DESIGN OF AN INFRARED SENSOR
TO ESTIMATE TARGET PARAMETERS

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Summary
This is the last of a group of papers on design of infrared sensors having large apertures and many detectors. The first described an eight step, high-level design procedure starting with the establishment of mission specifications. The second assumed a scanning sensor and treated design tradeoffs among aperture size, detector quality, number of detectors, and filter quality to achieve a specified performance at minimum cost. A description of the pertinent optical techniques is also available.

In this paper a frequency-domain error analysis is first presented for the infrared sensor. Based on models for the target, the noise, and the sensor, the explicit system response and the optimal parameter estimator are derived. These results are then applied to the design of optimum focal plane geometry.

Finally, the design of realizable, suboptimal, matched filters using computer-aided optimization techniques is presented. The results presented in this paper have been applied to the design of an infrared sensor system leading to a practical realization of the signal processing portion of the system.

Estimation
In an infrared sensor system, the parameter estimation problem is that of determining whether a signal exists in the neighborhood of given point P and has an intensity in the neighborhood of J0. This is a classical problem of hypothesis testing on a signal with unknown parameters. For the infrared sensor system, the unknown parameters to be estimated are the target position and magnitude. This classical problem has been treated extensively in the literature. The first step in the problem stated above is to obtain an estimate of position and magnitude.

The problem of estimating the magnitude and position (in time) of a signal of known shape is also treated in the literature. This is the one-dimensional counterpart of the problem stated above. The one-dimensional problem describes the function of a single detector cell channel in the infrared sensor system treated in this paper. To treat the full two-dimensional problem, one must consider an array of detector cells and solve a multi-channel estimation problem. Only the single-channel problem will be discussed here.

It has been shown that the estimate of signal magnitude and position is based on two quantities. One can be interpreted as the output of a filter which maximizes the signal-to-noise ratio SNR (hereafter referred to as a matched filter). This filter is well known and is defined in the frequency domain by

\[ F(u) = \frac{S^*(u)}{G(u)} \]  (1)

where \( * \) denotes complex conjugate. The second quantity used in the estimation logic is simply the derivative of the output of the matched filter. In practice this derivative can be obtained from an internal pick-off in the matched filter, or the estimation logic can be implemented digitally in such a way that it is not necessary to differentiate the noisy output of the matched filter. In any case, the basic filter necessary for the estimation logic is the matched filter as given above.

The variance of the estimates in position and magnitude can be derived by linearizing the estimation equations about the true parameter values. Let

\[ J = \text{true signal magnitude}, \]
\[ p = \text{true signal position (e.g., time of signal peak),} \]
\[ \hat{J} = \text{estimate of } J, \]
\[ \hat{p} = \text{estimate of } p. \]

Then linearization of the estimation equation yields...
It is noted that

\[ \sigma_J^2 \triangleq (J - J_0)^2 = J^2 \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{\hat{G}(\omega)}{G(\omega)} \right|^2 d\omega \right]^{-1}, \quad (2) \]

\[ \sigma_p^2 \triangleq (p - p_0)^2 = \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{\hat{\delta}(\omega)}{G(\omega)} \right|^2 d\omega \right]^{-1}. \quad (3) \]

where \( \sigma_J \) is the standard deviation of the signal, \( \sigma_p \) is the standard deviation of the noise, \( J_0 \) is the baseline flux, and \( p_0 \) is the baseline position.

It is noted that

\[ \sigma_J^2 = \frac{J^2}{\text{SNR}_0}, \quad (4) \]

or

\[ \text{SNR}_0 \triangleq \sigma_J. \quad (5) \]

where \( \text{SNR}_0 \) is the maximum signal-to-noise ratio as produced by the ideal matched filter. The quantities \( \sigma_J \) and \( \sigma_p \) (or \( \text{SNR}_0 \)) are taken as measures of the system performance in both evaluation of system designs and in optimization of certain system parameters.

### Target, Noise, and Sensor Models

The scanning infrared sensor considered here is represented by the block diagram in Figure 1. For low signal strengths the detector response and noise statistics are realistically represented as linear and time-invariant, so they are amenable to frequency domain analysis. This is desirable, since the signal processing algorithms are conveniently handled this way. Furthermore, the fast Fourier transform allows efficient computation in the frequency domain.

