ADAPTIVE IDENTIFICATION OF AND POLE PLACEMENT FOR STABLE PLANTS

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Abstract

The improved parameter identification and state observation technique of a recent paper by Nuyan and Carroll [1] is employed in this report to adaptively place the poles of an nth order single-output, linear time-invariant stable system having unknown parameters to arbitrarily specified locations in the open left half-plane. The proposed scheme uses only input and output measurements, has an arbitrarily fast exponential convergence and is totally stable.

1. Introduction

In applications of the automatic control theory, an engineer is frequently faced with the problem of designing a compensation device for a process for which very little knowledge may be available. Furthermore the behavior of most processes are slightly nonlinear and tend to depend upon their environment. If the engineer can perform tests, usually off-line, on the unknown process and determine a nominal linear model of it in a certain environment, the well-known linear feedback control theory can be applied for satisfactory compensation in that particular environment for which the acquired model adequately describes the process.

Although this is the typical approach in most applications, it has several disadvantages, and may not be justified, in situations where radical environmental changes occur and stringent requirements are placed on the system performance. Consequently a more sophisticated compensation device may be necessary. It would be desirable that this device be capable of compensating systems for which very little is known. It should also be capable of adapting to environmental changes as effectively as possible.

Recent developments in the adaptive observer theory appears to offer the desired features for a possible solution of this problem. The adaptive observer is presently applicable to linear time-invariant processes for which no knowledge may be available to the designer except an upper bound on process order. The technique not only generates asymptotic estimates of the plant states but it also identifies the parameters of the process. Therefore, the state estimation property combined with the identification of the process parameters may allow the design of a stable, on-line, adaptive controller where possibly only input and output measurements are required for the system control. This eliminates the complicated and costly off-line modeling which would be required for each environment the system may encounter. Such a system would adapt automatically to undesirable changes in the system attributes.

Despite the stipulation that the adaptive observer placed in a closed loop for system control can solve the adaptive feedback control problem in a unique way, neither a practical nor a theoretical solution has been formulated yet. There are several difficulties associated with using the adaptive observer in a closed loop control. The canonical form of the observer is simple for system control and the stability of the closed loop system is certainly one of the main concerns of the current research in this area. Another important question that is still to be resolved is that of the overall stability. Although the adaptive observer configuration is that of the Luenberger observer after adaption is essentially completed and comes to possess the important separation property of the non-adaptive Luenberger observer, the stability of the overall system during the adaptation remains yet to be shown for the general class of systems for which an adaptive observer can be constructed.

In this report, a discussion of canonical forms of the estimate states to be used in closed loop is presented. We also propose an adaptive control for a restricted class of systems. The proposed method applies to systems that are stable and remain stable for all environmental changes. For such systems it is shown that a stable adaptive control system can be designed to place the poles of the plant to arbitrary locations on the complex left-half-plane. The design philosophy is based on successful and rapid identification of the parameters [1] which together with some model states are used to perform a compensating control action on the plant. Unlike the model reference adaptive system employing dynamic compensation for system control [2] this method does not require that the plant have only left-half-plane zeros. The plant zeros can be anywhere on the complex plane and are not affected by the control algorithm proposed.

II. Problem Statement

Consider the single-input, single-output nth order linear time invariant process for which a
transfer function description is given as
\[ \frac{Y(s)}{U(s)} = \sum_{i=1}^{n} \frac{b_i}{s^i} \]
where \( y(t) \) is the single output and \( u(t) \) is the single input. All or some of the parameters \( a_i \) and \( b_i \) may be unknown.

It is desired to control (1) in the sense that the control system is able to have poles at arbitrarily chosen locations on the left half plane; i.e., the controlled system is to behave to an external command input \( R(s) \) as
\[ \frac{Y(s)}{R(s)} = \sum_{i=1}^{n} \frac{c_i}{s^i} \]
where \( c_i \) can be chosen by the designer to enable him to place the poles anywhere on the left half plane. Furthermore the controlled system is to adapt to the changes in the system parameters \( a \) in a manner as to retain the assigned pole locations during the entire operation.

The approach taken in this report to solve this problem is that of adaptive identification of process parameters followed by adaptive control. Therefore, a suitable adaptive identification method is discussed first to lay down the basis for the control strategy suggested.

