq v \leq t + T/2 \), which can be expressed as \( z(v) \text{rect}(v - t)/T \), where
\[
\text{rect}(v) = \begin{cases} 
1, & |v| \leq 1/2, \\
0, & \text{otherwise},
\end{cases}
\]
we can form the lag product
\[
L_2(\nu, v, \tau)_n = \prod_{j=1}^{n} z(v + \tau_j) \text{rect}(\frac{v + \tau_j - t}{T}),
\]
and the CTMF estimate
\[
R_2(\nu, v, \tau)_n \triangleq \frac{1}{T} \int_{-\infty}^{\infty} L_2(\nu, v, \tau)_n e^{-i2\pi v v} dv
\]
\[
= \frac{1}{T} \int_{t}^{t+T} \prod_{j=1}^{n} z(v + \tau_j) e^{-i2\pi v v} dv,
\]

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where \( t_i = t - T/2 - \min(\tau_j) \) and \( t_u = t + T/2 - \max(\tau_j) \). Clearly this estimate is strongly consistent (converges pointwise in \( \tau \) for every \( t \)) since the limit (2) is assumed to exist,

\[
\lim_{T \to \infty} R^2_{\varphi}(t, \tau)_n = R^2_{\varphi}(\tau)_n. \tag{16}
\]

The estimator for the cyclic cumulant is (6) with each CTMF replaced by the estimated CTMF (15),

\[
C^2_{\varphi}(t, \tau)_n = \sum_{p=0}^{T} k(p) \times \left[ \sum_{(\alpha_j) \alpha^{j=0} = 1} \prod_{j=1}^{p} R^2_{\varphi}(t, \tau_{\alpha_j}) \right]. \tag{17}
\]

The estimate of the reduced-dimension CTCF, denoted by \( C^2_{\varphi}(t, u)_n \), is given by (17) with \( u_i = \tau_i, i = 1, \ldots, n - 1 \), and \( \tau_n = 0 \). Since the limits defining all of the lower-order CTMFs are assumed to exist, the reduced-dimension cyclic cumulant estimator is strongly consistent,

\[
\lim_{T \to \infty} C^2_{\varphi}(t, u)_n = C^2_{\varphi}(u)_n. \]

Because we know that in the \( n = 2 \) case the limit of the transform of the estimated reduced-dimension CTCF does not exist unless the CTCF estimate is first tapered [1], we introduce a cumulant tapering window \( w_{1/\Delta f}(u) \) before transforming in \( u \) to obtain an estimate of the CP:

\[
P^2_{\varphi}(f')_n = \int_{-\infty}^{\infty} w_{1/\Delta f}(u) C^2_{\varphi}(t, u)_n e^{-i2\pi u f'} du \tag{18}
\]

The tapering window is assumed to be absolutely integrable with approximate width \( 1/\Delta f \) in each dimension. It is can be taken to be the product of \( n - 1 \) one-dimensional window functions such as

\[
w_{1/\Delta f}(u) = \prod_{j=1}^{n-1} \text{rect}(u_j A f).
\]

Since (18) can be expressed as the multidimensional convolution

\[
P^2_{\varphi}(t, f')_A f = W_{a}(f') \circ P^2_{\varphi}(t, f')_n, \tag{19}
\]

where

\[
W_{a}(f') = \mathcal{F}\{w_{1/\Delta f}(u)\},
\]

\[
P^2_{\varphi}(t, f')_n = \mathcal{F}\{C^2_{\varphi}(t, u)_n\}, \tag{20}
\]

we can propose the alternative time-domain estimation technique that first computes the cyclic cumulant estimate (17), then Fourier transforms to obtain (20), and finally frequency smooths as in (19). This is the generalization from the spectrum to the cyclic polyspectrum of the Wiener-Daniell method (cf. [1]). The estimator (18) is the generalization from the spectrum to the cyclic polyspectrum of the Blackman-Tukey method (cf. [1]).

As long as the tapering window \( w_{1/\Delta f}(\cdot) \) has finite support, we can approximate the Riemann integral in (18) to within any desired tolerance with a finite sum and then interchange the order of the limit as \( T \to \infty \) and the sum. The resultant limit then exists pointwise in \( u \) for every \( t \) and equals the reduced-dimension CTCF. Since the Fourier transform of the reduced-dimension CTCF is assumed to exist, then the Fourier transform of its product with the tapering window exists and equals the convolution of the CP with the transformed window. Finally, the effect of this convolution on the CP vanishes in the limit as the width parameter \( 1/\Delta f \) approaches infinity. Hence the CP estimate (18) is strongly consistent:

\[
P^2_{\varphi}(f')_n = \lim_{\Delta f \to 0} \lim_{T \to \infty} P^2_{\varphi}(t, f')_\Delta f. \tag{21}
\]

The order of the limits in (21) reflects the fact that \( T \gg 1/\Delta f \) is required to obtain an estimate with low temporal variance (cf. [1] for the \( n = 2 \) case).