#### Background Noise

![Block Diagram of Sensor](image)

In the following development the two-dimensional spatial Fourier transform of \( f(x, y) \) is defined by

\[ \mathcal{F}_{x,y}(f(x,y)) = \hat{f}(k_x, k_y) \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-j2\pi(k_xx + k_yy)} f(x,y) dx \, dy, \quad (6) \]

and its inverse is given by

\[ \mathcal{F}_{x,y}^{-1}(\hat{f}(k_x, k_y)) = f(x,y) \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j2\pi(k_xx + k_yy)} \hat{f}(k_x, k_y) dk_x \, dk_y. \quad (7) \]

The one-dimensional inverse \( \mathcal{F}_{x,y}^{-1} \) is defined similarly. The temporal Fourier transform of \( g(t) \) is defined by

\[ \mathcal{F}_{t}(g(t)) = G(\omega) = \int_{-\infty}^{\infty} e^{-j\omega t} g(t) dt, \quad (8) \]

and its inverse is given by

\[ g(t) = \mathcal{F}_{t}^{-1}(G(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} G(\omega) d\omega. \quad (9) \]

### Signal Output

The signal portion of the detector output is found from the detector response to the incident target power. For a point target located at the origin, the spatial Fourier transform of the spectral radiance is

\[ J_0 \delta(x) \delta(y) \rightarrow J_0 \frac{W}{sr \cdot u}. \quad (10) \]

The optical point spread function is assumed to be Gaussian, so the transform pair is

\[ \frac{1}{2\pi \sigma_x^2 \sigma_y^2} \exp\left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right) \leftrightarrow O(k_x, k_y) \]

\[ = \exp\left( -\frac{1}{2}(2\pi \sigma_x \sigma_y)^2 \right). \quad (11) \]

For a detector centered at \((x, y)\), the transform pair of the aperture function is

\[ \frac{\Delta x}{2} \leq x \leq \frac{\Delta x}{2} \quad \text{and} \quad \frac{\Delta y}{2} \leq y \leq \frac{\Delta y}{2} \rightarrow A(k_x, k_y) \]

\[ = \frac{\sin \frac{xk_x \Delta x}{\pi k_x}}{\sin \frac{yk_y \Delta y}{\pi \xi y}} \quad (12) \]

where \( \Delta x \) and \( \Delta y \) are the dimensions of the detector image. It will be convenient in subsequent computations to use the fact that \( O(k_x, k_y) \) and \( A(k_x, k_y) \) are separable; that is,

\[ O(k_x, k_y) = O_x(k_x)O_y(k_y), \quad (13) \]

where

\[ O_x(k_x) = \exp\left( -\frac{1}{2}(2\pi \sigma_x \sigma_y)^2 \right), \quad (14) \]
Now, if the target spectral radiance is assumed constant over the optical bandwidth, the spatial transform pair of the target power incident on the detector is

\[ A(k_x, k_y) = A_x(k_x)A_y(k_y), \quad (15) \]

where

\[ A_x(k_x) = \frac{1}{\pi k_x \Delta k} \sin \left( \frac{\pi k_x \Delta k}{k_x} \right) \quad (16) \]

and

\[ \Delta \lambda = \text{optical bandwidth}, \]
\[ A = \text{area of optical aperture}, \]
\[ R = \text{target range}, \]
\[ \eta = \text{optical efficiency}. \]

When the scanner moves at constant speed, \( u \), in the \( x \)-direction, the incident power is changed from a two-dimensional space variable to a one-dimensional time variable by

\[ x = ut; \quad y = y_D, \quad \text{a constant.} \quad (19) \]