III. Adaptive Identification of Process Parameters

The dynamics of the process given in (1) can alternatively be described as a set of \( n \) first order differential equations in the state variable form as
\[ \xi = A \xi + Bu, \quad \xi(0) = \xi_0 \]
where \( \xi \) is an \( n \)-th order state vector. The state equations are assumed to have the observable canonical form (output form) which is most suitable for the identification problem.

The significance of this form is that the unknown system parameters appear in the left-most column and multiply the observable output measurement \( y = c_1 \).

The design method of the parameter identification process should be stable. Preferably it should employ input – output measurements only and should not resort to signal differentiation. The parameter identification capability of the adaptive observers utilizing Lyapunov stability theory to guarantee global asymptotic stability and the state variable filter concept to facilitate a design without signal differentiation is a suitable approach in this context. In the control scheme presented in this report there is a need for rapid identification of the process parameters. However, the simple adaptive observers and identifiers discussed in [3,4], lack a fast rate of convergence for the parameters and will not be successful in a control philosophy where fast identification of the parameters is a prerequisite for system control. The convergence problem of the adaptive observers is discussed in depth in [1] and an improved adaptive observer and identifier with an arbitrarily fast rate of convergence is formulated in the same paper. In this report, the improved identification technique of [1] will be employed rather than those of the single error adaptive observers of [3] and [4] to identify the process parameters for constructing a compensating control input.

The key to obtaining rapid convergence seems to lay with using a sufficient number of linearly independent generalized equation errors to result in a positive definite criterion surface in the parameter space. For an arbitrarily fast exponential convergence, \( n \) linearly independent error equations \( e_i \) where \( e_i \) is the number of unknown parameters \( \theta \) of (1) are defined as
\[ e = \sum_{i=1}^{n} \frac{c_i}{s^i} \]
where \( e \) and \( c_i \) are \( v \) and \( 2n \) vectors, respectively, and
\[ W(t) = [w_1, w_2, ..., w_v] \]
\[ w_i = [v_{n-1+i}, v_{n-2+i}, ..., v_{n+i}], i = 1, 2, ..., v \]
\[ \rho = [\alpha_1, \beta_1, ..., \alpha_v, \beta_v] \]
In (4) \( \rho \) is the vector of adjustable parameters and is combined with the filtered output and input variables \( v_i \) and \( q_i \) which will be defined shortly in such a way that \( e = 0 \) iff \( \rho = \rho^* = \begin{bmatrix} \alpha^* & \beta^* \end{bmatrix}^T \), i.e. after the adaptation is completed the parameters \( a_i \) and \( b_i \) in (1) can be identified from \( a_i \) and \( -b_i \), respectively. To this end the filtered output and input variables \( v_i \) and \( q_i \) in (4) will be defined in terms of \( m \)th \( (m = n + v) \) order state equations and first order output equations as
\[ v = \lambda v + c_i v, \quad v_{m+1} = -\lambda v + y \]
\[ q = \lambda q + c_i q, \quad q_{m+1} = -\lambda q + u \]
where
\[ \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ 1 \end{bmatrix} \lambda \]
The vector \( \lambda \) must be chosen so that \( \Lambda \) is stable.
From the previously mentioned definitions it can easily be checked that $c = 0$ if and only if

$$Y(s) = \sum_{i=1}^{n} b_i s^{n-i}$$

$$U(s) = s^n + \sum_{i=1}^{n} a_i s^{n-i}$$

Comparing (6) with (1), we conclude that if an adaptive law to change $p$ can be formulated as to make $E = 0$, we can identify the parameters $a_i$ and $b_i$ from $a_i s^{n-i}$ and $b_i s^{n-i}$, respectively.

The stability of (7) for any bounded input $u$ is proved in [1] via an appropriate Lyapunov function and asymptotic stability is shown for a class of inputs characterized by a richness property of their frequency content. Moreover, the theorem established in [1] establishes that the exponential bounds for $\tilde{p}$ can be made arbitrarily small by a proper choice of $G$. Therefore, with this method, the identification process can be made as fast as a satisfactory control scheme would require.