4 Frequency-Domain Estimation Methods

A frequency-domain method is one in which the first step is to transform the data as in (9). The motivation for doing this is to utilize the power and speed of the FFT. This approach leads to the well-known computationally efficient methods of Bartlett and Welch (the time-averaged periodogram method) and Wiener and Daniel1 (the frequency-smoothed periodogram method) in the \( n = 2 \) case for \( \alpha = 0 \) (stationary time-series) (cf. [1]). For \( \alpha \neq 0 \) (for cyclostationary time-series) in the \( n = 2 \) case, these methods also can be computationally attractive [1][13]. However, for the \( n > 2 \) case, there is a serious computational impediment. The time-averaging approach follows from (10) and (11) and uses in (14) the estimates

\[
S^2_{\varphi}(t, f')_\Delta f \triangleq g_\Delta(t) \circ S^2_{\varphi}(t, f'), \tag{22}
\]

where \( g(\cdot) \) is a one-dimensional function with approximate width \( \Delta f \), which should be much greater than the reciprocal \( T = 1/\Delta f \) of the spectral resolution width \( \Delta f \) (to obtain low temporal variance, cf. [1] for
The frequency-smoothing approach replaces the time-following frequency-smoothed estimates:

\[ S_{x}(t, f') = \frac{1}{T} \sum_{k=1}^{n} X_{k}(t, \alpha - f') \]

The computational impediment in these two methods is the issue of the assumed temporal mean-square convergence in the case of a zero-mean stationary time-series with no additive sine-wave components, the polyspectra of the spectral lines and then estimating the corresponding sine-wave components in the data and subtracting them from the data. The second-order SMF that contain no Dirac deltas, the second-order CP results in all Dirac deltas cancelling. Neverthe-

\[ \tilde{S}_{x}(t, f') = S_{x}(t, f') - \sum_{\alpha_{1} + \alpha_{2} = \alpha} S_{x}^{\alpha_{1}} \tilde{S}_{x}^{\alpha_{2}} \delta(\alpha - f') \]  

(24)

where

\[ \tilde{S}_{x}^{\alpha} = < x(t)e^{-i\alpha t} >_{t}, \quad k = 1, 2. \]

For \( \alpha = 0 \), \( \tilde{S}_{x}^{(f_{1})} \) is the usual power spectrum. The spectral lines in \( \tilde{S}_{x}^{(f_{1})} \) correspond precisely to the terms \( S_{x}^{\alpha_{i}} \delta(\alpha_{i}) \) that are subtracted off (because of the assumed temporal mean-square convergence in (2), cf. [1] for this \( n = 2 \) case). Even though the CP in (24) contains no Dirac deltas, the second-order SMF \( \tilde{S}_{x}^{(f_{1})} \) to be estimated to form the CP does in general contain Dirac deltas. However, the linear combination of the products of SMFs that defines the CP results in all Dirac deltas cancelling. Nevertheless, when estimates of these SMFs must be made, a huge dynamic range for computation is required. This can be avoided in the \( n = 2 \) case by simply detecting the spectral lines and then estimating the corresponding sine-wave components in the data and subtracting them from the data. The second order SMFs then contain no Dirac deltas. However, for \( n > 2 \), this approach is not adequate. For example, for third-order SMFs, the sine-wave components present in second-order lag products but absent in the data itself cannot be removed by preprocessing the data.

In general, all frequency-domain-based methods of estimating the CP suffer from the problem of requiring excessive dynamic range for computation because all such methods entail the estimation of SMFs, which in general contain Dirac deltas that are supposed to cancel each other when combined to form the CP estimate. However, if the time-series does not exhibit lower-order cyclostationarity, then the following equivalence holds

\[ \tilde{S}_{x}^{(f')} = \tilde{P}_{x}^{(f')}, \quad \alpha \neq 0. \]  

(25)