Then the temporal transform pair of the target power incident on the detector is

\[ p_T(x, y) \leftrightarrow p_T(k_x, k_y) \]

\[ = cJ_0(k_x, k_y)A(k_x, k_y) \quad (17) \]

where

\[ c = \Delta \lambda \frac{A}{R} \eta, \quad (18) \]

and

\[ \Delta \lambda = \text{optical bandwidth}, \]
\[ A = \text{area of optical aperture}, \]
\[ R = \text{target range}, \]
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When the scanner moves at constant speed, \( u \), in the \( x \)-direction, the incident power is changed from a two-dimensional space variable to a one-dimensional time variable by

\[ x = ut; \quad y = y_D, \quad \text{a constant.} \quad (19) \]

Then the temporal transform pair of the target power incident on the detector is

\[ p_T(u, y) \leftrightarrow p_T(k_x, k_y) \]

\[ = cJ_0(k_x, k_y)A(k_x, k_y) \quad (17) \]

where

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and

\[ \Delta \lambda = \text{optical bandwidth}, \]
\[ A = \text{area of optical aperture}, \]
\[ R = \text{target range}, \]
\[ \eta = \text{optical efficiency}. \]

The expression for \( \Delta A \) follows from the fact that the (inverse) transform of a product is equal to the convolution of the (inverse) transforms.

Finally, we have

\[ S(\omega) = cJ_0 \frac{1}{u} O_x(\omega)A_x(\omega) \Lambda(y_D)R(\omega) \quad (28) \]

Background Noise

The background is assumed to be exponentially correlated and isotropic; therefore, the spatial autocorrelation function-power spectral density (PSD) transform pair is given by

\[ \Delta N^2 = \exp \left( -\frac{1}{k_c^2} \left( \frac{\tau x}{\gamma x} + \frac{\tau y}{\gamma y} \right) \right) \leftrightarrow \mathcal{F} \left[ \mathcal{F}_T(\omega) \right] \quad (29) \]

where

\[ \Delta N^2 = \text{variance of background spectral radiance}, \]
\[ k_c = \text{correlation length}. \]

The PSD of the structured background noise at the detector is given by

\[ \mathcal{F}_B(\omega) = \mathcal{F}_B(k_x, k_y) = B(k_x, k_y) e^{-2(2\pi k_x \xi_c)^2 + (2\pi k_y \xi_c)^2} \quad (30) \]

Since the \( y \) position is constant during scanning, the \( y \)-direction correlation is evaluated at \( \tau_y = 0 \).

Therefore,

\[ \mathcal{F}_B(\tau_x) = \mathcal{F}_B^{-1}(\tau_x, y) \quad (31) \]

and the temporal transform in the scan direction is given by

\[ \mathcal{F}_{BS}(\omega) = \mathcal{F}_B(\omega) \]

where

\[ \mathcal{F}_B(\omega) = \mathcal{F}_B^{-1}(\tau_x) \quad (32) \]

and

\[ \mathcal{F}_B^{-1}(\tau_x) \quad (33) \]
where
\[ B'(\omega, k_y) = B(\omega, k_y) \]
and
\[ A_y(k_y) \propto \Delta v^2. \] (36)

Then
\[ \gamma h^{-1} y y' y y'(k_y) A(k_y) \gamma_y = 0 \]
\[ \propto \Delta v^2 E_x^2 \Delta N^2_k (1 + \frac{(\omega \tau)}{u_c})^2. \] (37)

The PSD of the detector output noise voltage due to the structured background is given by
\[ G_B(\omega) = \frac{P^2_B}{2\pi \sigma_B} |R(\omega)|^2 \]
\[ \propto c^2 |\sigma_x(\omega)|^2 |A_y(\omega)|^2 |B(\omega)|^2 \]
\[ \times \frac{2k_B}{D^2_u} \Delta N^2_k \sqrt{1 + (\omega \tau)^2}. \] (38)

Detector Noise

In HgCdTe and PbS detectors, four internal noise sources are modeled: general-recombination noise, excess noise, Johnson-Nyquist noise, and amplifier noise. The PSD of the generation-recombination noise is
\[ G_{GR}(\omega) = \frac{1}{2} a \left( \frac{D}{D_u} \right)^2 \frac{2}{1 + (\omega \tau)^2}, \] (39)
where
\[ a = \text{area of the detector}, \]
\[ D^u = \text{background limited detectivity}. \]

The detectivity is a function of the total photon flux from the background target. To be amenable to a linear, time-invariant analysis, this photon flux must be assumed constant. This is possible if the flux from the background fluctuations and the target are small compared with the mean background radiance.