IV. Discussion of the Canonical Forms for the State Estimate

Nuyan and Carroll [1] have shown that the dynamics of the above identification scheme is sufficient to algebraically construct estimates for the plant states as defined in (2) without requiring further integrators. In fact, they have found the conditions and have developed the equations for algebraically constructing state variables $\tilde{x}$ in the form of the Luenberger observer.

$$\dot{\tilde{x}} = F\tilde{x} + g(a,b)Y + d(a,b)u$$

This is a more general state reconstruction scheme and provides the designer with considerable freedom in the choice of the canonical forms, the generation of the controllable state variables would be generated instead of the output or observable state variables as has been customary in the adaptive observer literature. Even though the state reconstruction approach described above provides some freedom in the choice of the canonical forms, the generation of the controllable state variables does not seem to be possible within the framework of this formulation. To see this let us take a second order system described in controllable form state variables as

$$\dot{x} = Mz + Mz_2$$

where $z_1$ and $z_2$ are obtained via the output equations $z_1 = r_1$ and $z_2 = r_2$. The variables $z_1$ and $z_2$ are shown that they actually satisfy the $n$th order dynamical equations of the form

$$\dot{z}_1 = Kz_1 + Ly$$

$$\dot{z}_2 = Kz_2 + Lu$$

if the characteristic polynomial $\lambda(s)$ of (5) contains the characteristic polynomial $K(s)$ of (10) as a factor and $\tilde{r}$ is chosen appropriately. $M_1$ and $M_2$ are found from $x_1, x_2, L, F, g$ and $d$.

The problem encountered in using an adaptive observer in feedback control would considerably be reduced if estimates for the controllable canonical state variables would be generated instead of the output or observable state variables as has been customary in the adaptive observer literature. Even though the state reconstruction approach described above provides some freedom in the choice of the canonical forms, the generation of the controllable state variables does not seem to be possible within the framework of this formulation. To see this let us take a second order system described in controllable form state variables as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Adding and subtracting $Fx$ to the right hand side of (11a), one obtains

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + f_{12} u$$

It is now desired to find unique vectors $g(a,b)$ and $d(a,b)$ in (8) such that (8) can estimate (11) correctly when the parameters have converged to make $c = 0$, i.e., $\tilde{x}$ when

$$g(a,b) = g(a,-b) = g^*$$
$$d(a,b) = d(a,-b) = d^*$$

Thus, the vectors $g^*$ and $d^*$ should satisfy

$$\begin{bmatrix} f_{11} & f_{12} \\ -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u = \begin{bmatrix} g^* \\ d^* \end{bmatrix}$$

If they exist. Inserting (11b) for $y$ and replacing $x_1$ and $x_2$ by their transfer functions one obtains

$$\begin{bmatrix} f_{11} & f_{12} \\ -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u = \begin{bmatrix} g^* \\ d^* \end{bmatrix}$$

Unfortunately this set of equations does not allow a unique solution for $g^*$ and $d^*$. It is seen from the first equation that $d^* = 0$. This, however, results in contradicting values for $g^*$. A similar argument holds for the second equation.
where $d^* = 1$ results in nonunique solutions for $g^*$. Thus $g(a, b)$ and $d(a, b)$ do not exist to allow the construction of the controllable state estimates.

As the controllable state estimates satisfying (8) is not easily constructable in the framework of the state reconstruction technique formulated in [1], it becomes difficult to deal with the pole placement problem. A more sophisticated state reconstruction technique is strongly desired to facilitate the estimation of the controllable form of state variables.

**V. Control Strategy**

The controllable state variable description of the system in (1) can be obtained from (2) by a similarity transformation $x = P_0 x$ as

$$
\dot{x} = A_c x + b_c u \\
y = c^T x
$$

where the subscript c stands for controllable and

$$
A_c = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \\
0 & 0 & 0 & \cdots & 1
\end{bmatrix},
$$

$$
c = \begin{bmatrix} b_n \\
b_{n-1} \\
\vdots \\
b_2 \\
b_1
\end{bmatrix}
$$

(14)

As the adaptive scheme described in Section III identifies $a_i$ and $b_i$ as $a_i^*$ and $b_i^*$, a possible approach to estimate the controllable state variables in (13) would be through the equation

$$
\dot{x} = \hat{A} \dot{x} + b_c u \quad x(0) = x_0
$$

$$
y = c^T \dot{x}
$$

(15a)

(15b)

where $\hat{A}$ and $c$ are of the same form as in (14) with $a_i^*$ and $b_i^*$ replaced by $a_i$ and $b_i$ respectively.