In this case, the CP can be estimated in the frequency domain using either the time-averaging method (22) or the frequency-smoothing method (23). Estimation of the polyspectrum for stationary stochastic processes is treated in [9], where the frequency-domain technique (23) with \( \alpha = 0 \) is proposed. The authors of [9] state that this can be done only if the set of frequencies \( f \) in (11) is such that no proper subset sums to zero. Since the set itself must sum to \( \alpha = 0 \) (zero is the only cycle frequency for stationary signals), this condition is equivalent to stating that there can be no partition of the frequencies for which each subset sums to zero. Thus, none of the terms in (12) that contain more than one factor are nonzero. Or, in other words, (25) holds with \( \alpha = 0 \). In the case of a zero-mean stationary time-series with no additive sine-wave components, the polyspectra of orders 1, 2, and 3 can all be estimated for any \( f \) using the frequency-smoothed higher-order periodogram method (23) of Brillinger and Rosenblatt because no lower-order SMF products are nonzero. This explains the common appearance of frequency-domain techniques for estimation of the bispectrum in the literature. However, for estimation of cyclic polyspectra, restriction to specific sets of frequencies \( f \) for which equality in (25) holds can be excessively limiting in practice.

5 Examples

In this section we provide examples of the estimation of the CTCF \( C_{x}(n) \) and CP \( P_{x}(f') \) for a PAM time-series in two corruptive environments for the case of \( n = 4 \). The two environments are additive white Gaussian noise and cyclostationary interference. The PAM time-series is modeled by

\[ x(t) = \sum_{m=-\infty}^{\infty} a_{m}p(t + mT_{b}), \]  

(26)

where \( \{a_{m}\} \) is an iid symbol sequence, and \( p(t) \) is a pulse function. It can be shown that the CTCF for \( x(t) \) is given by

\[ C_{x}(n) = \frac{C_{a,n}}{T_{b}} \int_{-\infty}^{\infty} p(v) \prod_{j=1}^{n-1} p(v + \tau_{j}) e^{-i2\pi \alpha v} dv, \]

for \( \alpha = k/T_{b} \) for all integers \( k \), where \( C_{a,n} \) is the nth-order cumulant for the symbol variables \( \{a_{m}\} \).

Estimation for a discrete-time version of (26) was carried out on a digital computer. The PAM signal that was generated is considered to be a sampled version of the continuous model (26), with sampling increment \( T_{s} \), which we take to be equal to unity for
convenience. The symbol interval $T_0$ is equal to 8, the symbols are binary-valued ($\pm 1$), and occur with equal probability. The pulse function is a rectangle with unity height and width equal to $T_0$.

Estimates (15) of the CTMF are produced by applying an FFT to the lag product time-series, and choosing the bin that corresponds to the desired value of $\alpha$. The sets of $\alpha$ values $\{\alpha_j\}$ corresponding with $n_j = 2$ used in (17) is $\{k/T_0 : |k| \leq 3\}$. Partitions with an element $\nu_i$ such that $n_j = 1$ are not used to form the CTCF estimate, since the signal is known to have no additive sine-wave components (i.e., no sinusoidal components of the time-series). In the latter case, the cyclic polyspectrum is seen to be a linear combination of products of spectral moment functions which include Dirac deltas, all of which cancel out in the cyclic polyspectrum. Because of the impracticality of this cancellation using finite precision arithmetic, it is concluded that the cyclic polyspectrum is best measured by first measuring the cyclic temporal cumulant, tapering it, and then transforming it. The noise tolerance of the cyclic polyspectrum, which results from the tolerance to Gaussian noise exhibited by any cumulant (of order greater than 2) combined with the tolerance to any stationary noise exhibited by any cyclic statistic is illustrated qualitatively by considering estimation for a noisy pulse-amplitude-modulated time-series. Also, the tolerance to cyclostationary interference with cycle frequencies unequal to those of the signal of interest is illustrated by considering estimation for a time-series consisting of the sum of two PAM signals.

6 Conclusions

We have considered the problem of estimating cyclic polyspectra, which are higher-order statistics of a particular class of nonstationary time-series, namely cyclostationary time-series. The cyclic polyspectrum can be viewed as the Fourier transform of a higher-order cyclic temporal cumulant, or as a cumulant of spectral components of the time-series. In the latter case, the cyclic polyspectrum is seen to be a linear combination of products of spectral moment functions which include Dirac deltas, all of which cancel out in the cyclic polyspectrum. Because of the impracticality of this cancellation using finite precision arithmetic, it is concluded that the cyclic polyspectrum is best measured by first measuring the cyclic temporal cumulant, tapering it, and then transforming it. The noise tolerance of the cyclic polyspectrum, which results from the tolerance to Gaussian noise exhibited by any cumulant (of order greater than 2) combined with the tolerance to any stationary noise exhibited by any cyclic statistic is illustrated qualitatively by considering estimation for a noisy pulse-amplitude-modulated time-series. Also, the tolerance to cyclostationary interference with cycle frequencies unequal to those of the signal of interest is illustrated by considering estimation for a time-series consisting of the sum of two PAM signals.

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