The excess noise PSD is
\[ G_E(\omega) = 4\pi e^2 c^2 \tau / |\omega|, \] (40)

where
\[ E = \text{experimental parameter}, \]
\[ c = \text{bias electric field}, \]
\[ r = \text{aspect ratio of detector}. \]

It is assumed here that the electrodes are connected across the longer dimension of the detector, so that
\[ r = \max(\lambda x / \omega y, \omega y / \lambda x). \]

The PSD of the Johnson-Nyquist noise is
\[ G_{JN}(\omega) = 2k_B T R_D, \] (41)
where
\[ k_B = \text{Boltzmann's constant}, \]
\[ T = \text{operating temperature of the detector}, \]
\[ R_D = \text{detector resistance}. \]

The PSD of the amplifier noise is
\[ G_A(\omega) = \frac{1}{2} M_1, \] (42)
for HgCdTe, and
\[ G_A(\omega) = \frac{1}{2} M_1 + \pi M_2 / |\omega|, \] (43)
for PbS, where \( M_1 \) and \( M_2 \) are determined experimentally.

The PSD of the total internal detector noise is given by
\[ G_D(\omega) = G_{GR}(\omega) + G_E(\omega) + G_{JN}(\omega) + G_A(\omega), \] (44)
and for the structured background noise combined with the internal detector noise, the PSD is given by
\[ G(w) = G_B(\omega) + G_D(\omega). \] (45)

Focal Plane Optimization

Focal plane optimization refers to the process of determining the size, the shape, and layout of detector cells on the focal plane so as to optimize the system performance in some sense. As in the case of parameter estimation, this is actually a two-dimensional problem involving arrays of detector cells; see Figure 2. Because only a single detector cell channel is considered here, the optimization process will be discussed only in conceptual terms.

Figure 2. Detector Array and Target Image
In the optimization process $\sigma_f$ (or $\text{SNR}_p$) and $\sigma_p$ are among the quantities used to measure system performance. Note that $\text{SNR}_p$ and $\sigma_p$ are functions of the detector size, since both the signal spectrum, $S(u)$, and the noise power spectral density, $G(u)$, are functions of the detector size. The performance quantities as given by (2) and (3) are based on the assumption that an ideal matched filter (which is also a function of the detector size) is used in the signal processing. By using (2) and (3) in the focal plane optimization, it becomes unnecessary to consider the details of the signal processing. In effect, it is merely assumed that the processing is close to the ideal.

When the focal plane optimization has been completed and the detector cell size has been chosen, then the ideal matched filter is determined by (1). The task then remains of trying to implement this ideal filter with realizable components. This is done by fitting $F(u)$ given in (1) as closely as possible with polynomial functions.

Note that the condition of causality was not imposed upon the ideal filter in arriving at system performance quantities. The reason for this is that sufficient delay is acceptable in the signal processing so that the ideal filter can be made nearly causal. Mathematically this means that the filter spectrum to be fitted is actually described by

$$F'(u) = e^{-j\tau} F(u),$$  

where $\tau$ is an arbitrary delay. $F(u)$ and $F'(u)$ have the same performance, and $F'(u)$ can be made nearly causal by allowing $\tau$ to be sufficiently large.

In designing a filter to fit $F'(u)$, $\tau$ may be considered a free parameter which can be chosen to make $F'(u)$ easiest to fit with polynomial functions. This means that the phase angle of the ideal matched filter may be modified by an arbitrary linear term, $\omega \tau$. In most cases, particularly for narrow bandwidth filters, this arbitrary degree of freedom in the phase angle allows the filter designer to ignore the phase angle and concentrate only on fitting the amplitude response of the ideal filter. An amplitude fit only is usually capable of achieving a performance level (as measured by $\text{SNR}_p$ and $\sigma_p$) within 90% of the ideal matched filter. Some wider band cases have been encountered, however, where ignoring the phase angle will result in a performance level of 75-80% of the ideal. In these cases the phase angle must be included in the design process to achieve the 90% performance level.