Although serious questions about the stability of (15) and the effect of initial conditions are difficult problems to deal with, they need not be answered for the time varying equations (15) in the particular control strategy suggested in this report, as Eq. (15) will not be implemented in an open loop. In a way we get around these problems by obtaining a time invariant system in the closed loop. However, the resulting scheme has a serious restriction: It can only be applied to stable plants. This restriction has to be put on the plant because $\dot{x}$ in (15) will not estimate $x$ asymptotically, i.e. when the adaptation is completed, if the plant is unstable.

To develop the equations for the overall system, suppose the control input $u$ is chosen as

$$
u = k(t) T x + r
$$

where $r$ is the command input and $k(t) = [k_n, k_{n-1}, \ldots, k_1, k_0]$ is chosen such that the poles of the controlled system are placed to the desired locations given by $s^* + s_1, s^* + s_2, \ldots, s^* + s_n = 0$.

Defining $\vec{e}^T = [\vec{a}_n, \vec{a}_{n-1}, \ldots, \vec{a}_1]$ and $\vec{a}(t) = [a_n, a_{n-1}, \ldots, a_1]$, let us choose

$$
k(t) = \vec{a}(t) - \vec{e}
$$

(17)

with the choice of $u$ and $k$ as in (16) and (17)

Equation (15a) becomes

$$
\dot{x} = \hat{A} \dot{x} + b_c \hat{r}
$$

$$
\dot{x} = \hat{A} \dot{x} + b_c (\dot{x} - x)
$$

(18)

where

$$
A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \\
0 & 0 & 0 & \cdots & 1
\end{bmatrix},
$$

$$
c = \begin{bmatrix} b_n \\
b_{n-1} \\
\vdots \\
b_2 \\
b_1
\end{bmatrix}
$$

(14)

Hence the closed loop estimate system has been converted to a linear time invariant system. Eq. (18) is asymptotically stable on account of the asymptotic stability of $A$ and can be implemented to generate $u = k \dot{x} + r$ as an output equation to control (13). The overall system structure is shown in Fig. 1.

The basic problem in putting the adaptive observer in the closed loop is the stability of the overall system. The resulting equations are nonlinear nonhomogeneous differential equations for which it is difficult to obtain a stability proof. The difficulty arises from the fact that one cannot treat the parameter identification and state observation process independently from the control process if the state estimates $x$ are obtained in the form of (8). In the control strategy of Fig. 1, the estimates are obtained from a model which has asymptotically stable system matrix $A$ and the entire model is totally stable. It is further known that the parameters $a_i$ and $b_i$ are bounded if the plant is stable. Therefore, the entire scheme will be stable if the controlled plant is stable.

It is evident that $\lim x(t) = x(t)$ or equivalently $\lim x(t) = 0$ after the parameters have converged if the plant poles are in the open left half plane. Setting $e = x - x$ and subtracting (13a) from (18)

$$
\dot{e} = A_c \hat{e} + (A_c + b_c k \hat{r}) x = A_c \hat{e} + b_c (\hat{a} - a) x
$$

(19)

where

$$
A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \\
0 & 0 & 0 & \cdots & 1
\end{bmatrix},
$$

$$
c = \begin{bmatrix} b_n \\
b_{n-1} \\
\vdots \\
b_2 \\
b_1
\end{bmatrix}
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(19)

where

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A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \\
0 & 0 & 0 & \cdots & 1
\end{bmatrix},
$$

$$
c = \begin{bmatrix} b_n \\
b_{n-1} \\
\vdots \\
b_2 \\
b_1
\end{bmatrix}
$$

(14)

VI. Discussion

The success of the pole placement method of Fig. 1 is dependent on the success of the identification of the process parameters. Indeed the control output $u(t)$ becomes effective after the parameters are in the neighborhood of the matching point. As an alternative to this approach, one might design constant feedback on the system if an off-line identification can be carried out.
mental changes. For these changes, the on-line
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are under way and will be the topic of a forth-
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The restriction of the proposed method to
stable plants and environments is the main dis-
advantage. It seems that in order to be able to
relax this condition to include unstable systems,
the model parameters are those of the plant after adaptation. The response to
the initial conditions and the difference between
model and plant states during adaptation do
not die out for unstable plants. This makes even
an asymptotic state estimation impossible.