**Design of Realizable Suboptimal Matched Filters**

For the infrared sensor system treated in this paper, the theoretical basis for the design of the ideal optimum matched filters, based on models of the target, the noise, and the infrared sensor, has been presented in the previous sections. Since the ideal matched filter is noncausal and hence unrealizable, the problem treated in this section is that of designing suboptimal and physically realizable filters which will approximate the ideal matched filter characteristics. The criteria used in evaluating the suboptimal filters are the signal to noise ratio and the centroid errors with respect to the theoretical performance limits of the ideal matched filters.

The acceptable performance level of the suboptimal matched filters as measured by $\text{SNR}_p$ and $\sigma_p$ is selected to be within 90% of the ideal matched filter. In the computer-aided design of the suboptimal matched filters, the prescribed optimal matched filter characteristics in the frequency domain are approximated by realizable filter functions of various degrees of complexity. Although the approximation of the magnitude or attenuation is emphasized, the phase and delay characteristics have also been considered. In some wide bandwidth cases, all-pass or delay equalizers must be used to meet the 90% performance level.

**Medium Wavelength Filter**

The optimized filter transfer functions are realized in the forms of active RC filters using monolithic linear integrated circuits. The active elements of these filters are monolithic integrated high-performance operational amplifiers. For IC realizations, the filter complexity becomes a factor of paramount importance and it is used as a design parameter in the trade-off studies of the various filter realizations. Ease of IC realizations and element value spread have also been considered.

For the HgCdTe detectors, the normalized magnitude characteristics of the ideal matched filter for the medium wavelength channel are shown in Table 1. Using computer-aided optimization techniques, a realizable filter transfer function is given by

$$T(s) = H T_1(s) T_2(s),$$

where $H$ is a constant scale factor, $T_1(s)$ is a second-order low-pass (LP) section and $T_2(s)$ is a first-order high-pass (HP) section. The optimized results are given by

$$H = 14.982,$$

$$T_1(s) = \frac{1}{s^2 + 4.003s + 24.051},$$

$$T_2(s) = \frac{s}{s + 1.305}.$$

**Table 1**

<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>IDEAL</th>
<th>[T(jw)]</th>
<th>[T(jw)]</th>
<th>$\phi(w)$</th>
<th>$\tau(w)$</th>
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<tbody>
<tr>
<td>1.3287 kHz</td>
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<td>0.469</td>
<td>0.542</td>
<td>0.573</td>
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<td>1.7756 kHz</td>
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<td>0.566</td>
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<td>0.493</td>
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<tr>
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<td>0.099</td>
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</tr>
<tr>
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<tr>
<td>3.1 kHz</td>
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<td>0.725</td>
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<td>0.770</td>
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<tr>
<td>3.9861 kHz</td>
<td>0.775</td>
<td>0.794</td>
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<td></td>
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<tr>
<td>4.4429 kHz</td>
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<td>0.785</td>
<td>-1.047</td>
<td>0.584</td>
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</tr>
<tr>
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<td>0.731</td>
<td>0.762</td>
<td>-1.293</td>
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<td>-1.525</td>
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<tr>
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<tr>
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<td>0.209</td>
<td>0.291</td>
<td>-2.300</td>
<td>0.156</td>
<td></td>
</tr>
</tbody>
</table>
The characteristics of the ideal matched filter are given by

$$T(s) = s^2 + 6.49s + 94.5 \quad \text{T}(j\omega) = 3.33 \omega + 1.136$$

However, the performance of this filter does not meet the 90% specification. The SNR and σ errors are given by

$$\text{SNR/SNR}_o = 0.98, \quad \sigma/\sigma_o = 1.08.$$ 

The implementation of this filter is shown in Figure 3.