Unless a more sophisticated method to recon-
struct the states as pointed out in Section IV is
found which can account for these aspects, the
control of unstable plants do not appear to be
feasible. Further investigation into this area
are under way and will be the topic of a forth-
coming report.

VII. Computer Simulation Results

Using the proposed scheme for placing the
poles of an unknown system to arbitrary locations
on the left half plane, computer simulations were
carried out to control the second order system
specified by its transfer function

\[ Y(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2} = \frac{s + 2}{s^2 + 5s + 10} \]  \[ (20) \]

It is desired that the control process has a trans-
fer function from the command input \( R(s) \) to the
controlled output \( Y(s) \)

\[ Y(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2} = \frac{s + 2}{s^2 + 6s + 5} \]  \[ (21) \]

The command input is  \( r(t) = 5 \sin \frac{2\pi t}{48} \) and it was assumed that
all parameters of (2) except \( b_2 = 2 \) are unknown.
For the fast adaptive identification of the three
unknown parameters \( a_1, a_2, b_1 \), the method of
Section III was used with \( n = 5, \nu = 4 \) and \( A(s) = s^4 + 12s^3 + 57s^2 + 126s + 104 \).
The state variable filters had an input gain of 10 and the value of
adaptive gains were chosen as \( \alpha = 10^{-2} \). The
initial estimates of the parameters \( a_1, a_2, b_1 \)
were

\[ a_1 = 1, \quad a_2 = 5, \quad b_1 = 1 \]

\[ b_2 = 7, \quad \alpha_2 = 10, \quad b_3 = -2, \quad \alpha_3 = 1 \]

\[ e_1 = x_1 - x, \quad e_2 = e_1 - y_1 \]

Fig. 3 also shows the output error \( e_1 = y - y_1 \). It is seen that the
parameters converge rapidly and all of the errors diminish in less than 4 sec.

To demonstrate the usefulness of the control
algorithm for stable environments, the parameter
\( a_1 \) is made subject to step and sinusoidal dis-
turbances after 6 seconds of operation. Letting
\( u(t-t_1) \) denote a unit step function applied
at time \( t_1 \), two simulation runs were carried out
with

\[ (i) \quad a_2 = a_{20} + A \sin \omega t \]

\[ (ii) \quad a_2 = a_{20} + A \sin \omega t u(t-t_6) \]

For comparison purposes with the performance of a
constant feedback system, the process is simu-
lated separately with

\[ u(t-t_1) = (a_{20} - \delta_2) x_1 + (a_{20} - \delta_2) d \]

Designating the states and output of the constant
feedback system by \( x_C \) and \( y_C \), the errors \( e_C = x - x_C \)
and \( e_Y = y - y_C \) are compared with \( e \) and \( e_Y \).

The results for a step disturbance with \( A_2 = 12.25 \)
and \( \omega = \frac{\pi}{48} \) in Fig. 9-11. Figures 4 and 9 show the dis-
turbed parameter \( a_2 \) and its estimate \( a_2 \).
Figs. 4 and 9 show the parameter estimates vs. time.
Fig. 5 shows the parameter estimates vs. time.
Fig. 6 and 9 also show the desired and actual controllable states; i.e.,

\[ u_c(t) = (a_{20} - \delta_2) x_1 + (a_{20} - \delta_2) d x_2 + r \]

VIII. References

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Figure 1. Adaptive control of stable plants. (Pole placement).

Figure 3. The errors between the desired and actual controllable state variables and the output error.

Figure 4. A step disturbance in $a_2$ and the response $a_2'$.

Figure 5. The response of $a_1$ and $\bar{a}_1$ to a step disturbance in $a_2$.

Figure 6. $e_0$ and $e_{oc}$ vs. time for a step disturbance in $a_2$.

Figure 7. $e_1$ and $e_{1c}$ vs. time for a step disturbance in $a_2$.

Figure 8. $e_2$ and $e_{2c}$ vs. time for a step disturbance in $a_2$.

Figure 9. A sinusoidal disturbance in $a_2$ and the response $a_2$.

Figure 10. The response of $a_1$ and $\bar{a}_1$ to a sinusoidal disturbance.

Figure 11. $e_0$ and $e_{oc}$ vs. time for a sinusoidal disturbance in $a_2$. 

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