**Short Wavelength Filter**

For the short wavelength channel, the magnitude characteristics of the ideal matched filter are shown in Table 2. Based on fitting the amplitude characteristics only, the optimized transfer function is given by

$$T(s) = \frac{1}{s^2 + 6.49s + 94.5} \quad \text{SNR/\text{SNR}}_o = 2.955, \quad \sigma/\sigma_o = 1.06.$$ 

The transfer function of the all-pass factor is given by

$$T_{AP}(s) = \frac{s^2 - (0.475)(9.74)s + (0.74)^2}{s^2 + (0.475)(9.74)s + (0.74)^2}.$$

The characteristics of the equalizer are given in Table 3, and the implementation is shown in Figure 5. The phase of the short wavelength filter, both with and without the equalizer, are shown in Figure 6, and the resulting delays are shown in Figure 7.

**TABLE 2**

| FREQUENCY | $|T(j\omega)|$ | $|\text{T}(j\omega)|$ | $\sigma(\omega)$ | $\tau(\omega)$ |
|-----------|---------------|---------------------|-----------------|---------------|
| 1 kHz     | 0.0306        | 0.0435              | 3.300           | 1.136         |
| 2 kHz     | 0.193         | 0.2061              | 1.388           | 0.799         |
| 3 kHz     | 0.465         | 0.4497              | 1.651           | 0.704         |
| 4 kHz     | 0.730         | 0.7266              | 0.948           | 0.704         |
| 5 kHz     | 0.914         | 0.9288              | 0.258           | 0.662         |
| 6 kHz     | 0.9925        | 0.9942              | -0.358          | 0.569         |
| 6.333 kHz | 1.000         | 0.9943              | -0.542          | 0.542         |
| 6.666 kHz | 0.996         | 0.9878              | -0.719          | 0.520         |
| 7 kHz     | 0.984         | 0.9764              | -0.890          | 0.504         |
| 8 kHz     | 0.910         | 0.9157              | -1.380          | 0.462         |
| 9 kHz     | 0.798         | 0.8119              | -1.854          | 0.463         |
| 10 kHz    | 0.664         | 0.6654              | -2.294          | 0.409         |
| 11 kHz    | 0.53          | 0.5120              | -2.664          | 0.329         |
| 12 kHz    | 0.4           | 0.3838              | -2.954          | 0.253         |
| 13 kHz    | 0.286         | 0.2882              | -3.177          | 0.194         |
| 14 kHz    | 0.188         | 0.2197              | -3.348          | 0.152         |

$T(s) = H T_1(s) T_2(s)$,

where

$$H = 400,$$

$$T_1(s) = \frac{1}{s^2 + 6.49s + 94.5} \quad \text{and} \quad T_2(s) = \frac{s^2}{s^2 + 6.49s + 25.8}.$$ 

The implementation of this filter is shown in Figure 4.

By adding an optimized second-order all-pass (AP) phase equalizer, the performance is improved to

$$\text{SNR/\text{SNR}}_o = 2.955, \quad \sigma/\sigma_o = 1.06.$$ 

The transfer function of the all-pass factor is given by

$$T_{AP}(s) = \frac{s^2 - (0.475)(9.74)s + (0.74)^2}{s^2 + (0.475)(9.74)s + (0.74)^2}.$$ 

The characteristics of the equalizer are given in Table 3, and the implementation is shown in Figure 5. The phase of the short wavelength filter, both with and without the equalizer, are shown in Figure 6; and the resulting delays are shown in Figure 7.

**TABLE 3**

<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>$\phi(\omega)$</th>
<th>TOTAL $\phi(\omega)$</th>
<th>$\tau(\omega)$</th>
<th>TOTAL $\tau(\omega)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kHz</td>
<td>3.300</td>
<td>3.292</td>
<td>1.136</td>
<td>1.236</td>
</tr>
<tr>
<td>2 kHz</td>
<td>2.388</td>
<td>2.186</td>
<td>0.799</td>
<td>0.908</td>
</tr>
<tr>
<td>3 kHz</td>
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**Acknowledgement**

The authors wish to acknowledge the very large contribution to this work of Dr. Richard V. Board of The Aerospace Corporation.

**References**


Figure 3. Filter for Medium Wavelength Channel

Figure 4. Filter for Short Wavelength Channel

Figure 5. Equalizer for Short Wavelength Channel
Figure 6. Phase Characteristics of Short Wavelength Filter

Figure 7. Delay of Short Wavelength Filter